NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Faculty of Computer Science
Bachelor's Programme "Data Science and Business Analytics"

UDC XXXXXXX

Software Project Report on the Topic:

Option Pricing Systemg

Fulfilled by:

Student of the group БПАД212
Ilin Artem Alexeevich

(signature) 07.06.2023 (date)

Assessed by the Project Supervisor:

Lukyanchenko Pyotr Pavlovich
Research Fellow
Faculty of Computer Science, HSE University

(signature) 07.06.2023 (date)

Moscow 2023

Contents

Annotation

Since the very beginning of the history of one of the world's greatest inventions – money, there have always been risks and opportunities which came along with that. Depreciation of the assets and potential downside of investments is what institutional and private investors fear the most. Thus, it's essential for market participants to have a clear vision of asset management processes and its hedging, so it is where the whole class of derivative market instruments serviceable at its beauty.

In November 2022 on the Moscow Stock Exchange individuals received a new derivative to operate – standard option. It raised the interest on the Russian financial market and affected its volatility. As part of participation in such historical event the project is dedicated to understanding the nature of options trading, volatility, and solution of delta hedge problem under different market conditions and modes of operation.

In this paper we'll observe and implement in software several option pricing models, volatility models, including the stochastic ones, address the delta hedge problem and will try to predict structural breaks in time series of option prices additionally using implied volatility as a feature for the long short-term memory network.

Аннотация

С самого начала истории одного из величайших изобретений человечества - денег, его всегда преследовали различные сопутствующие риски и возможности. Обесценивание активов и потенциальная просадка - это то чего институционные и частные инвесторы боятся как больше всего. Таким образом, для участников рынка необходимо иметь четкое видение на процессы управления активами и хеджирования, вот где весь класс финансовых деривативов выступает в своей красе.

В ноябре 2022 года Московская биржа представила новый инструмент доступный физическим лицам - стандартный опцион, это вызвало интерес на российском финансовом рынке и повлияло на волатильность. Как участники данного исторического события этот проект посвящен пониманию опционного трейлинга, волатильности и дельта хеджированию в различных условиях рынка и операционных моделях.

В данной работе мы рассмотрим и имплементируем в качестве программного обеспечения различные модели прайсинга опционов, модели волатильности, включая стохастические. Рассмотрим задачу дельта хеджирования и попробуем предсказать структурные сдви-

ги временного ряда цен опционов, дополнительно используя волатильность как признак для предсказательной модели - LSTM.

Keywords

Options, pricing, greeks, volatility, modeling, stochastic processes, delta hedge, structural breaks, data mining, LSTM, BSM, Heston, SABR, finance, python, API

1 Terms & definitions

Derivative financial instrument – an agreement under which the parties obtain the right or obligation to perform some action in relation to the underlying asset.

Exchange – marketplace where securities, commodities, derivatives, and other financial instruments are traded.

Order book – a list of orders that the trading platform uses to register the interest of buyers and sellers in a particular financial instrument.

Market maker – a broker/dealer company assumes the risk of acquiring and holding securities of a certain issuer on its accounts in order to organize their sale.

Broker – a party that facilitates transactions between traders, sellers, or buyers.

Transaction costs – costs incurred in connection with the conclusion of contracts; costs accompanying the relationship of economic agents.

Bid – the price or the maximum price at which the buyer is willing to buy the asset.

Ask/Offer – the price or the lowest price at which the seller is willing to sell the asset.

OTC – the over-the-counter trading system.

Risk-management – the process of determining the price of accepted financial risks and controlling fair compensation for risks taken as a result of economic activity.

Hedging – opening trades in one market to offset the impact of price risks from an equal but opposite position in another market.

Call/Put option contract – the right but not obligation to buy/sell an asset at specific exercise price at some time in the future.

European option – option contract with defined exercise date.

American option – an option that may be exercised on any trading day on or before expiration.

Vanilla option – standard European or American option.

Underlying asset – an asset on which a derivative financial instrument is based, a financial derivative.

Spot – current price of an asset.

Strike of an option – the price set in an option at which the buyer of the option can buy or sell the underlying asset under the active option, and the seller of the option is accordingly obliged to sell or buy the corresponding amount of the underlying asset.

ITM, in the money – a phase when option possesses intrinsic value.

OTM, out of the money – phase of option when an option has no intrinsic value.

ATM, at the money – phase when option's strike price is equal to current market price of the underlying asset.

BSM – Black-Scholes-Merton Model.

Greeks – financial measures of the sensitivity of an option's price to its underlying determinants.

Stochastic process – a collection of random variables indexed by some parameter, most often playing the role of time or coordinate.

Shock – economical unexpected or unpredictable event.

Delta-hedging – hedging with regard to changes in the delta, one of the Greeks.

PL, PnL, P/L – profit and loss on a position or portfolio.

V, F - value of a derivative security/portfolio.

K – strike price.

S, U – spot price of underlying asset.

r – interest rate.

σ, vol – volatility.

ρ – correlation.

2 Data Mining

As one of The Economist articles stated "The world's most valuable resource is no longer oil, but data", since then it is more than reasonable to begin this project with techniques which will lead us to gain information from several sources and allow us to leverage data and base the project on it.

2.1 Yahoo Finance

Yahoo Finance is one tool presented in a given project dedicated to providing financial information. It is one of the leading providers of such data in the US with about 70 million visitors as of May 2018 according to eBizMBA. In the software created we use python open source library to connect to the such service - $yfinance$, since then function was programmed in such way it allows user to access option chain (list of all accessible options) for given underlying ticker.

Figure 2.1: JPM option chain

```
ticker = 'JPM'
```

```
jpm_yf = yf_get_chains(ticker)
jpm_yf.head()
```

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMoney	contractSize	currency	optionType	expDate	daysToExp
0	JPM230203C00100000	2023-01-03 19:38:02+00:00	100.0	34.05	38.80	39.00	0.0	0.0	100.0	0	2.718753	True	REGULAR	USD	call	2023-02-03 23:59:59	1
1	JPM230203C00105000	2023-01-03 18:15:30+00:00	105.0	29.37	33.60	34.25	0.0	0.0	1.0	0	3.054690	True	REGULAR	USD	call	2023-02-03 23:59:59	1
2	JPM230203C00110000	2023-02-02 16:27:29+00:00	110.0	29.74	0.00	0.00	0.0	0.0	10.0	0	0.000010	True	REGULAR	USD	call	2023-02-03 23:59:59	1
3	JPM230203C00115000	2022-12-30 17:53:27+00:00	115.0	19.00	25.15	25.60	0.0	0.0	6.0	0	3.248049	True	REGULAR	USD	call	2023-02-03 23:59:59	1
4	JPM230203C00117000	2023-01-27 16:08:18+00:00	117.0	22.90	0.00	0.00	0.0	0.0	7.0	0	0.000010	True	REGULAR	USD	call	2023-02-03 23:59:59	1

2.2 Moscow Exchange

However Yahoo Finance at the moment does not have a capacity to provide data regarding option chains for Russian companies. Since then this problem had to be overcome by addressing the Moscow Exchange API. By creating sessions and pushing requests we are now able to get option chains directly from the exchange they are listed in.

