University of Massachusetts Dartmouth
Department of Electrical and Computer Engineering

ECE 321 Continuous-time Linear Systems

Spring 2022

MATLAB **Project 2**

Amplitude Modulation

Issued: Sunday, 20 March 2022 **Due:** Friday, 8 April 2022

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1 Ground Rules for Matlab Projects

This is a group project. Each group will hand in only one report, signed by each member of the group to certify their participation and understanding. Only members of the group who actively participated in the work should sign the report. All members of the group will receive the same grade for the report based on the level of understanding demonstrated by the written work. You are welcome to divide the work up within the group, but all group members must contribute to the project.

The report must include three sections.

- The main body which includes text and plots answering the question for each part.
- An appendix that includes the complete Matlab code used to do the project.
- A breakdown indicating clearly what each group member contributed to the project, i.e., which parts they wrote Matlab code for or derived equations, which plots they made, and which parts of the report they wrote.

The main body of your report should be primarily text answering the questions given, with the figures integrated into the body of the report. The report should include the most important Matlab commands in the body of the report as you discuss your work, but you should not include every command to create and label plots in the main body. Your report should include an appendix that is all of the Matlab commands in your script for the project. For the parts that require analytic solutions, you can solve them **neatly** with paper and pencil, and include a scan of the handwritten equations or figures in your report. You do not need to spend time typing equations if you prefer to work by hand.

Your report should include an appendix that is a complete listing of all Matlab programs, scripts or functions that you wrote. Also, all graphs in the body of the report should have all axes labeled and a title clearly indicating what the graph shows and the part of the project. I will not spend time trying to figure out which graphs are for which problems. Any graph not labeled will be considered not handed in.

Each group will consist of two or three students. You may choose your own group members if you wish by signing up on MyCourses under the "Groups for Matlab Project" tab. Any students who have not chosen a group by Friday 11 February will be assigned to a random group.

As discussed in the syllabus, non-participating group members may be fired from the group after one warning. Students fired from a group must either convince another group to hire them, or complete the entire project on their own.

Remember: You may not share programs, plots, or figures among groups. You can discuss the project with other students in general terms about your approach or strategy for solving the problem.

Reports must submitted to MyCourses by 11:59 PM on the due date.

Introduction

In this project, you will use the modulation and filtering properties of the CT Fourier Transform to decode a message in Morse Code. A simple amplitude modulation system can be described by

$$x(t) = m(t)\cos(2\pi f_0 t), \tag{1}$$

where $m(t)$ is called the message waveform and f_0 is the modulation frequency. The continuous-time Fourier transform (CTFT) of a cosine of frequency f_0 is

$$C(j\omega) = \pi\delta(\omega - 2\pi f_0) + \pi\delta(\omega + 2\pi f_0), \tag{2}$$

which can confirmed by substituting $C(j\omega)$ into the CTFT Synthesis Equation to yield

$$\cos(2\pi f_0 t) = \frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right). \tag{3}$$

Using $C(j\omega)$ and the multiplication property of the CTFT, you can obtain the CTFT of $x(t)$,

$$X(j\omega) = \frac{1}{2}M\big(j(\omega - 2\pi f_0)\big) + \frac{1}{2}M\big(j(\omega + 2\pi f_0)\big), \tag{4}$$

where $M(j\omega)$ is the CTFT of $m(t)$. Since the CTFT of a sinusoid can be expressed in terms of impulses in the frequency domain, multiplying the signal $m(t)$ by a cosine places copies of $M(j\omega)$ at the modulation frequency.

The remainder of this exercise will involve the signal,

$$x(t) = m_1(t)\cos(2\pi f_1 t) + m_2(t)\sin(2\pi f_2 t) + m_3(t)\sin(2\pi f_1 t), \tag{5}$$

and several parameters that can be loaded into MATLAB from the file `ctftmod.mat`. This file is in the Computer Explorations Toolbox, which can be obtained from The MathWorks at the address listed in the Preface. If the file is in one of the directories in your MATLABPATH, type `load ctftmod.mat` to load the required data. The directories contained in your MATLABPATH can be listed by typing `path`. If the file has been successfully loaded, then typing `who` should produce the following result:

```
>> who

Your variables are:

af        dash      f1        t
bf        dot       f2        x
```

In addition to the signal $x(t)$, you also have loaded:

- a lowpass filter, whose frequency response can be plotted by `freqs(bf,af)`,
- modulation frequencies `f1` and `f2`,
- two prototype signals `dot` and `dash`,
- a sequence of time samples `t`.

