Time Series Labb 1

Time Series Analysis, Umeå University

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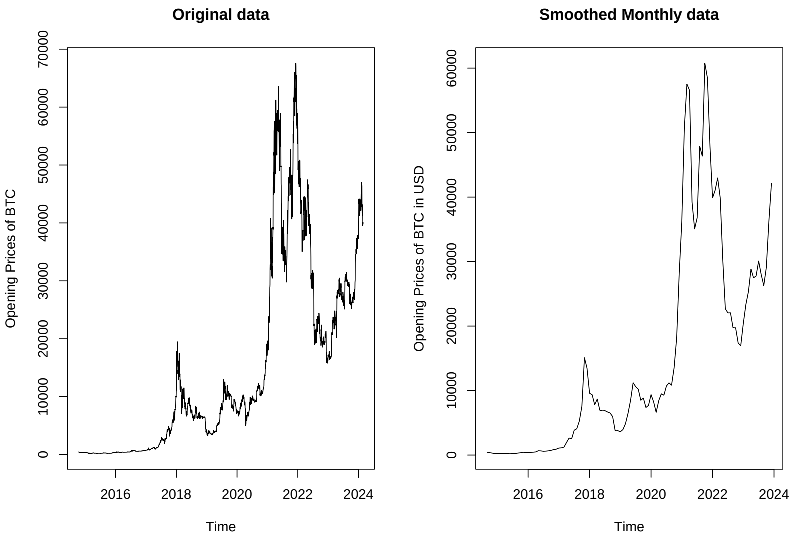
Problem Forumlation

- Time Series Forecasting
- Investment Decision based on Accuracy of Models
- Exploration of data

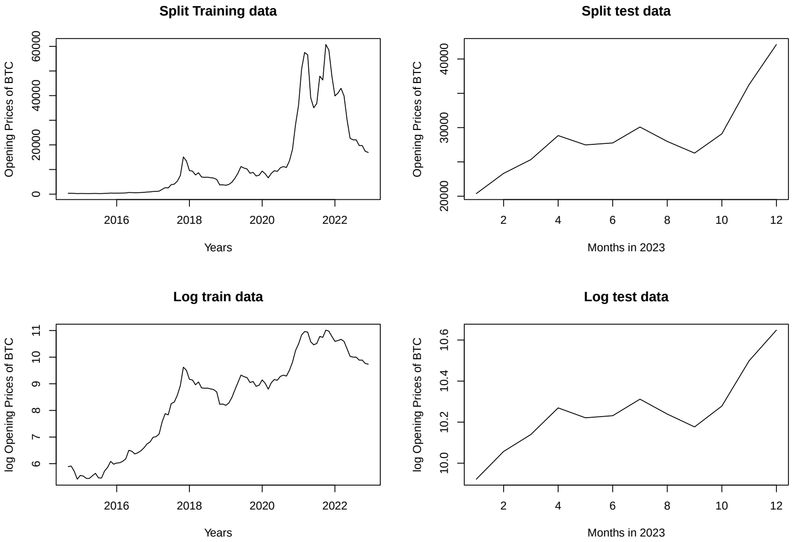
Data Presentation

- Yahoo Finance
- Smoothed to Monthly Prices

Data Presentation



Data Presentation



Statistical Methods Applied

- S1 Method

$$X_t = m_t + s_t + Y_t, \quad t = 1, \ldots n, \quad \text{where }, \mathbf{E}[Y_t] = 0 \tag{1}$$

$$m_t = \frac{0.5x_{t-3} + x_{t-2} + x_{t-1} + x_t + x_{t+1} + x_{t+2} + 0.5x_{t+3}}{6} \tag{2}$$

$$s_t = w_k - \frac{1}{d}\Sigma_i^d w_i \quad , i, k = 1, 2, \ldots d \tag{3}$$

$$d_t = x_t - s_t \tag{4}$$

$$\text{Then re-estimate the means using the de-seasonalized data} \tag{5}$$

$$\hat{m}_t = \frac{0.5d_{t-3} + d_{t-2} + d_{t-1} + d_t + d_{t+1} + d_{t+2} + 0.5d_{t+3}}{6} \tag{6}$$

$$\hat{Y}_t = x_t - \hat{m}_t - s_t \tag{7}$$

Statistical Methods Applied

- S2 Method

Method S2 consist of elimination of trend and seasonal component by differencing.