Figure 2.2: Accessing MX option chain

```
mx_get_chains('SBER', login, password).head(10)
```

2.3 Theta Data

Despite success with obtaining option chains we still need the historical data of derivatives' parameters. There is when Theta Data provider comes in handy, we use Yahoo finance to find all available option for given underlying - option chain, then choose needed product and parse history of its price using coded function:

Figure 2.3: Theta data historical prices

```
prices = get_prices(2024, 1, 19, 150, 'JPM', 'call') # get_prices(y, m, d, strike_, underlying, option_type)
```

```
prices.head(5)
```

	DataType.OPEN	DataType.HIGH	DataType.LOW	DataType.CLOSE	DataType.VOLUME	DataType.COUNT	DataType.DATE
0	26.12	26.12	26.12	26.12	1	1	2021-09-13
1	27.86	27.89	25.20	25.20	48	7	2021-09-14
2	26.00	26.00	26.00	26.00	1	1	2021-09-16
3	22.60	22.65	22.60	22.65	2	2	2021-09-20
4	23.21	23.38	23.21	23.38	12	3	2021-09-21

2.4 Historical Moex Data

Indeed Theta Data appears to be a great source for mining data for financial derivatives and more, but in terms of our project it has a disadvantage: it does not provide data for stock options listed in Moscow Exchange. To deal with it due to the technical uniqueness of MOEX API some historical information needed to be parsed by hand, gladly their website does provide such opportunity. Nonetheless, due to relatively new instruments for individuals, options do not seem to have a lot of liquidity and several historical time series are just a couple weeks long. Since then to be fully immersed into the nature of the Russian financial market, Artificial Intelligence was used to generate data with identical structure based on a given parsed historical one. We used OpenAI's ChatGPT to extend the length of the time series with the feature of preserving the complexion of the original one. It allows us to get into detail of Russian financial market conditions and more fully immerse ourselves into it.

3 Black-Scholes-Merton and Greeks

3.1 BSM

Black-Scholes-Merton formula is one of the most important in derivatives theory and in the whole world of financial mathematics. It was originated in 1973 by joint efforts of three economists: Fischer Black, Myron Scholes, and Robert Merton, and later in 1997 that led them to a Nobel prize for “a new method to determine the value of derivatives”. Such method of pricing found a wide usage in variety of financial institutions thanks to clear parameters and closed for solution of model’s equation.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r\frac{\partial V}{\partial S} - rV = 0$$

$$C = N(d_1)S - N(d_2)Ke^{-rt}$$
$$P = N(-d_2)Ke^{-rt} - N(-d_1)S$$
$$where:$$
$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

In project the corresponding functions are presenting: BSM(r, spot, strike, time, sigma, type=’call’), d1(r, spot, strike, time, sigma), d2(r, spot, strike, time, sigma)

3.2 Greeks

Important part of option derivatives are risk measures for it and most common of them are greeks:

- Delta (Δ) - measure of sensitivity of option price to change in underlying.
- Gamma (Γ) - measure of sensitivity of option price to change in Delta.
- Rho (ρ) - measure of sensitivity of option price to change in interest rate.
- Theta (Θ) - measure of sensitivity of option price to change in time, also known as time decay of option.
- Vega (ν) - measure of sensitivity of option price to change in volatility.

Since then Black-Scholes-Merton equation can be rewritten as:

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma + rS\Delta - rV = 0$$

Programmed software provides not only ability of evaluation of such risk metrics but also plotting them in the following manner:

Figure 3.1: Δ, Γ, ρ

```
for greek in ['delta', 'gamma', 'rho', 'theta', 'vega']:
    OptionLib.greek_plot(df_prices, greek_name=greek, type='both')
```

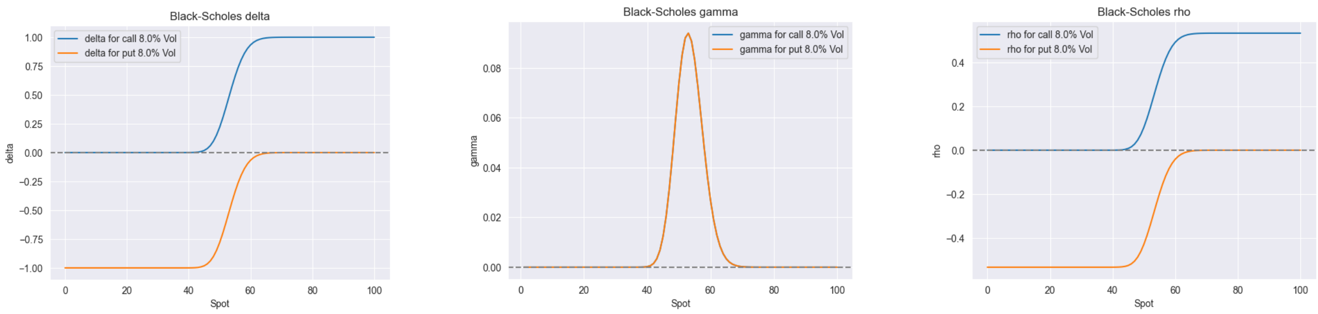
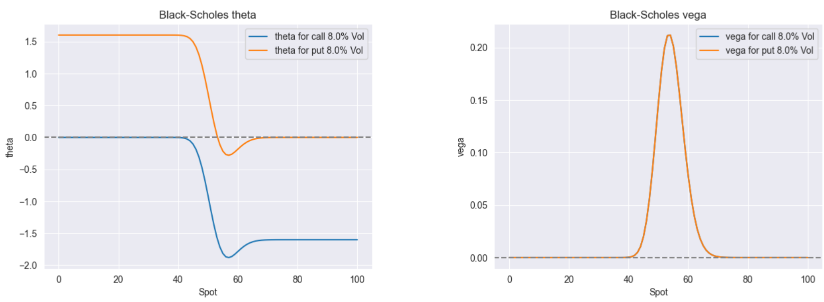


Figure 3.3: Θ, $Vega$



4 Volatility

A loud Italian option trader was known to start singing "volare, volare," an Italian popular song, every time the market experienced a panic, to the great annoyance of his peers. When asked for the reason (by the angry author) he explained that volatility came from the Latin volare, which means "to fly."

