Faculty of Computer Science

Data Science & Business Analytics

Moscow 2023

Individual software project on topic: Option Pricing System (Система Прайсинга Опционов)

Created by: Ilin Artem Alexeevich DSBA212
Supervisor: Lukyanchenko Pyotr Pavlovich, HSE, Moscow Exchange

Description and Actualization

Option contact is the derivative financial instrument which gives a right but not obligation to buy/sell underlying asset from/to the issues. This instrument opens a whole class of model and techniques in quantitative finance.

In November 2022 on the Moscow Stock Exchange individuals received a new derivative to operate – standard option. It raised the interest on the Russian financial market and affected its volatility. As part of participation in such historical event the project is dedicated to understanding the nature of options trading, volatility, and solution of delta hedge problem under different market conditions and modes of operation.

Objective and Goals

The objective of this project is to describe how the pricing, valuation, and risk management of vanilla stock options may be performed, as well as to present corresponding software.

Goals:

- Creating data parser of option chains and stocks
- Examine pricing models
- Understand stochastic integral basics and Ito’s calculus
- Perform research on risk-management models
- Run analyses dedicated to hedging and implement such processes
- Implement approaches in Python programming language
- Access Russian financial market and its volatility
- Carrying out numerical experiments and predictions

Analysis of Solutions and Functional Requirements

1. Being able to perform evaluation of price via covered models.
2. Ability to model volatility via several techniques.
3. Ability to use the Heston model.
4. Ability to use the SABR model.
5. Ability to use the Crank–Nicolson model.
6. Modeling volatility as well as its smile, surface.
7. Ability to calibrate models.
8. Ability to plot corresponding graphs of modeled volatility, Greeks, and several other parameters.
9. Ability to parse data from Yahoo Finance.
10. Ability to parse data from Moscow Stock Exchange.
11. Ability to perform delta-hedging.
12. Ability to carry out numerical experiments and predictions.

Analogues: Bloomberg Terminal, Refinitiv Eikon

Data Mining

1. Yahoo Finance
2. Theta Data
3. Moscow Exchange

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMoney	contractSize	currency	optionType	expDate	daysToExp
0	JPM230203C00100000	2023-01-03 19:38:02+00:00	100.0	34.05	38.80	39.00	0.0	0.0	100.0	0	2.718753	True	REGULAR	USD	call	2023-02-03 23:59:59	1
1	JPM230203C00105000	2023-01-03 18:15:30+00:00	105.0	29.37	33.60	34.25	0.0	0.0	1.0	0	3.054690	True	REGULAR	USD	call	2023-02-03 23:59:59	1
2	JPM230203C00110000	2023-02-02 16:27:29+00:00	110.0	29.74	0.00	0.00	0.0	0.0	10.0	0	0.000010	True	REGULAR	USD	call	2023-02-03 23:59:59	1
3	JPM230203C00115000	2022-12-30 17:53:27+00:00	115.0	19.00	25.15	25.60	0.0	0.0	6.0	0	3.248049	True	REGULAR	USD	call	2023-02-03 23:59:59	1
4	JPM230203C00117000	2023-01-27 16:08:18+00:00	117.0	22.90	0.00	0.00	0.0	0.0	7.0	0	0.000010	True	REGULAR	USD	call	2023-02-03 23:59:59	1

```
prices.head(5)
```

	DataType.OPEN	DataType.HIGH	DataType.LOW	DataType.CLOSE	DataType.VOLUME	DataType.COUNT	DataType.DATE
0	26.12	26.12	26.12	26.12	1	1	2021-09-13
1	27.86	27.89	25.20	25.20	48	7	2021-09-14
2	26.00	26.00	26.00	26.00	1	1	2021-09-16
3	22.60	22.65	22.60	22.65	2	2	2021-09-20
4	23.21	23.38	23.21	23.38	12	3	2021-09-21