The **lag-d** difference operator ∇_d is defined as

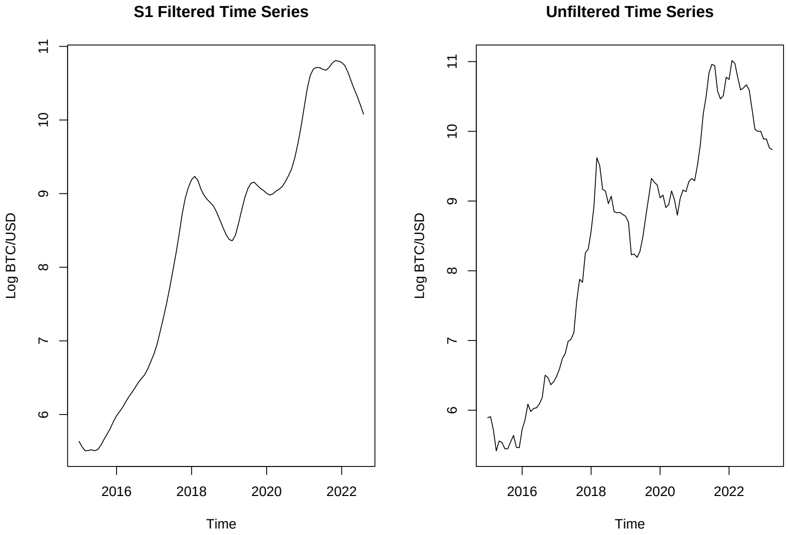
$$\nabla_d X_t = X_t - X - t - d = (1 - \mathcal{B}^d)X_t$$

Using special property of

$$\mathcal{B}X_t = X_{t-1}$$

$$\nabla_d X_t = m_t - m_{t-d} + Y_t - Y_{t-d}$$

Statistical Methods Results: S1

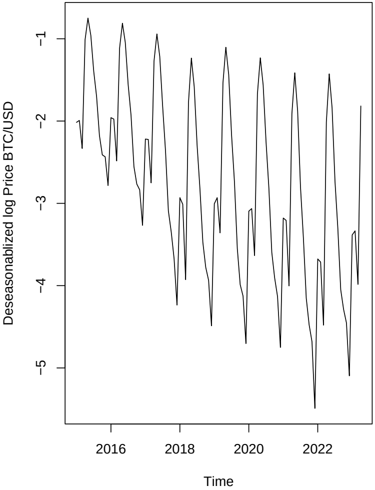


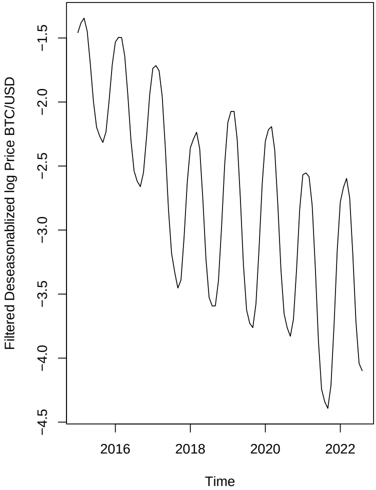
Statistical Methods Results: S1

Seasonal components:

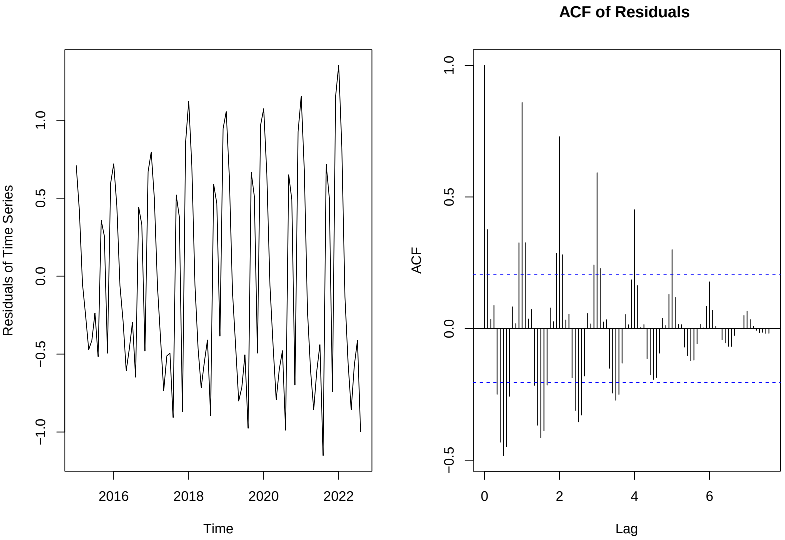
Months	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
s_t	-0.34	-0.33	-0.41	-0.18	-0.14	-0.17	-0.25	-0.31	-0.39	-0.43	-0.45	-0.51