— Taleb, Dynamic Hedging

Volatility is considering as a risk metric of an asset and often refers as standard deviation or variance, in other words how big is fluctuation from the mean of data set. In our project we will address implied volatility which is one predicted/calculated by some method or algorithm, which contrasts with historical volatility which is derived from past movements of an asset.

4.1 Newton-Raphson method

This numerical method to determining volatility, which starts with initial guess and by using several iterations minimizes the difference between calculated and observed in the market prices. It is leverages the value of $Vega$ and BSM price in updating its best guess:

$$Vol_{x+1} = Vol_x + \frac{Observed\ Price - Calculated\ Price}{Vega\ Value}$$

4.2 Bisection method

Bisection method is the technique to evaluate the implied volatility with leveraging Black-Scholes-Merton (BSM) function, however due to its flexibility its also widely used for other models such as Heston one, which will be addressed later on. It uses range for the possible volatility and proposes lower vol, upper vol for call and put respectfully as well as mid vol, which later on used to calculate option price using given model and update best guess with guarantied convergence.

Noticeable, in some cases it appears to be more robust than Newton-Raphson one.

4.3 Skew, Term Structure, Surface

Volatility Skew, Term Structure, and Surface are important instruments in understanding option pricing and movements.

• Volatility Skew describes dependence between different option strikes for fixed underlying asset and implied volatility. Depending on its form it is distinguished volatility smile, smirk.

• Volatility Term Structure describes dependence between different option expiration dates for fixed underlying asset and implied volatility.

• By combining previous two relationships we obtain three dimensional representation called Volatility Surface.

In our software project we are able to present such graphical representations for real parsed conditions:

Figure 4.1: IV skew and term structure

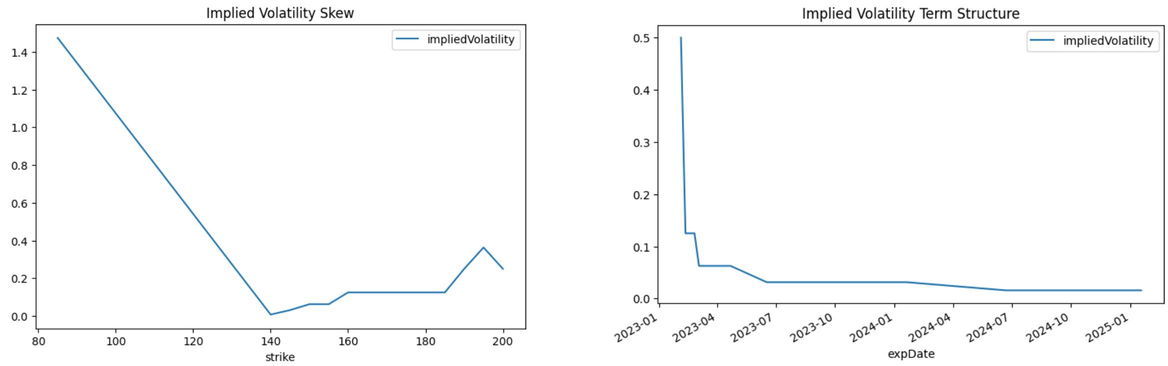
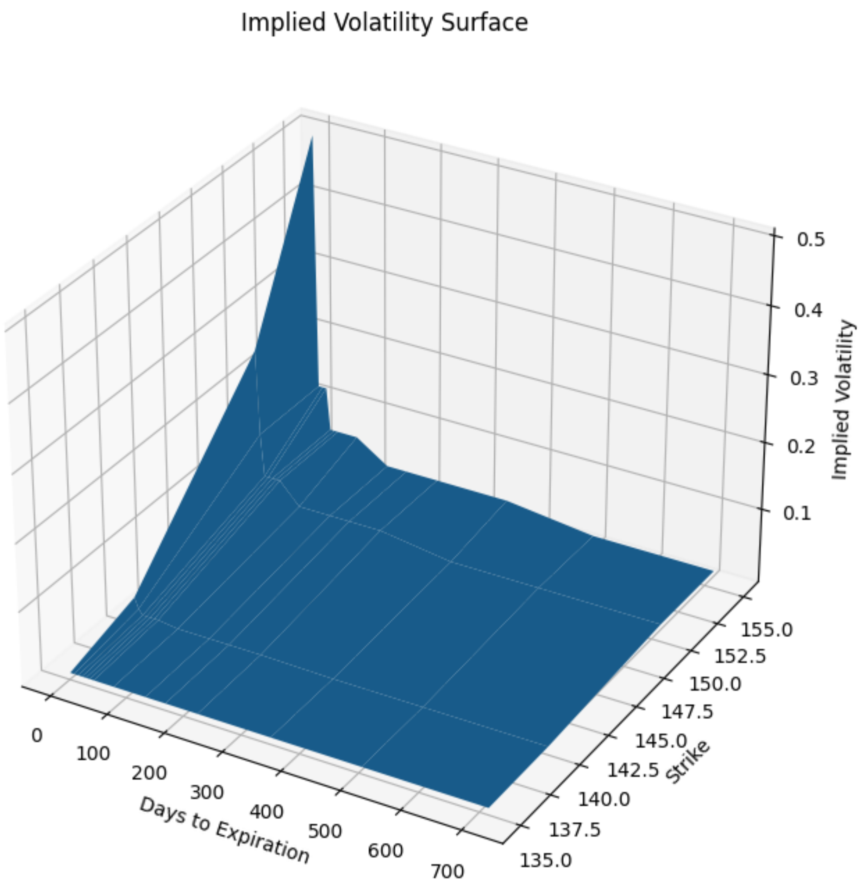


Figure 4.2: Volatility surface



5 Heston Model

Options and poker theory are fun because they prove that math works in the real world.

— Steven L. Heston

The next model will implement in our software was developed by Steven L. Heston in 1993, and in contrast with Black-Scholes-Merton takes volatility of underlying asset not as a constant but models its evolution through the time.

5.1 Definition

The model defined by the series of equations, and uses two separate stochastic processes for asset and volatility behaviours:

$$dS = \mu S_t dt + \sqrt{v(t)} S_t dW_t^{(1)}$$
$$dv(t) = \kappa[\theta - v(t)]dt + \sigma\sqrt{v(t)} dW_t^{(2)}$$
$$dW_t^{(1)} dW_t^{(2)} = \rho dt$$
$$2\kappa\theta > \sigma^2 - Feller\ condition$$

determined by the following factors:

$$S_t = spot\ price$$
$$W_t^{(i)} \ = \ a\ Wiener\ process$$
$$v(t) = variance$$
$$\mu = (risk\ neutral)\ drift$$
$$\sigma = volatility\ of\ the\ volatility$$
$$0 < \theta = long - term\ variance$$
$$\theta < \kappa = rate\ of\ mean\ reversion$$
$$\rho\ is\ the\ correlation\ value$$

In code we provide python class for the model: $heston\_model = Heston(...)$

To acknowledge the correctness of the model we will compare it with Black-Scholes results, when we eliminate the trend ($v_0 = \theta = \sigma^2_{BSM}$), and fixing $vol\ of vol = 0$ computational result should not differ. Notice that we will consider results accurate to 5 decimal places since it is already more accurate than one basis point change.