To make this exercise interesting, the signal $x(t)$ contains a simple message. When loading the file, you should have noticed that you have been transformed into Agent $0.007 + j$, the (mostly) imaginary code-breaking sleuth. The dying words of your predecessor, the aged Agent $0.007 - j$[1] were "The future of technology lies in. . . " at which point Agent $0.007 - j$ produced a thumb drive and keeled over. The floppy disk contained the MATLAB file `ctftmod.mat`. Your job is to decipher the message encoded in $x(t)$ and complete Agent $0.007 - j$'s prediction.

Here is what is known. The signal $x(t)$ is of the form of Eq. (5), where f_1 and f_2 are given by the variables `f1` and `f2`, respectively. It is also known that each of the signals $m_1(t)$, $m_2(t)$ and $m_3(t)$ correspond to a single letter of the alphabet which has been encoded using International Morse Code, as shown in the following table:

[1] Note that $0.007 - j$ was in a negative phase of their career, while you are in the positive phase.

A	· -	H	····	O	- - -	V	··· -
B	- ···	I	··	P	· - - ·	W	· - -
C	- · - ·	J	· - - -	Q	- - · -	X	- ·· -
D	- ··	K	- · -	R	· - ·	Y	- · - -
E	·	L	· - ··	S	···	Z	- - ··
F	·· - ·	M	- -	T	-		
G	- - ·	N	- ·	U	·· -		

Basic Problems

(a). Using the signals `dot` and `dash`, construct the signal that corresponds to the letter 'Z' in Morse code, and plot it against `t`. As an example, the letter C is constructed by typing `c = [dash dot dash dot]`. Store your signal $z(t)$ in the vector `z`.

(b). Plot the frequency response of the filter using `freqs(bf,af)`.

(c). The signals `dot` and `dash` are each composed of low frequency components such that their Fourier transforms lie roughly within the passband of the lowpass filter. Demonstrate this by filtering each of the two signals using

```
>> lpf = tf(bf,af);
>> ydash=lsim(lpf,dash,t(1:length(dash)));
>> ydot=lsim(lpf,dot,t(1:length(dot)));
```

Plot the outputs `ydash` and `ydot` along with the original signals `dash` and `dot`.

(d). When the signal `dash` is modulated by $\cos(2\pi f_1 t)$, most of the energy in the Fourier transform will move outside the passband of the filter. Create the signal $y(t)$ by executing `y=dash.*cos(2*pi*f1*t(1:length(dash)))`. Plot the signal $y(t)$. Also plot the output `yo=lsim(lpf,y,t)`. Do you get a result that you would have expected?

Intermediate Problems

(e). Determine analytically the Fourier transform of each of the signals

$$m(t)\cos(2\pi f_1 t)\cos(2\pi f_1 t),$$

$$m(t)\cos(2\pi f_1 t)\sin(2\pi f_1 t),$$

and

$$m(t)\cos(2\pi f_1 t)\cos(2\pi f_2 t),$$

in terms of $M(j\omega)$, the Fourier transform of $m(t)$.

(f). Using your results from Part (e) and by examining the frequency response of the filter as plotted in Part (b), devise a plan for extracting the signal $m_1(t)$ from $x(t)$. Plot the signal $m_1(t)$ and determine which letter is represented in Morse code by the signal.

(g). Repeat Part (f) for the signals $m_2(t)$ and $m_3(t)$. Agent $0.007 + j$, where does the future of technology lie?

This project is based on a project in the *Computer Explorations in Signals and Systems, Second Edition* book by Buck, Daniel and Singer.