Black-Scholes-Merton and Greeks

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r\frac{\partial V}{\partial S} - rV = 0$$

$$C = N(d_1)S - N(d_2)Ke^{-rt}$$

$$P = N(-d_2)Ke^{-rt} - N(-d_1)S$$

$$where:$$

$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Figure 3.1: $\Delta,\ \Gamma,\ \rho$

```
for greek in ['delta', 'gamma', 'rho', 'theta', 'vega']:
    OptionLib.greek_plot(df_prices, greek_name=greek, type='both')
```

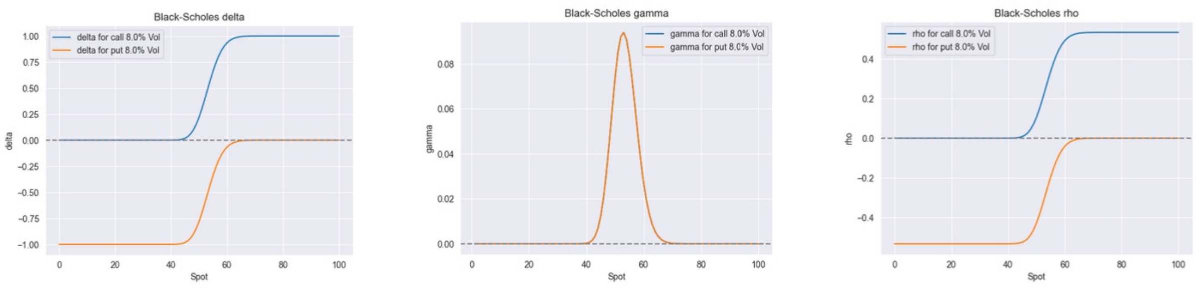
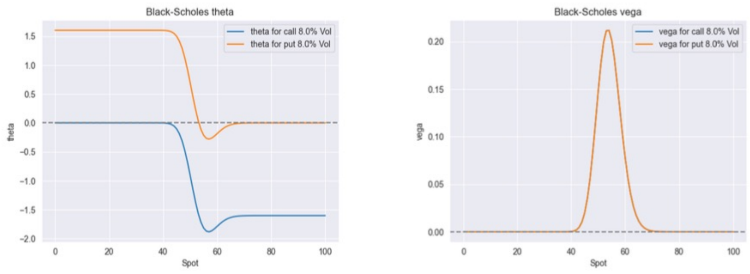


Figure 3.3: $\Theta,\ Vega$



Volatility

1. Newton-Raphson method
2. Bisection method
3. Skew
4. Term Structure
5. Surface

Figure 4.1: IV skew and term structure

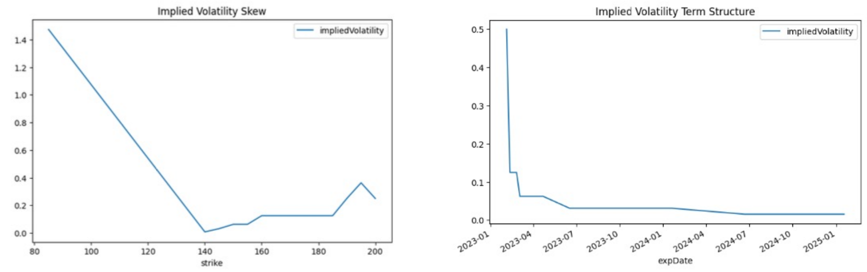
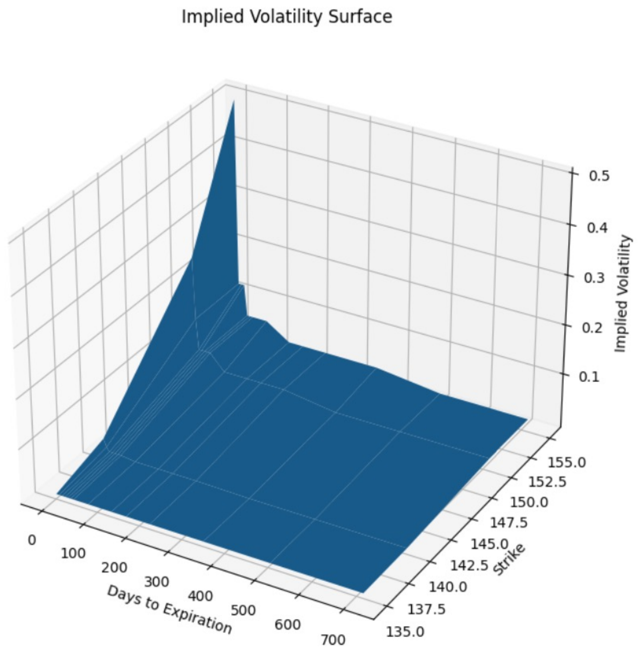