Table 5.1: Validation of coded Heston pricing model

	Call		Put		
	BSM	Heston	BSM	Heston	difference
Exp 1	6.80496	6.80496	1.92790	1.92790	0
Exp 2	17.43902	17.43902	15.29970	15.29970	0
Exp 3	8.3751	8.3751	2.30958	2.30958	0

Results of zero differences show us that model was implemented correctly.

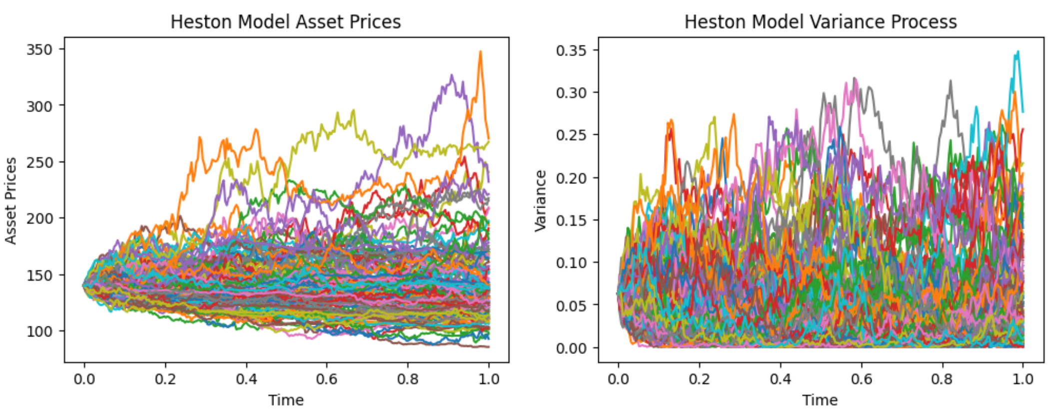
5.2 Calibration

Nonetheless, we still provide function "calibrate" in "Heston" class for determining correct parameters corresponding to the market conditions, we use "minimize" function from the "scipy.optimize" and "method='Nelder-Mead'" due to its robustness and absence of need in function's gradient information.

5.3 Monte Carlo Simulation

To be able to continue experimentation with model given under different conditions Monte Carlo Simulation were created with function "*heston_simulation*" provided.

Figure 5.1: Heston simulations




```
Spots.mean(), variances.mean()
```

```
(137.88027977364598, 0.04288228948616884)
```

6 SABR Model

6.1 Definition and Calibration

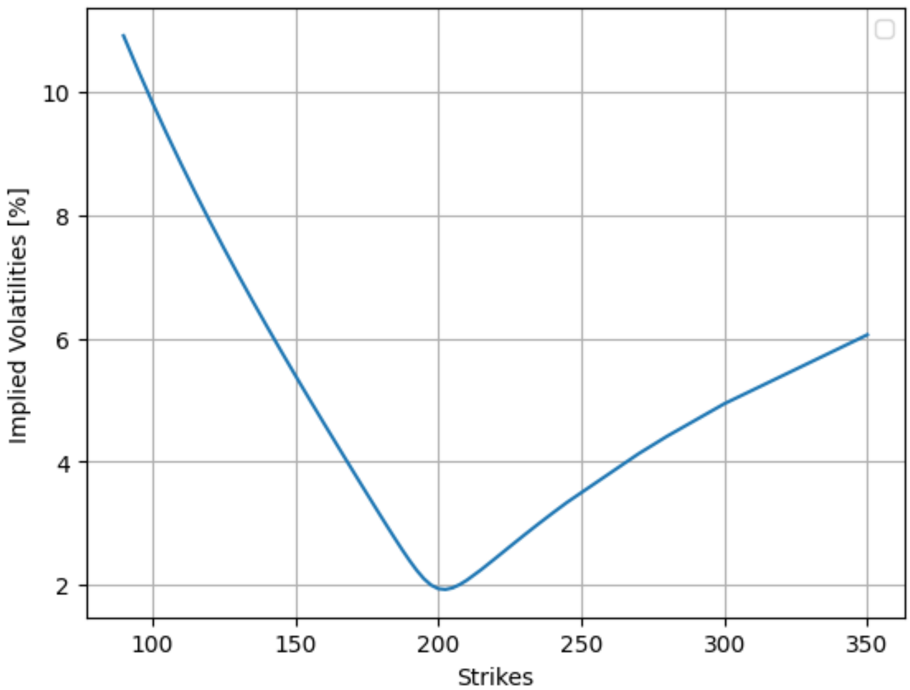
SABR or Stochastic alpha, betta, rho – which are parameters of such model aimed to capture stochastic volatility of the derivative. It is originated in 2002 by Patrick S. Hagan.

The SABR model of Hagan is described by the following 3 equations:

$$dF_t = \alpha_t F^{\beta} dW_t^{(1)}$$
$$d\alpha_t = \nu \alpha_t dW_t^{(2)}$$
$$dW_t^{(1)} W_t^{(2)} = \rho$$

where α - initialvolatility, β - forward rate exponent, ρ - correlation, ν - vol of volatility, which satisfy the following conditions satisfy conditions $\alpha \geq 0$; $0 \leq \beta \leq 1$; $-1 \leq \rho \leq 1$; $\nu \geq 0$.
In presented software function for volatility *hagan_iv* as well as class SABR was implemented which also includes calibration of its initial parameters. With usage of python package *scipy.optimize*, *minimize* we find best suitable values to match market's implied volatility. Function to observe implied volatility under sabr model was coded - *sabr_smile*, let's take a look at one of such results for parsed option chain for Tesla ("TSLA").

Figure 6.1: TSLA IV smile



6.2 Experiments

Additionally, we provide capability for user (be it portfolio manager or sophomore researcher) to experiment with data and parameters of the model in particular. Since then several conclusions can be made:

• α is the core parameter, derived from all others. It cannot be observed or approximated from historical data.

• β is the "shape" parameter, in practice, setting $\beta = 0.5$ is s quite adequate for most purposes.

• ρ affects the "tilt", or skewness, a relatively flat tilt indicates a low ρ parameter, and a relatively sharp tilt indicates a high ρ parameter of the volatility smile in the market.

