## Euribor (January 1994 - October 2024) Univariate DL TSA

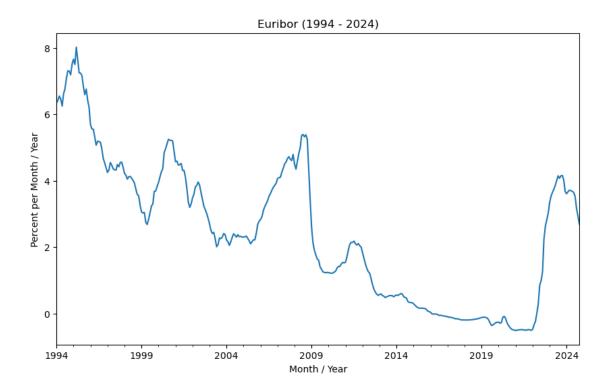
## November 20, 2024

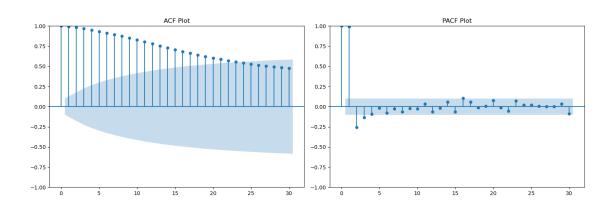
```
[1]: #Long Short-Term Memory Model
    import os
    import random
    import numpy as np
    import pandas as pd
    import matplotlib.pyplot as plt
    import statsmodels.api as sm
    from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
    from sklearn.preprocessing import MinMaxScaler
    from tensorflow.keras.models import Sequential
    from tensorflow.keras.layers import LSTM, Dense, Input
    from tensorflow.keras.callbacks import EarlyStopping
    from sklearn.metrics import mean_squared_error
    random.seed(42)
    new_directory = 'C:/Users/artem/Desktop'
    os.chdir(new_directory)
    euribor_data = pd.read_csv('ECB Data Portal_20241114135109.csv')
    euribor_data.rename(columns={'Euribor 1-year - Historical close, average of_
      ⇔observations through period (FM.M.U2.EUR.RT.MM.EURIBOR1YD_.HSTA)':⊔
     columns_to_remove = ['DATE', 'TIME PERIOD']
    euribor_data = euribor_data.drop(columns=columns_to_remove)
    dates = pd.date_range(start='1994', periods=len(euribor_data), freq='M')
    euribor_data.index = dates
    plt.figure(figsize=(10, 6))
    euribor_data['Euribor'].plot()
```

```
plt.title('Euribor (1994 - 2024)')
plt.xlabel('Month / Year')
plt.ylabel('Percent per Month / Year')
plt.show()
def acf_pacf_fig(series, both=True, lag=30):
   fig, axes = plt.subplots(1, 2 if both else 1, figsize=(15, 5))
   if both:
       plot_acf(series, lags=lag, ax=axes[0])
       plot_pacf(series, lags=lag, ax=axes[1])
        axes[0].set_title('ACF Plot')
        axes[1].set_title('PACF Plot')
   else:
       plot_acf(series, lags=lag, ax=axes[0])
        axes[0].set_title('ACF Plot')
   plt.tight_layout()
   plt.show()
acf_pacf_fig(euribor_data['Euribor'], both=True, lag=30)
train = euribor_data.iloc[:300]
valid start = euribor data.index[300]
test_start = euribor_data.index[324]
valid = euribor_data[(euribor_data.index >= valid_start) & (euribor_data.index_
test = euribor_data[test_start:]
scaler = MinMaxScaler()
train.loc[:, 'Euribor'] = scaler.fit_transform(train[['Euribor']])
valid.loc[:, 'Euribor'] = scaler.transform(valid[['Euribor']])
test.loc[:, 'Euribor'] = scaler.transform(test[['Euribor']])
T = 12
HORIZON = 3
def prepare_shifted_data(data, T, horizon):
   data_shifted = data.copy()
   y_columns = []
   for h in range(1, horizon + 1):
        data_shifted[f'y_t+{h}'] = data_shifted['Euribor'].shift(-h)
        y_columns.append(f'y_t+{h}')
   for t in range(1, T + 1):
```

```
data_shifted[f'Euribor_t-{t}'] = data_shifted['Euribor'].shift(t)
   data_shifted.dropna(inplace=True)
   y = np.array(data_shifted[y_columns])
   X = np.array(data_shifted[[f'Euribor_t-{t}' for t in range(1, T + 1)]])
   return X.reshape(X.shape[0], T, 1), y
X_train, y_train = prepare_shifted_data(train, T, HORIZON)
X_valid, y_valid = prepare_shifted_data(valid, T, HORIZON)
X_test, y_test = prepare_shifted_data(test, T, HORIZON)
print(f"X_train shape: {X_train.shape}, y_train shape: {y_train.shape}")
latent_dim = 6
batch_size = 16
epochs = 50
model = Sequential()
model.add(Input(shape=(X_train.shape[1], X_train.shape[2])))
model.add(LSTM(latent_dim, return_sequences=True))
model.add(LSTM(units=64, return sequences=False))
model.add(Dense(HORIZON))
model.compile(optimizer='adam', loss='mean_squared_error')
LSTM_earlystop = EarlyStopping(monitor='val_loss', patience=5)
model_fit = model.fit(
   X_train, y_train,
   batch_size=batch_size,
   epochs=epochs,
   validation_data=(X_valid, y_valid),
    callbacks=[LSTM earlystop],
   verbose=1
preds = model.predict(X_test)
evdta = pd.DataFrame(preds, columns=[f't+{t}' for t in range(1, 4)])
predicted_values = evdta[['t+1', 't+2', 't+3']].values
```

```
scaled_predicted_values = scaler.inverse_transform(predicted_values)
evdta[['t+1', 't+2', 't+3']] = scaled_predicted_values
evdta['Average Prediction'] = evdta[['t+1', 't+2', 't+3']].mean(axis=1)
evdta.columns = [f'November 2024' if col == 't+1' else
                 f'December 2024' if col == 't+2' else
                 f'January 2025' if col == 't+3' else
                 'Average Prediction' if col == 'Average Prediction' else col
                 for col in evdta.columns]
evdta = evdta.drop(columns=['actual'], errors='ignore')
print(evdta[['November 2024', 'December 2024', 'January 2025', 'Average_
 →Prediction']])
y_test_flattened = y_test.reshape(-1, 1)
avg_nov_2024 = evdta['November 2024'].mean()
avg dec 2024 = evdta['December 2024'].mean()
avg_jan_2025 = evdta['January 2025'].mean()
print(f"Average Prediction for November 2024 (t+1): {avg_nov_2024}")
print(f"Average Prediction for December 2024 (t+2): {avg_dec_2024}")
print(f"Average Prediction for January 2025 (t+3): {avg_jan_2025}")
mse_avg = mean_squared_error(y_test_flattened[:len(evdta)], evdta['Average_\'
⇔Prediction'])
mse_t1 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['November_u
⇒2024'])
mse_t2 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['December_
mse_t3 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['Januaryu
 →2025'])
rmse_avg = np.sqrt(mse_avg)
rmse_t1 = np.sqrt(mse_t1)
rmse_t2 = np.sqrt(mse_t2)
rmse_t3 = np.sqrt(mse_t3)
print(f"Root Mean Squared Error for November 2024 (t+1): {rmse_t1}")
print(f"Root Mean Squared Error for December 2024 (t+2): {rmse_t2}")
print(f"Root Mean Squared Error for January 2025 (t+3): {rmse_t3}")
print(f"Root Mean Squared Error for Average Prediction: {rmse_avg}")
```





```
X_train shape: (285, 12, 1), y_train shape: (285, 3)
```

