$$d = \frac{\left|(\vec{r}_c - \vec{r}_1) \wedge (\vec{r}_c - \vec{r}_2)\right|}{\left|\vec{r}_2 - \vec{r}_1\right|}$$

gradient

$$\begin{pmatrix} x_k \\ y_k \\ m_k \end{pmatrix} \quad \underbrace{\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} \begin{pmatrix} a_k \\ b_k \\ c_k \end{pmatrix}}$$

$$\vec{r}_c \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} \quad \underset{\vec{r}_1}{\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix}} - \underset{\vec{r}_2}{\begin{pmatrix} x_k + a_k \\ y_k + b_k \\ z_k + c_k \end{pmatrix}}$$

$$d^2 = \frac{\left[(x_k - x_c)^2 + (y_k - y_c)^2 + (z_k - z_c)^2\right]\cancel{(\quad)}}{\cancel{(a_k^2 + b_k^2 + c_k^2)}}$$

$$\vec{r}_2 \quad \sqrt{a_k^2 + b_k^2 + c_k^2}$$

$$- \frac{\left[a_k(x_k - x_c) + b_k(y_k - y_c) + c_k(z_k - z_c)\right]^2}{(\quad)}$$

$$\left|\vec{r}_1 - \vec{r}_c\right|^2$$

$$=$$

$$\vec{r}_1 \vec{r}_2 - \left|\vec{r}_1\right|^2 - \vec{r}_c \vec{r}_2 + \vec{r}_c \vec{r}_1$$

$$\begin{pmatrix} x_k - x_c \\ y_k - y_c \\ z_k - z_c \end{pmatrix} \cdot \begin{pmatrix} a_k \\ b_k \\ c_k \end{pmatrix} = \Big[a_k(x_k - x_c) + b_k(y_k - y_c) + c_k(z_k - z_c)\Big]^2$$

$$d^2 = x_k^2 - 2x_k x_c + x_c^2 + y_k^2 - 2y_k y_c + y_c^2 + z_k^2 - 2z_k z_c + z_c^2$$

$$- \frac{a_k^2(x_k^2 - 2x_k x_c + x_c^2) + b_k^2(y_k^2 - 2y_k y_c + y_c^2) + c_k^2(z_k^2 - 2z_k z_c + z_c^2)}{a_k^2 + b_k^2 + c_k^2}$$

$$- \frac{2a_k b_k (x_k y_k - x_c y_k - y_c x_k + x_c y_c)}{a_k^2 + b_k^2 + c_k^2}$$

$$- \frac{2a_k c_k (x_k z_k - x_c z_k - z_c x_k + x_c z_c)}{a_k^2 + b_k^2 + c_k^2}$$

$$- \frac{2b_k c_k (y_k z_k - y_c z_k - z_c y_k + y_c z_c)}{a_k^2 + b_k^2 + c_k^2}$$

$$\sum d_k^2 w_k = \chi^2 \qquad \partial_{x_c}\chi^2 = 0 \qquad \partial_{y_c}\chi^2 = 0 \qquad \partial_{z_c}\chi^2 = 0$$

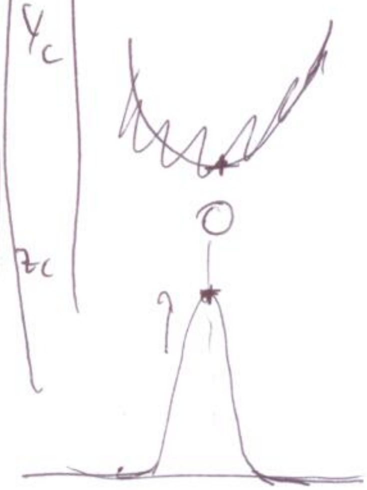
$$0 = \sum_k w_k \left(-\cancel{2} x_k + \cancel{2} x_c + \frac{\cancel{2} a_k^2 x_k - \cancel{2} a_k^2 x_c}{m_k^2} + \frac{\cancel{2} a_k c_k z_k - \cancel{2} a_k c_k z_c}{m_k^2} + \frac{\cancel{2} a_k b_k y_k - \cancel{2} a_k b_k y_c}{m_k^2} \right)$$

$$0 = \cancel{\sum_k} x_c \left(\sum_k w_k \left(1 - \frac{a_k^2}{m_k^2} \right) \right) + \sum_k w_k \left(-x_k + \frac{a_k^2 x_k}{m_k^2} + \frac{a_k c_k (z_k - z_c)}{m_k^2} + \frac{a_k b_k (y_k - y_c)}{m_k^2} \right)$$