• ν affects the overall "height" of the volatility smile.

Figure 6.2: α and β experiments

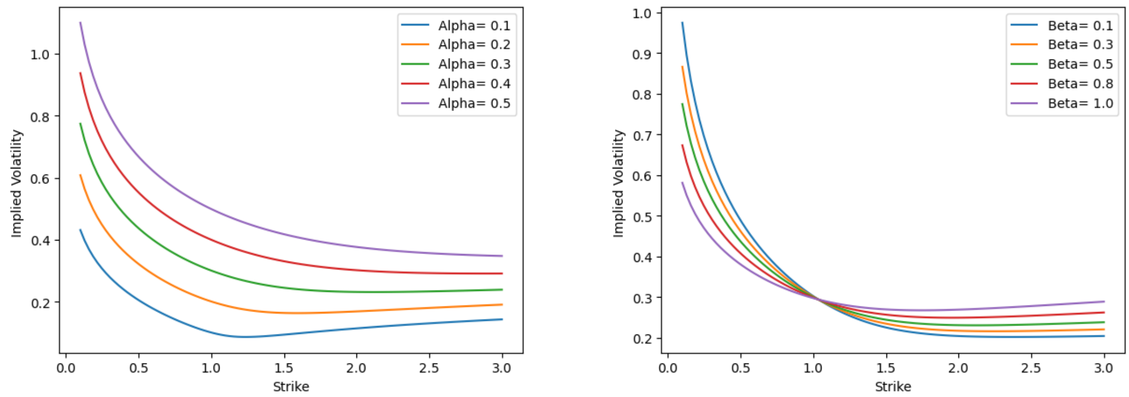
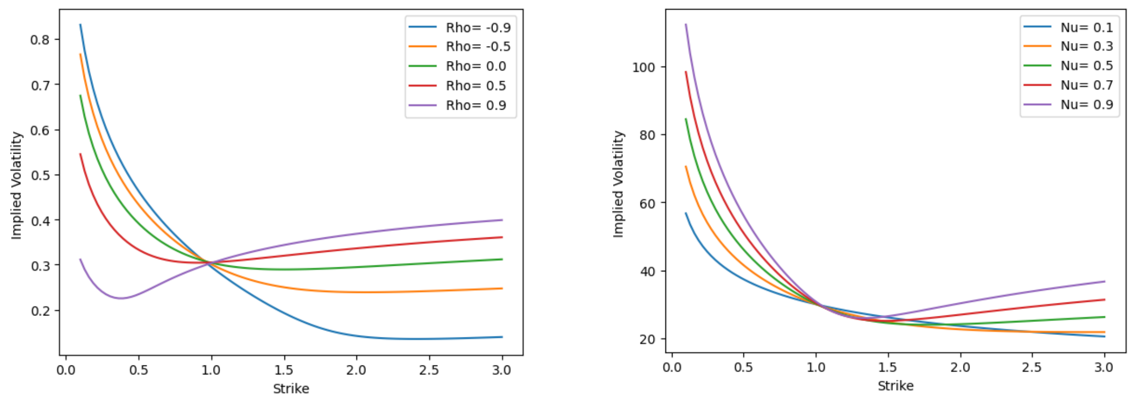


Figure 6.3: ρ and ν experiments



7 Crank–Nicolson

Crank–Nicolson is the numerical method for solving Black-Scholes Partial Differential Equation (PDE) and deriving the option price from it. Crank–Nicolson has its similarities with pricing by binomial or trinomial trees models.

$$rf = \frac{df}{dt} + rS\frac{df}{dS} + \frac{1}{2}\sigma^2 S^2 \frac{d^2 f}{dt^2}$$

For such purpose $M \times N$ discrete-time grid is introduced, whose purpose is to contemplate prices in each iteration which followed by backward movements for the next step. Thus, S, and t will satisfy following conditions: $S \in [0, 2\partial S,\ 3\partial S, ...,\ (M1)\partial S,\ S_{max}]$ and $t \in [0, \partial t,\ \ 2\partial t,\ 3\partial t, ...,\ (N1)\partial t,\ T]$.

$$\begin{gathered}
\frac{1}{2} r f_{i,j-1} + \frac{1}{2} r f_{i,j} = \\
\frac{f_{i,j} - f_{i,j-1}}{dt} \frac{1}{2} ridS \left(\frac{f_{i+1,j-1} - f_{i-1,j-1}}{2dS} \right) + \\
\frac{1}{2} ridS \left(\frac{f_{i+1,j} - f_{i-1,j}}{2dS} \right) + \frac{1}{4}\sigma^2 i^2 dS^2 \left(\frac{f_{i+1,j-1} - 2f_{i,j-1} + f_{i-1,j-1}}{dS^2} \right) + \\
\frac{1}{4}\sigma^2 i^2 dS^2 \left(\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{dS^2} \right)
\end{gathered}$$

After re-assignment of variables in such way that:

$$\begin{aligned}
\epsilon_i &= \frac{dt}{4}\left(\sigma^2 i^2 - ri\right) \\
\zeta_i &= \frac{dt}{2}\left(\sigma^2 i^2 + ri\right) \\
\xi_i &= \frac{dt}{4}\left(\sigma^2 i^2 + ri\right)
\end{aligned}$$

$$-\epsilon_i f_{i-1,j-1} + (1-\zeta_i) f_{i,j-1} - \xi_i f_{i+1,j-1} = \epsilon_i f_{i-1,j} + (1-\zeta_i) f_{i,j-1} - \xi_i f_{i+1,j}$$

It follows by constructing the following system of linear equations:

$$M_1 f_{j-1} = M_2 f_j$$

$$M_1 = \begin{bmatrix} 1-\zeta_1 & -\xi_1 & 0 & 0 & 0 & 0 \\ -\epsilon_2 & 1-\zeta_2 & -\xi_2 & 0 & 0 & 0 \\ 0 & -\epsilon_3 & 1-\zeta_3 & -\xi_3 & 0 & 0 \\ \\ 0 & 0 & ... & ... & ... & 0 \\ 0 & 0 & 0 & -\epsilon_{M-2} & 1-\zeta_{M-2} & -\xi_{M-2} \\ \\ 0 & 0 & 0 & 0 & -\epsilon_{M-1} & 1-\zeta_{M-1} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1+\zeta_1 & \xi_1 & 0 & 0 & 0 & 0 \\ \epsilon_2 & 1+\zeta_2 & \xi_2 & 0 & 0 & 0 \\ 0 & \epsilon_3 & 1+\zeta_3 & \xi_3 & 0 & 0 \\ \\ 0 & 0 & ... & ... & ... & 0 \\ 0 & 0 & 0 & \epsilon_{M-2} & 1+\zeta_{M-2} & \xi_{M-2} \\ \\ 0 & 0 & 0 & 0 & \epsilon_{M-1} & 1+\zeta_{M-1} \end{bmatrix} f_i = \begin{bmatrix} f_{1,j} \\ f_{2,j} \\ . \\ . \\ . \\ f_{M-1,j} \end{bmatrix}$$

Thus, class $CrankNicolson()$ was created, however it does differ from Black-Schloes results, since we same inputs were produced with certain gap:

$$BSM\ price: \ 4.075980984787783\ ;\ CN\ price\ 4.072254507998117$$

Such result exhaustively explains Duffy in "Finite difference methods" by criticizing Crank–Nicolson method for the following reasons:

• "The Crank-Nicolson method is second-order accurate on uniform meshes only."

