Problem Statement

1) Let the measurements of the rotation angle $\Theta(t_k)$ be made at moments in time $t_k = \Delta k$, $k = 1, 2, ..., N$, over the time interval $[0, T]$, where $T = \Delta N$, i.e.

$$\Theta(t_k) = \Theta(\Delta k) = A_0 \sin(\omega_0 \overbrace{\Delta k}^{t_k}) + \varepsilon_k \tag{1}$$

where ε_k is a random variable with mean zero and variance σ^2, and $\varepsilon_k \sim N(0, \sigma^2)$.

Our goal is to estimate (A_0, ω_0), which are the parameters that exactly define the system on the rotational axis powered by $a_0 \sin(\omega_0 t)$.

2) Consider the sequential estimate of ξ_N, $N = 1, 2, ...$, given by the rule:

$$\xi_N = \frac{1}{\Delta N}\left(\sum_{k=1}^{N}(\theta^2(\Delta k)\cdot\Delta)\right) = \frac{1}{N}\sum_{k=1}^{N}(A_0 \sin(\omega_0 \Delta k) + \varepsilon_k)^2 \tag{2}$$

Computational Tasks

Task 1: Compute $E[\xi_N]$ and find $\lim_{N\to\infty} E[\xi_N] =?$.

Task 2: Compute $E[(\xi_N - E[\xi_N])^2]$ and $\lim_{N\to\infty} E[(\xi_N - E[\xi_N])^2] =?$.