Figure 4.2: Volatility surface



Heston Model

$$dS = \mu S_t dt + \sqrt{v(t)} S_t dW_t^{(1)}$$

$$dv(t) = \kappa[\theta - v(t)]dt + \sigma\sqrt{v(t)}dW_t^{(2)}$$

$$dW_t^{(1)} dW_t^{(2)} = \rho dt$$

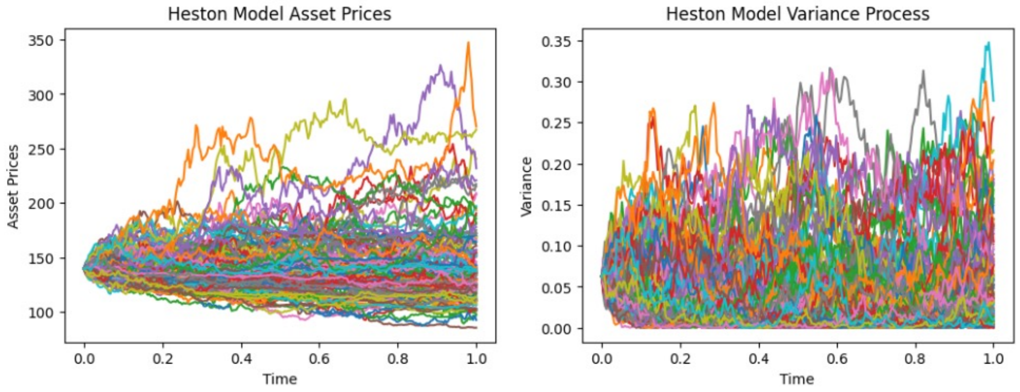
$$2\kappa\theta > \sigma^2 - Feller\ condition$$

1. Calibration
2. Monte Carlo Simulation

Table 5.1: Validation of coded Heston pricing model

	Call		Put		
	BSM	Heston	BSM	Heston	difference
Exp 1	6.80496	6.80496	1.92790	1.92790	0
Exp 2	17.43902	17.43902	15.29970	15.29970	0
Exp 3	8.3751	8.3751	2.30958	2.30958	0

Figure 5.1: Heston simulations




```
Spots.mean(), variances.mean()
```

```
(137.88027977364598, 0.04288228948616884)
```