• "It gives terrible results near the stike price for approximations to the first and second derivatives in the space direction."

8 Delta Hedge

Delta hedging is crucial technique in risk management of derivatives. It requires taking offsetting position in underlying asset in order to counterpoise risk coming from option. When hedging with respect to delta, which is sensitivity of option prise with respect to changes in underlying asset, we need to consider such shifts and make according adjustments. To hedge option properly we need to quantify the Delta Hedge Quantity, taking into consideration that standard option include bet on 100 stocks:

$$DHQ = |\Delta| * number\_of\_option\_contracts * 100$$

Then our position of DHQ (in this case we stick to assumption each security is perfectly divisible) in underlying is determined by the sign of delta: long if $\Delta < 0$, and short if $\Delta > 0$.

In our software we use functions to create portfolio for concrete option and rebalancing it for each step of time series given (row of data frame). As a result we come into possession of option portfolio history. For instance for classic BSM delta and portfolio of eight European calls on "JPM" (JPMorgan Chase) "JPM230428C00145000" we have:

Figure 8.1: Delta hedge

	underlying_price	option_price	delta	delta_pos	DH_quantity	underlying_needed	underlying_pos
0	120.000000	25.000000	1.000000	800.000000	800.000000	6.666667	-1.0
1	110.080640	24.780115	0.747941	598.352480	598.352480	5.435583	-1.0
2	116.122338	27.083789	0.829282	663.425445	663.425445	5.713160	-1.0
3	113.456048	27.919812	0.769023	615.218568	615.218568	5.422528	-1.0
4	110.983356	28.541619	0.736963	589.570399	589.570399	5.312242	-1.0
...	...	...	...	...	...	...	...
116	61.418184	24.251426	0.642706	514.164786	514.164786	8.371540	-1.0
117	61.399870	23.375081	0.632300	505.840114	505.840114	8.238456	-1.0
118	62.095566	23.821407	0.635551	508.441182	508.441182	8.188043	-1.0
119	61.225318	24.921022	0.651187	520.949711	520.949711	8.508730	-1.0
120	62.351163	25.261175	0.651590	521.271995	521.271995	8.360261	-1.0

8.1 BSM Delta

Under Black-Scholes-Merton model's conditions delta has the following form:

$$\Delta_{C,P} = \frac{dV}{dS} = \pm N(\pm d_1)$$

8.2 Heston Delta

For Heston model we got the following equations:

$$\frac{dC}{dS_0} = e^{-q(T-t)}P_1 > 0$$
$$\Delta_{C,t} = \frac{C_t}{S_t} = e^{-q(T-t)}P_1$$

and as follows from Put-Call parity $\Delta_{P,t} = \frac{P_t}{S_t} = \frac{C_t}{S_t} - e^{-q(T-t)} = e^{-q(T-t)}(P_1 - 1)$

The *heston_delta* function is also a part of *Heston* class.

8.3 SABR Delta

This method was proposed by Peter Barret and have tendency to provide high accuracy approximations. Here *skew* is a representation of asymmetry in implied volatility.

$$skew = \beta * \alpha * F^{1-\beta} + O(\epsilon)$$
$$\Delta_{SABR} = \Delta_{BSM} + Vega_{BSM} * skew$$

Every delta function here is implemented in project's software.

9 Structural breaks

When we refer to the structural breaks (bifurcations, change points) we connote points of time series data where happens a significant shift in statistical properties. Identification of such breaks is important to obtaining better understanding regarding the nature of data dependencies and properties. The branch of mathematical theories and studies colluded in the theory of catastrophes, which addresses the way dramatic changes occur due to slight changes of input parameters. It found its usage in variety of fields including finance and economics.

9.1 Detection

To distinguish such breaks we present several techniques:

Ruptures python library is a utility for an offline detection of structural breaks. It presents the following models in change points detection:

• Dynamic Programming – given technique uses evaluation of different partitioning schemes by several iterations, and after optimizing the criterion given (for instance sum of squared errors) provides user with solution.

• Pelt (Pruned Exact Linear Time) – method uses pruning techniques as well as dynamic programming, since then it presents exact solution, and at the same time works a linear complexity.

• Binary Segmentation – this method uses recursively splitting the data and after that applying test and criteria to define if structural break is occur.

• Bottom-up Segmentation – in this technique we treat each observation as a separate segment after that to minimize error criterion we join neighboring segments.

