Multistage Bidding Model with Elements of Bargaining. Extension for a Countable State Space*

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We consider a simplified model of a financial market with two players bidding for one unit of a risky asset (a share) for $n \leq \infty$ consecutive stages. Player 1 (an insider) is informed about the liquidation price s of the asset while Player 2 knows only its probability distribution $\mathbf{p}$. At each stage players place integral bids. The higher bid wins and a share is transacted to the winning player. Each player aims to maximize the value of her final portfolio.

A model where the price s has only two possible values $\{0, m\}$ is considered in [1]. It is reduced to a zero-sum game $G_n(\mathbf{p})$ with incomplete information on one side as in Aumann, Maschler [2]. In this model uninformed Player 2 uses the history of Player 1's moves to update the posterior probabilities over the liquidation price. Thus Player 1 should find a strategy controlling posterior probabilities in such a way that on one hand allows her to use the private information, and on the other hand doesn't reveal too much of it to Player 2. The main results in [1] are explicit optimal strategies and the value of the game $G_\infty(\mathbf{p})$. In [3] the model is extended so that the liquidation price can take any value $s \in S = \mathbb{Z}_+$ according to a probability distribution $\mathbf{p} = (p_0, p_1, \ldots)$. It is shown that when $\mathbb{E}\mathbf{p}^2 < \infty$ a game $G_\infty(\mathbf{p})$ is properly defined. For this game the value and optimal players strategies are found.

In both [1] and [3] the transaction price equals to the highest bid. Instead we could consider a transaction rule proposed in [4], and define a price at which the asset is transacted equal to a convex combination of proposed bids with some coefficient $\beta \in [0, 1]$. A model with such transaction rule and two possible values of the liquidation price is analyzed in [5]. Here these results are futher extended for the case of a countable state space.

Formally the model is defined as follows. At stage 0 a chance move chooses a state of nature $s \in S$ according to the distribution $\mathbf{p}$. At each stage $t = \overline{1, n}$ players make bids $i_t \in I, j_t \in J$ where $I = J = \mathbb{Z}_+$. A

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stage payoff in state s equals to

$$a^s(i,j) = \begin{cases} (1-\beta)i + \beta j - s, & i < j, \\ 0, & i = j, \\ s - \beta i - (1-\beta)j, & i > j. \end{cases}$$

Player 1 strategy is a sequence of actions $\sigma = (\sigma_1, \ldots, \sigma_n)$ where $\sigma_t : \ S \times I^{t-1} \to \Delta(I)$ is a mapping to the set of probability distributions $\Delta(I)$ over I. That is, at each stage Player 1 randomizes his bids depending on the history up to stage t and the liquidation price s. Similarly, Player 2 strategy $\tau = (\tau_1, \ldots, \tau_n)$ where $\tau_t : \ J^{t-1} \to \Delta(J)$. Player 1's payoff then defined as $K_n(\mathbf{p}, \sigma, \tau) = \mathbb{E}_{(\mathbf{p},\sigma,\tau)} \sum_1^n a^s(i,j)$. Player 2's payoff equals to $-K_n(\mathbf{p}, \sigma, \tau)$.

Following [3], Player 1's strategy σ in n–stage game can be represented as a pair $(\sigma_1, \sigma(i))$ where σ_1 is a one stage action and $\sigma(i)$ is a strategy in $(n-1)$–stage game dependent on the actual first bid. Similarly Player 2's strategy τ can be represented as a pair $(\tau_1, \tau(i))$. Denoting $q = (q_0, q_1, \ldots)$ a marginal distribution of the first bid, and $p(i)$ – a posterior distribution of the liquidation price given a bid i, the following recursive formula takes place

$$K_n(\mathbf{p}, \sigma, \tau) = K_1(\mathbf{p}, \sigma_1, \tau_1) + \sum_{i \in I} q_i K_{n-1}(p(i), \sigma(i), \tau(i)).$$

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