SABR Model

$$dF_t = \alpha_t F^{\beta} dW_t^{(1)}$$

$$d\alpha_t = \nu \alpha_t dW_t^{(2)}$$

$$dW_t^{(1)} W_t^{(2)} = \rho$$

Figure 6.1: TSLA IV smile

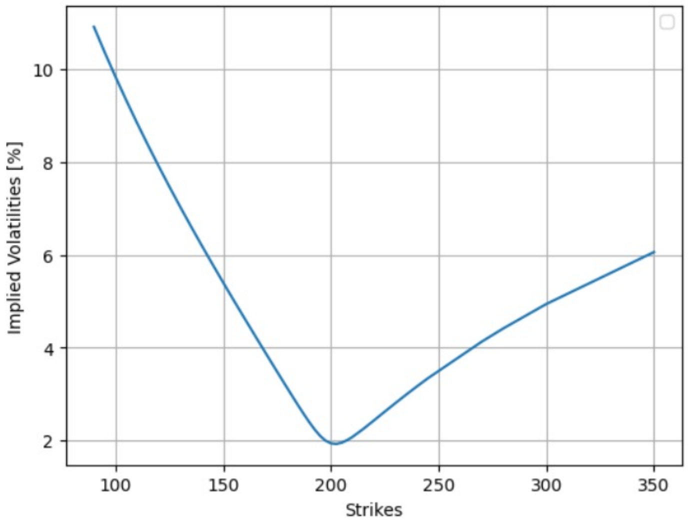


Figure 6.2: α and β experiments

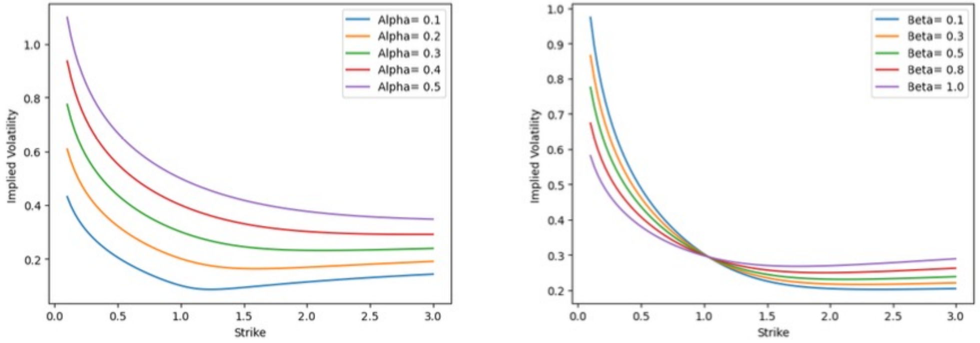
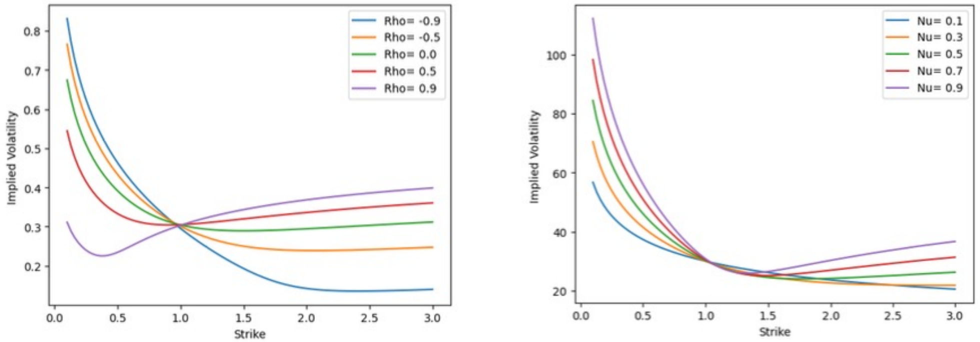