Figure 9.1: Dynamic Programming & Pelt change points

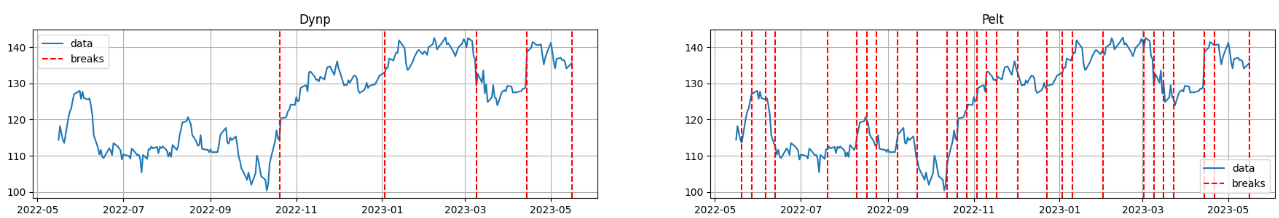
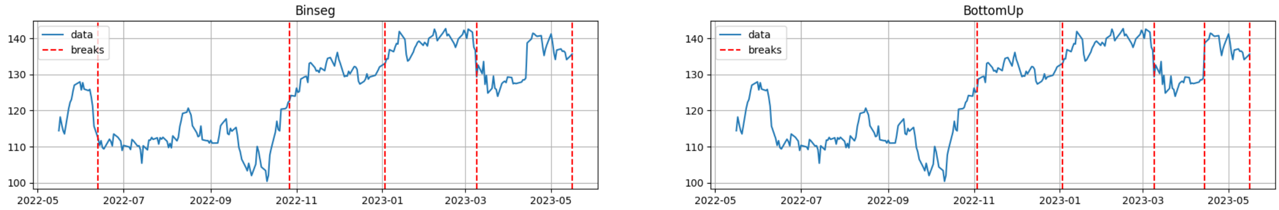
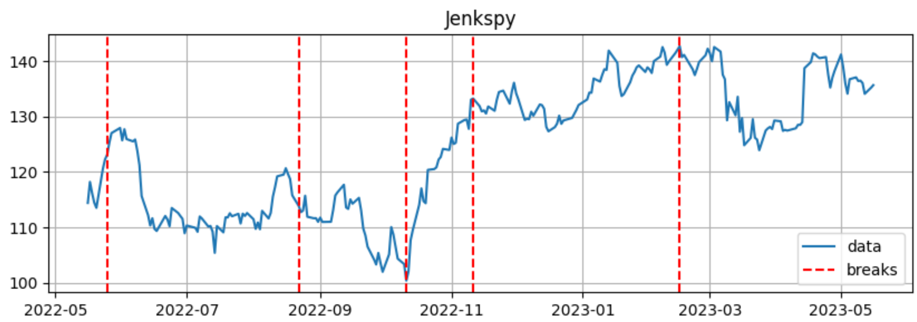


Figure 9.2: Binary Segmentation & Bottom-up Segmentation change points



Jenkspy is another python library which as its core uses Fisher-Jenks algorithm, which is a clustering-based method, since then it divides the time series given into several parts and provides results along with trying to minimize the sum of squared deviations metric in each one of them.

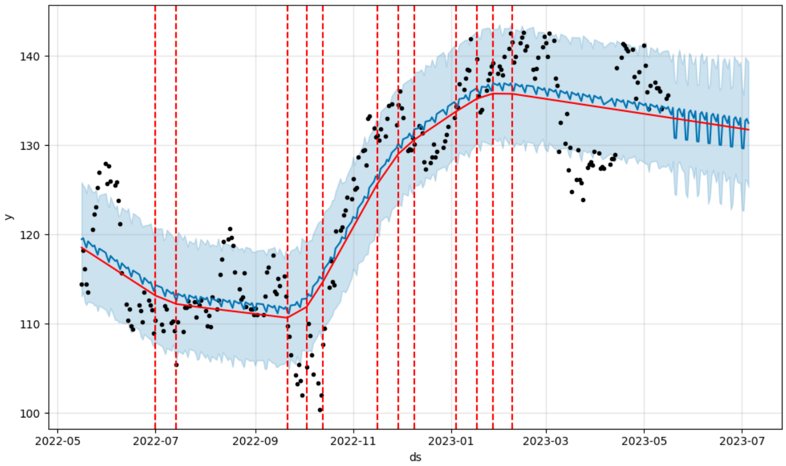
Figure 9.3: Jenkspy change points



Chow Test – was developed in 1960 by a econometrician Gregory Chow. The test might be viewed as a special case of F-test, it's statistic is distributed as F-distribution. In time series it checks if particular date is a responsible for a structural break by assuming there is no change point as a Null Hypothesis, and there is a change point at dived date as an Alternative Hypothesis.

Prophet is python library developed by Facebook/Meta and known for its machine learning algorithms, however in this project we are able to detect a structural breaks using it. Prophet leverages linear and logistic growth models to identify such changes.

Figure 9.4: Prophet change points



9.2 Dependence IV from option prices' changing points

The usage in pair with flexibility of bisection method of volatility determination takes it place in BSM, and Heston models, forth noticing that after calibration of SABR - we minimize difference between market volatility and the one it provides, since then we can omit such calculations. In the following manner the IV's for call on SBER (SR190CE3A) might be quantified.

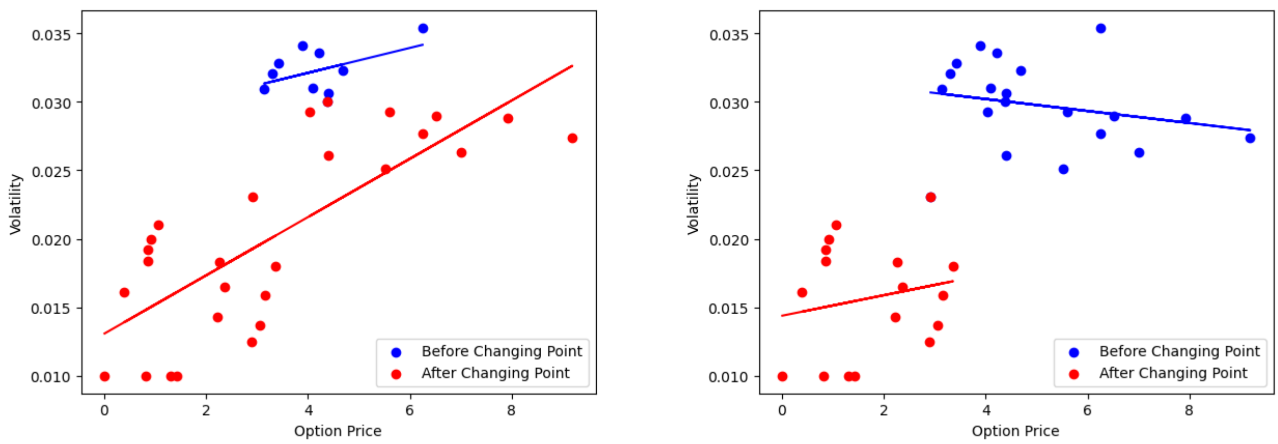
Figure 9.5: Implied Vols

date	price	underlying	daysToExp	bsm_iv	heston_iv
2023-04-11	30.29	218.60	22	0.3130	0.313027
2023-04-12	34.66	219.22	21	0.6288	0.628852
2023-04-13	32.91	219.85	20	0.4905	0.490480
2023-04-14	33.21	221.87	19	0.3269	0.000100
2023-04-17	37.64	228.10	16	0.0914	0.000100
2023-04-18	42.86	232.66	15	0.1042	0.000100
2023-04-19	44.74	232.95	14	0.6105	0.610498
2023-04-20	46.40	236.28	13	0.1199	0.000100
2023-04-21	47.08	235.17	12	0.7151	0.715137
2023-04-24	46.31	235.10	9	0.7225	0.722531

In the following experiments we make our focus on PELT from the ruptures Python library as a model for detecting change points due to its linear complexity of computations, which makes it efficient for analyzing large datasets, it also provides an exact and robust solutions. Additionally, PELT has three models to operate: "rbf", which stands for Radial Basis Function, L1, which stands for L1-norm or Manhattan norm, and L2-norm which is a Euclidean norm.

