Multistage Bidding Model with Elements of Bargaining. Extension for a Countable State Space*

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We consider a simplified model of a financial market with two players bidding for one unit of a risky asset (a share) for $n \leq \infty$ consecutive stages. Player 1 (an insider) is informed about the liquidation price s of the asset while Player 2 knows only its probability distribution p. At each stage players place integral bids. The higher bid wins and a share is transacted to the winning player. Each player aims to maximize the value of her final portfolio.

A model where the price s has only two possible values $\{0, m\}$ is considered in [1]. It is reduced to a zero-sum game $G_n(p)$ with incomplete information on one side as in Aumann, Maschler [2]. In this model uninformed Player 2 uses the history of Player 1's moves to update the posterior probabilities over the liquidation price. Thus, Player 1 should find a strategy controlling posterior probabilities in such a way that allows her to use the private information without revealing too much of it to Player 2. The main results in [1] are explicit optimal strategies and the value of the game $G_\infty(p)$. In [3] the model is extended so that the liquidation price can take any value $s \in S = \mathbb{Z}_+$ according to a probability distribution $p = (p_0, p_1, \ldots)$. It is shown that when $\mathbb{D}p < \infty$ a game $G_\infty(p)$ is properly defined. For this game the value and optimal players strategies are found.

In both [1] and [3] the transaction price equals to the highest bid. Instead we could consider a transaction rule proposed in [4], and define a price at which the asset is transacted equal to a convex combination of proposed bids with some coefficient $\beta \in [0, 1]$. A model with such transaction rule and two possible values of the liquidation price is analyzed in [5]. Here these results are futher extended for the case of a countable state space.

Formally the model is defined as follows. At stage 0 a chance move chooses a state of nature $s \in S$ according to the distribution p. At each stage $t = \overline{1, n}$ players make bids $i_t \in I, j_t \in J$ where $I = J = \mathbb{Z}_+$. A

*The reported study was funded by RFBR according to the research project No.16-01-00353a.

stage payoff in state s equals to

$$a^s(i,j) = \begin{cases} (1-\beta)i + \beta j - s, & i < j, \\ 0, & i = j, \\ s - \beta i - (1-\beta)j, & i > j. \end{cases}$$

Player 1's strategy is a sequence of actions $\sigma = (\sigma_1, \ldots, \sigma_n)$ where $\sigma_t : S \times I^{t-1} \to \Delta(I)$ is a mapping to the set of probability distributions $\Delta(I)$ over I. That is, at each stage Player 1 randomizes his bids depending on the history up to stage t and the liquidation price s. Similarly, Player 2's strategy $\tau = (\tau_1, \ldots, \tau_n)$ where $\tau_t : J^{t-1} \to \Delta(J)$. Player 1's payoff then defined as $K_n(p,\sigma,\tau) = \mathbb{E}_{(p,\sigma,\tau)} \sum_{t=1}^{n} a^s(i_t, j_t)$. Player 2's payoff equals to $-K_n(p,\sigma,\tau)$.

Following [3], Player 1's strategy σ in n–stage game can be represented as a pair $(\sigma_1, \sigma(i))$ where σ_1 is a one stage action and $\sigma(i)$ is a strategy in $(n-1)$–stage game dependent on the actual first bid. Similarly Player 2's strategy τ can be represented as a pair $(\tau_1, \tau(i))$. Denoting $q = (q_0, q_1, \ldots)$ a marginal distribution of the first bid, and $p(i)$ – a posterior distribution of the liquidation price given a bid i, the following recursive formula holds

$$K_n(p,\sigma,\tau) = K_1(p,\sigma_1,\tau_1) + \sum_{i \in I} q_i K_{n-1}(p(i), \sigma(i), \tau(i)).$$

Thus to define a strategy in $G_n(p)$ it is suffice to define a stage action for any posterior probability p. Let's define a pure strategy τ^k as

$$\tau_1^k = k, \quad \tau_t^k(i_{t-1}, j_{t-1}) = \begin{cases} j_{t-1}, & i_{t-1} < j_{t-1}, \\ j_{t-1}, & i_{t-1} = j_{t-1}, \\ j_{t-1}, & i_{t-1} > j_{t-1}. \end{cases}$$

It can be shown that for p such that $\mathbb{E}p = k - 1 + \beta + \xi$, $\xi \in [0,1)$ Player 2 can guarantee a payoff not more than $H^\infty(p) = \mathbb{D}p + \beta(1-\beta) - \xi(1-\xi)$. Function $H^\infty(p)$ is piecewise linear with breakpoints at $p \in \Theta(k+\beta)$ where $\Theta(x) = \{p : \mathbb{E}p = x\}$ and domains of linearity $\Lambda(k-1+\beta, k+\beta)$ where $\Lambda(a,b) = \{p : a \le \mathbb{E}p \le b\}$.

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