Figure 6.3: ρ and ν experiments



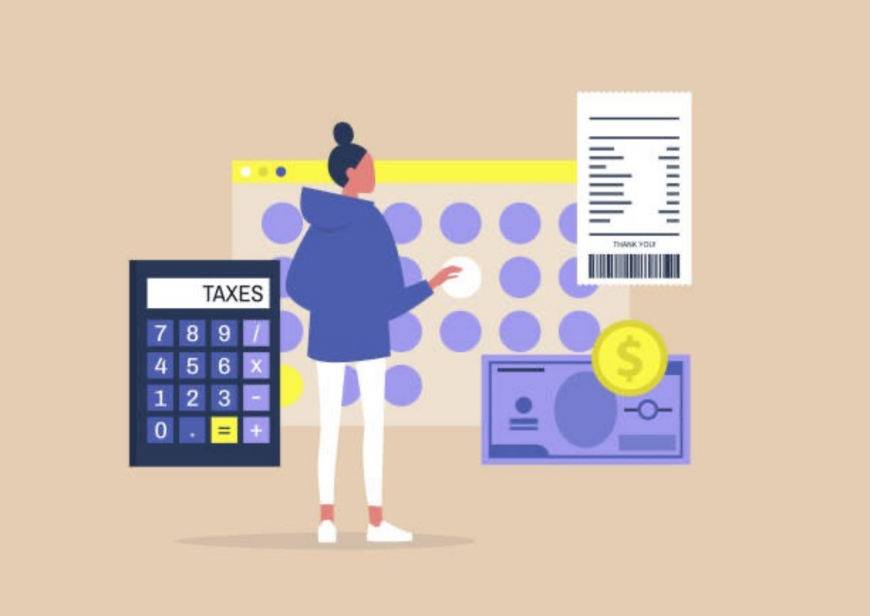
Crank–Nicolson

BSM price : 4.0759809847877 83 ;

CN price 4.072254507998117

Such result exhaustively explains Duffy in "Finite difference methods" by criticizing Crank–Nicolson method for the following reasons:

1. "The Crank-Nicolson method is second-order accurate on uniform meshes only."
2. "It gives terrible results near the strike price for approximations to the first and second derivatives in the space direction."



Delta Hedge

Delta hedge quantity: $DHQ = |\Delta| * number\_of\_option\_contracts * 100$

BSM delta

$$\Delta_{C,P} = \frac{dV}{dS} = \pm N(\pm d_1)$$

Heston delta

$$\frac{dC}{dS_0} = e^{-q(T-t)}P_1 > 0$$
$$\Delta_{C,t} = \frac{C_t}{S_t} = e^{-q(T-t)}P_1$$

SABR delta

$$skew = \beta * \alpha * F^{1-\beta} + O(\epsilon)$$
$$\Delta_{SABR} = \Delta_{BSM} + Vega_{BSM} * skew$$

Figure 8.1: Delta hedge

	underlying_price	option_price	delta	delta_pos	DH_quantity	underlying_needed	underlying_pos
0	120.000000	25.000000	1.000000	800.000000	800.000000	6.666667	-1.0
1	110.080640	24.780115	0.747941	598.352480	598.352480	5.435583	-1.0
2	116.122338	27.083789	0.829282	663.425445	663.425445	5.713160	-1.0
3	113.456048	27.919812	0.769023	615.218568	615.218568	5.422528	-1.0
4	110.983356	28.541619	0.736963	589.570399	589.570399	5.312242	-1.0
...	...	...	...	...	...	...	...
116	61.418184	24.251426	0.642706	514.164786	514.164786	8.371540	-1.0
117	61.399870	23.375081	0.632300	505.840114	505.840114	8.238456	-1.0
118	62.095566	23.821407	0.635551	508.441182	508.441182	8.188043	-1.0
119	61.225318	24.921022	0.651187	520.949711	520.949711	8.508730	-1.0
120	62.351163	25.261175	0.651590	521.271995	521.271995	8.360261	-1.0

Structural breaks

Ruptures:

1. Dynamic Programming
2. Pelt (Pruned Exact Linear Time)
3. Binary Segmentation
4. Bottom-up Segmentation

Jenkspy

Chow Test

Prophet

Figure 9.1: Dynamic Programming & Pelt change points

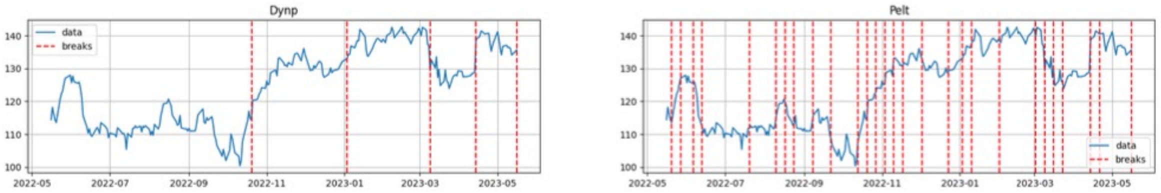


Figure 9.2: Binary Segmentation & Bottom-up Segmentation change points

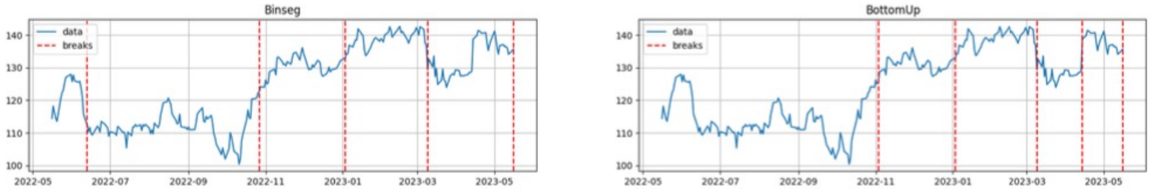


Figure 9.3: Jenkspy change points

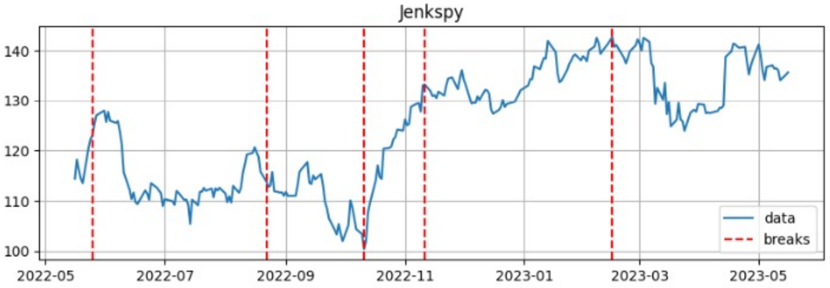
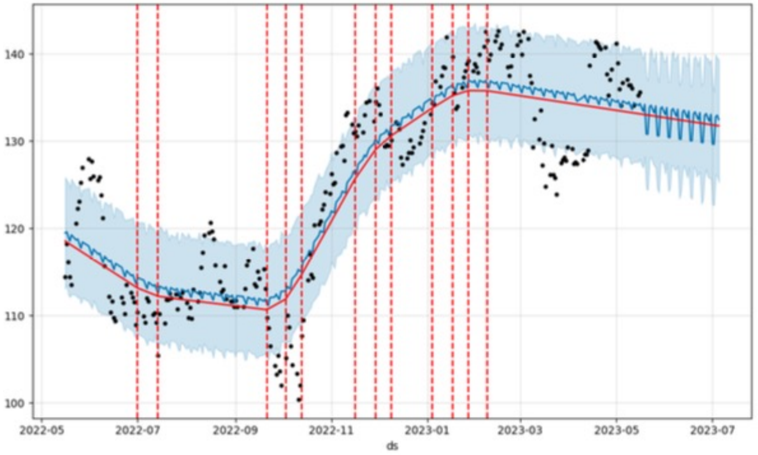


Figure 9.4: Prophet change points



Predictions of Structural Breaks

PELT: rbf, L1, L2

Dependence IV from option prices' changing points:

1. T-test
2. Regressions

LSTM

Figure 9.6: Regressions: option price, IV

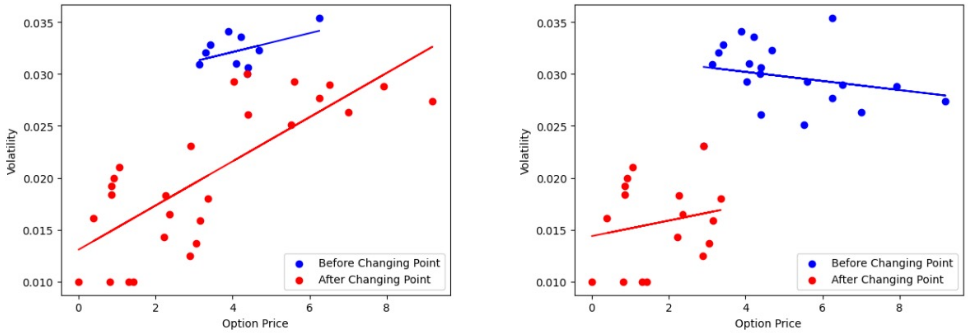
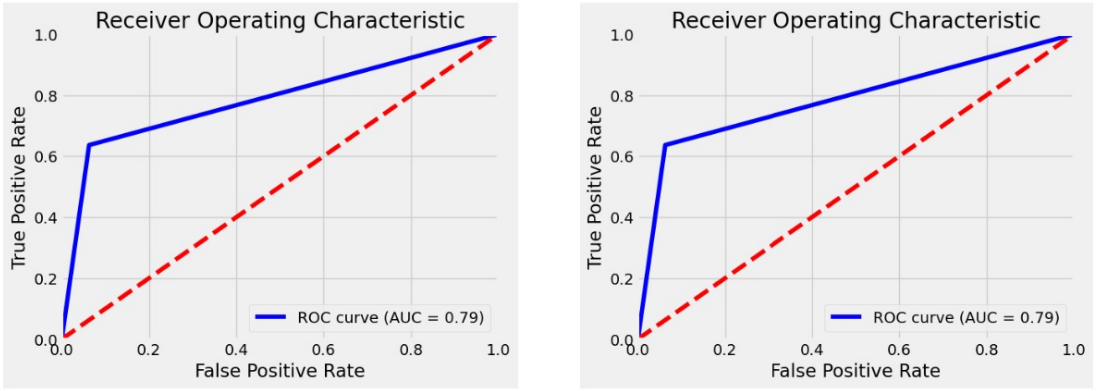


Figure 9.7: Description of data

	price	underlying	daysToExp	bsm_iv	rbf_breaks	l1_breaks	l2_breaks
count	550.000000	550.000000	550.00000	550.000000	550.000000	550.000000	550.000000
mean	21.490073	150.252214	416.80000	0.045253	0.032727	0.109091	0.141818
std	9.125141	14.683202	230.47865	0.054189	0.178084	0.312037	0.349181
min	3.550000	118.310265	17.00000	0.010000	0.000000	0.000000	0.000000
25%	14.912500	140.819153	219.25000	0.010000	0.000000	0.000000	0.000000
50%	20.325000	148.702873	416.50000	0.010000	0.000000	0.000000	0.000000
75%	26.867500	162.408073	616.75000	0.079975	0.000000	0.000000	0.000000
max	44.500000	180.434311	814.00000	0.230700	1.000000	1.000000	1.000000

Figure 9.8: AUC ROC without IV and with



Conclusions and Future Development

All goals of the project are successfully completed, was created a software with several models for pricing, volatility modelling, hedging, and structural breaks detections.

Successfully run different experiments regarding various aspects of project and presented software so approaches might be leveraged by individuals.

Software in a form of python library is uploaded on GitHub and available to leveraging by enthusiasts.

In the future we plan: expandation of models' span, real-time pricing and arbitrage finder, deploying connection with broker, option trading strategies implementation.

References

[1] Paul Wilmott On Quantitative Finance – by Paul Wilmott.

[2] Finite difference methods in financial engineering. A Partial Differential Equation Approach – by Daniel J. Duffy.

[3] Swaps and other derivatives – by Richard Flavell.

[4] OPTIONS, FUTURES, AND OTHER DERIVATIVES – by John C.

[5] Dynamic Hedging Managing vanilla and exotic options – by Nassim Taleb.

HSE UNIVERSITY • EST 1992 •