After structural breaks detection we do provide technical capacity in given software for a regression analysis of dependence of implied volatility form option price. However for presented in graphs changing point for dates 2022-10-19 and 2022-11-02 coefficients of before and after the break are 0.00090949, 0.00212789, and -0.00043845, 0.0007493, this pattern continues for every other point tested, which allows us to conclude that there is no linear dependency between two variables or other factors needed to be taken into consideration.

Figure 9.6: Regressions: option price, IV



We also provide a T-test ability to determine either or not the current date is a bifurcation for time series, in particular of option prices.

9.3 Predictions

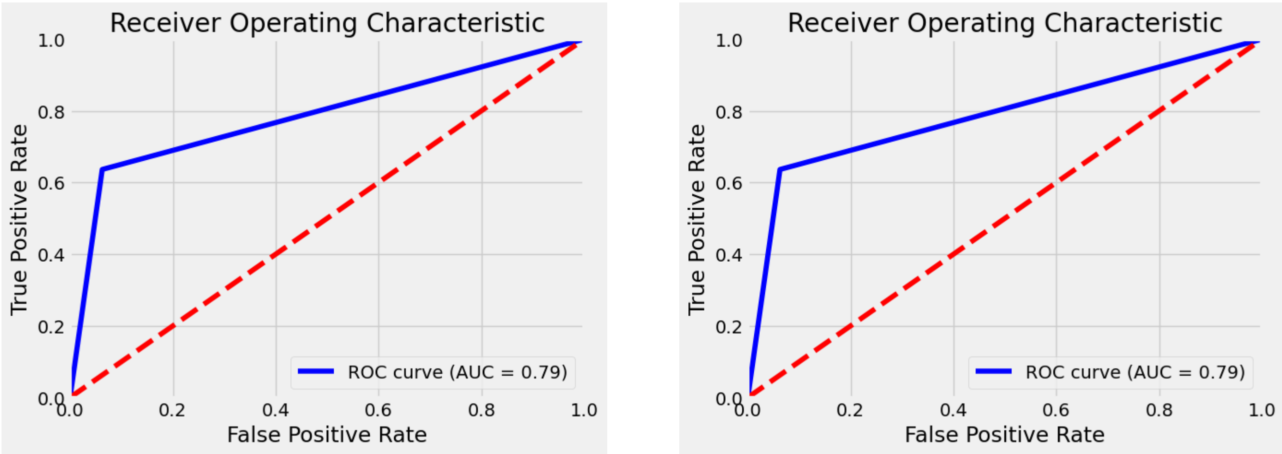
The additional goal is to acquire the knowledge of the potential ability to predict the bifurcation of option prices time series by leveraging the implied volatility as helping feature. For such purpose we addressed PELT with L2-norm model, as experimentally obtained it provided maximum of bifurcation detections, which is crucial for large volumes of data. In this binary classification problem 1 - presence of structral break, 0 - absence.

Figure 9.7: Description of data

	price	underlying	daysToExp	bsm_iv	rbf_breaks	l1_breaks	l2_breaks
count	550.000000	550.000000	550.00000	550.000000	550.000000	550.000000	550.000000
mean	21.490073	150.252214	416.80000	0.045253	0.032727	0.109091	0.141818
std	9.125141	14.683202	230.47865	0.054189	0.178084	0.312037	0.349181
min	3.550000	118.310265	17.00000	0.010000	0.000000	0.000000	0.000000
25%	14.912500	140.819153	219.25000	0.010000	0.000000	0.000000	0.000000
50%	20.325000	148.702873	416.50000	0.010000	0.000000	0.000000	0.000000
75%	26.867500	162.408073	616.75000	0.079975	0.000000	0.000000	0.000000
max	44.500000	180.434311	814.00000	0.230700	1.000000	1.000000	1.000000

As a result after several numerical experiments and usage of Long short-term memory (LSTM) machine learning model we are able to conclude that there is no significant increase of predictability power in model coming from leveraging implied volatility to predict change points in option prices. Despite increasing accuracy of the model from average of 0.75 to 0.86 confusion matrices and AUC ROC stayed the same.

Figure 9.8: AUC ROC without IV and with



10 Conclusions

All goals of the project are successfully completed, was created a software with several models for pricing, volatility modeling, hedging, and structural breaks detections. Successfully run different experiments regarding various aspects of project and presented software so approaches might be leveraged by individuals.

Despite relatively new option market availability for individuals in the Russian market there were no significant differences from other data presented, it might be partially explained by the same rules of economic theory such as invisible hand of the market and similar pricing and volatility models.

11 References

[1] Paul Wilmott On Quantitative Finance – by Paul Wilmott.

Author is a one of the biggest figures in quantitative finance, provides knowledge of derivatives evacuation techniques and covers options fundamentals as well as more advanced approaches.

[2] Finite difference methods in financial engineering. A Partial Differential Equation Approach – by Daniel J. Duffy.

Covers numerical methods to price derivative products in quantitative finance and dedicated to developing robust system to perform such processes. Also covers stochastic calculus, SDEs, and Heston's model.

[3] Swaps and other derivatives – by Richard Flavell.

Covers usage of derivatives and options in real world trading and hedging.

[4] OPTIONS, FUTURES, AND OTHER DERIVATIVES – by John C.

Provides risks-evaluation techniques and covers the fundamentals of options trading and management.

[5] Dynamic Hedging Managing vanilla and exotic options – by Nassim Taleb.

Covers liquidity and risk management, Greeks, models calibration and hedging as well as delta-hedging.

[6] Software interface of the Information and Statistical Server of the Moscow Exchange Developer Guide Version 1.4

[7] The pricing of commodity contracts 81 (1976) - by F. Black

[8] Options, Futures, and Other Derivative Securities, Prentice Hall, 1997. - by John C. Hull,

[9] Equivalent Black volatilities - by Patrick S. Hagan and Diana E. Woodward

[10] Geometric optics in finance, in preparation. - by P. S. Hagan, A. Lesniewski and D. E. Woodward

[11] The pricing of options on assets with stochastic volatilities, Journal of Finance, 42 (1987) - by J. Hull and A. White,

[12] A closed-form solution for options with stochastic volatility with applications to bond and currency options - by S.L. Heston

[13] Option Valuation Under Stochastic Volatility, Financial Press - by A. Lewis

[14] Course Notes on Computational Finance - by Rudiger Seydel

[15] Full and fast calibration of the Heston stochastic volatility model - by Yiran Cuia, Sebastian del Bano Rollinb, Guido Germano