Statistical Methods Results: S1





Statistical Methods Results: S1



Statistical Methods Results: S2

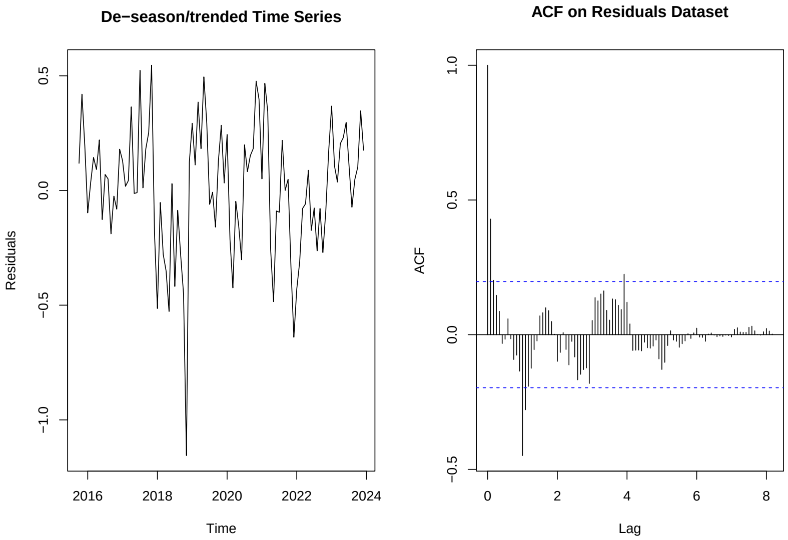


Figure 1: Differenced Method for de-seasonlized and de-trending Time

Checking Assumptions

- Visually checking the sample autocorrelation function

$$\mathrm{H}_0 = \text{The Time Series is iid Noise}$$
$$\mathrm{H}_1 = \text{The Time Series is NOT iid Noise}$$

- Protmanteau test
- Turning point test
- Difference-sign test
- Mann-Kendall Rank test
- Checking for normality
  - Histogram
  - qq plot
  - Normality test
    - Shapiro-Wilks test
    - Shapiro–Francia test