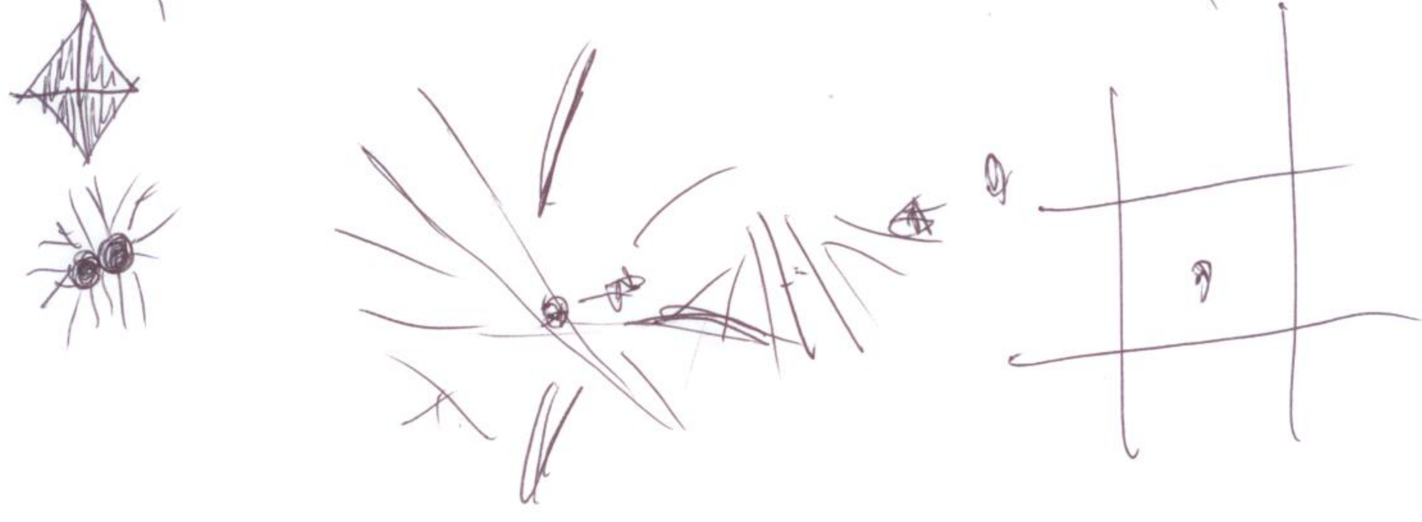
$$M \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = V \qquad \begin{pmatrix} \\ \end{pmatrix} = M^{-1} V$$

$$x_c \left(\sum_k w_k \left(1 - \frac{a_k^2}{m_k^2} \right) \right) + y_c \left(\frac{-a_k b_k}{m_k^2} \right) + z_c \left(\frac{-a_k c_k}{m_k^2} \right) = \sum_k w_k \left(x_k - \frac{a_k^2 x_k}{m_k^2} - \frac{a_k c_k z_k}{m_k^2} - \frac{a_k b_k y_k}{m_k^2} \right)$$

$$
\begin{pmatrix}
\sum w_k\left(1-\frac{a_k^2}{m_k^2}\right) & -\frac{wab}{m^2} & -\frac{wac}{m^2} \\
-\frac{wab}{m^2} & \sum w\left(1-\frac{b^2}{m^2}\right) & -\frac{wbc}{m^2} \\
-\frac{wac}{m^2} & -\frac{wbc}{m^2} & \sum w\left(1-\frac{c^2}{m^2}\right)
\end{pmatrix}
\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}
$$



$$
= \begin{pmatrix}
\sum w\left(x\left(1-\frac{a^2}{m^2}\right)\right) - \frac{wacz}{m^2} - \frac{waby}{m^2} \\
\sum w\left(y\left(1-\frac{b^2}{m^2}\right)\right) - \frac{waby}{m^2} - \frac{wbcz}{m^2} \\
\sum w\left(z\left(1-\frac{c^2}{m^2}\right)\right) - \frac{wacx}{m^2} - \frac{wbcy}{m^2}
\end{pmatrix}
$$



SCALING

$$\begin{pmatrix} x \\ u \\ v \end{pmatrix} \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

measured isotropic

Position

$$\underbrace{\frac{\partial I}{\partial u}}_{\text{measured}} = \frac{\partial I}{\partial y}\frac{\partial y}{\partial u} = \frac{1}{\alpha}\frac{\partial I}{\partial y}$$

Gradient

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1/\alpha \\ 1/\beta \end{pmatrix} \begin{pmatrix} s \\ n \\ t \end{pmatrix}$$

isotropic measured

$$d^2 = x^2 - 2xx_c + x_c^2 + \alpha^2\left(u^2 + 2uu_c + u_c^2\right) + \beta^2\left(v^2 + 2vv_c + v_c^2\right)$$

$$\cancel{= s^2\left(x^2 - 2xx_c + x_c^2\right) + n^2\left(y^2 + 2yy_c + y_c^2\right) + t^2(}$$

$$- \frac{s^2\left(x^2 - 2xx_c + x_c^2\right) + n^2\left(u^2 - 2uu_c + u_c^2\right) + t^2\left(v^2 - 2vv_c + v_c^2\right)}{s^2 + \frac{n^2}{\alpha^2} + \frac{t^2}{\beta^2}}$$

$$- \frac{2\,sn\left(xy - x_c y - y_c x + x_c y_c\right)}{s^2 + \frac{n^2}{\alpha^2} + \frac{t^2}{\beta^2}} - \frac{2\,st\left(xv - x_c v - xv_c + x_c v_c\right)}{s^2 + \frac{n^2}{\alpha^2} + \frac{t^2}{\beta^2}}$$

$$- \frac{2\,nt\left(uv - u_c v - uv_c + u_c v_c\right)}{s^2 + \frac{n^2}{\alpha^2} + \frac{t^2}{\beta^2}}$$