Checking Assumptions

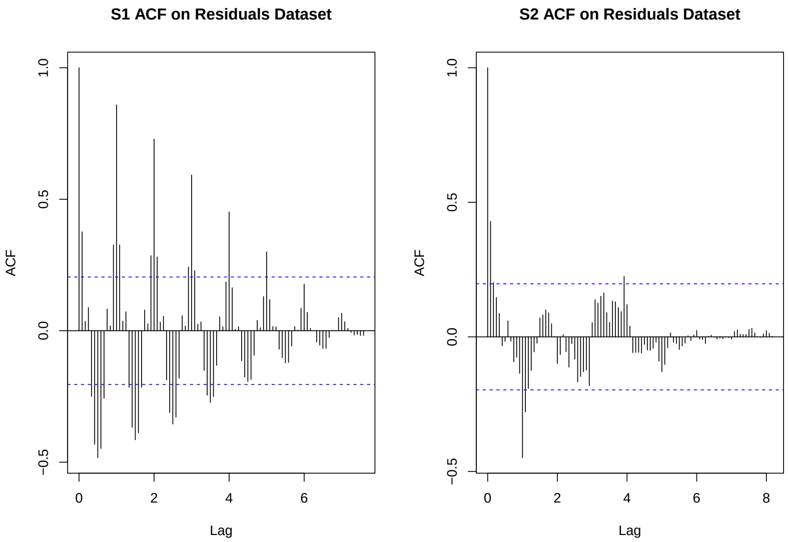


Figure 2: ACF for S1 and S2 methods on time series

Checking Assumptions

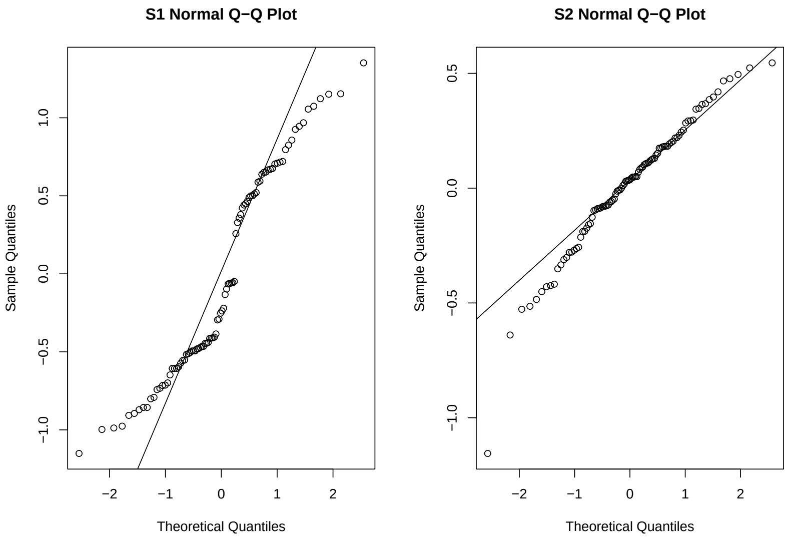


Figure 3: Q-Q plots for Normality of Residuals of S1 and S2 Method

Checking Assumptions

```r
# Box-Pierce Version of iid Sequence test
Box.test(S1.Residuals,type = "Box-Pierce", lag = 1)
```

```
##
##  Box-Pierce test
##
## data:  S1.Residuals
## X-squared = 13.004, df = 1, p-value = 0.0003109
```

```r
Box.test(S2.Residuals,type = "Box-Pierce", lag = 1)
```

```
##
##  Box-Pierce test
##
## data:  S2.Residuals
## X-squared = 18.228, df = 1, p-value = 1.96e-05
```

Forecast of 2023

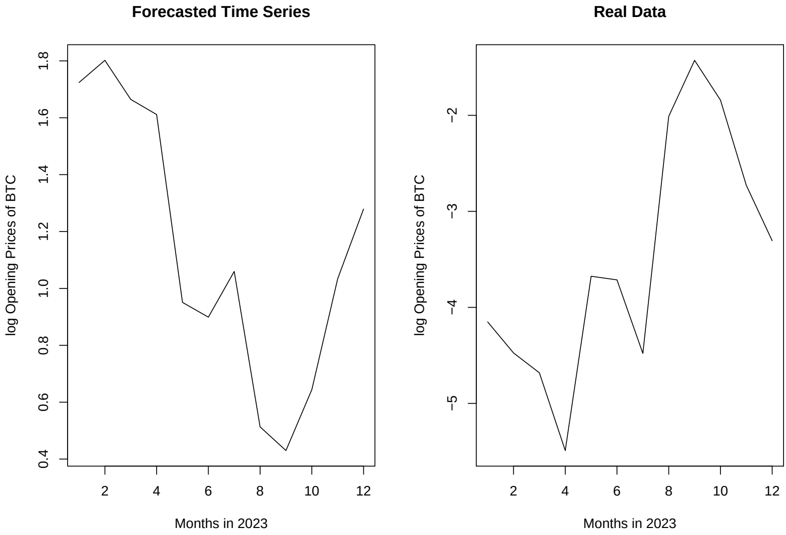


Figure 4: 12 Month Forecast of Monthly BTC/USD Prices in 2023

Conclusion

- Poor performance, linear time series model is insufficient or poorly specified for using on this kind of data set
- Stationary assumption and normality assumption violated
- Intuitively the assumption of the model that $s_t = s_{t+d}$ may be too strong
- Classical decomposition being unsuitable to forecast accurately monthly prices of Bitcoin

$$X_t = m_t + s_t + Y_t, \;\; t = 1, \ldots n, \;\; \text{where } , \mathbf{E}[Y_t] = 0$$

Questions?

Thank you for listening.