Epoch 1/50

18/18 6s 38ms/step - loss: 0.1186 - val\_loss: 0.0454

Epoch 2/50

0.0214 - val\_loss: 0.0159

Epoch 3/50

0.0120 - val\_loss: 0.0064

Epoch 4/50

```
Os 7ms/step - loss:
18/18
0.0076 - val_loss: 0.0018
Epoch 5/50
18/18
                  Os 8ms/step - loss:
0.0065 - val_loss: 7.7963e-04
Epoch 6/50
18/18
                  Os 10ms/step -
loss: 0.0070 - val_loss: 7.2944e-04
Epoch 7/50
18/18
                  Os 7ms/step - loss:
0.0077 - val_loss: 8.7984e-04
Epoch 8/50
18/18
                  Os 7ms/step - loss:
0.0073 - val_loss: 5.4222e-04
Epoch 9/50
18/18
                  Os 21ms/step -
loss: 0.0069 - val_loss: 6.8102e-04
Epoch 10/50
18/18
                  Os 12ms/step -
loss: 0.0065 - val_loss: 4.8734e-04
Epoch 11/50
                  Os 9ms/step - loss:
18/18
0.0056 - val_loss: 4.7078e-04
Epoch 12/50
18/18
                  Os 10ms/step -
loss: 0.0068 - val_loss: 5.3361e-04
Epoch 13/50
18/18
                  Os 10ms/step -
loss: 0.0063 - val_loss: 4.5304e-04
Epoch 14/50
18/18
                  Os 10ms/step -
loss: 0.0062 - val_loss: 3.9673e-04
Epoch 15/50
18/18
                  Os 9ms/step - loss:
0.0060 - val loss: 4.3649e-04
Epoch 16/50
18/18
                  Os 11ms/step -
loss: 0.0068 - val_loss: 4.0960e-04
Epoch 17/50
18/18
                  Os 9ms/step - loss:
0.0061 - val_loss: 4.1501e-04
Epoch 18/50
18/18
                  Os 11ms/step -
loss: 0.0054 - val_loss: 4.2010e-04
Epoch 19/50
18/18
                  Os 14ms/step -
loss: 0.0045 - val_loss: 4.4987e-04
1/1
                0s 486ms/step
```

```
0
             -0.390768
                             -0.492114
                                           -0.543599
                                                                 -0.475493
    1
             -0.389007
                             -0.490106
                                           -0.541366
                                                                 -0.473493
    2
                                                                 -0.449226
             -0.367369
                             -0.466700
                                           -0.513609
    3
                                           -0.470464
                                                                 -0.411446
             -0.333657
                             -0.430218
    4
             -0.266700
                             -0.357849
                                           -0.385228
                                                                 -0.336592
    5
             -0.165177
                             -0.248301
                                           -0.257169
                                                                 -0.223549
    6
              0.014270
                             -0.056222
                                           -0.034188
                                                                 -0.025380
    7
              0.201180
                              0.143381
                                            0.193654
                                                                  0.179405
    8
              0.412638
                              0.368099
                                            0.446346
                                                                  0.409028
    9
              0.772453
                              0.742941
                                            0.865622
                                                                  0.793672
    10
              1.172292
                              1.154593
                                             1.316570
                                                                  1.214485
    11
              1.566791
                                                                  1.623920
                              1.557704
                                             1.747267
    12
              1.945648
                              1.942766
                                             2.148731
                                                                  2.012382
    13
              2.327644
                              2.328063
                                             2.543222
                                                                  2.399643
    14
              2.681913
                              2.683908
                                             2.900262
                                                                  2.755361
    15
              2.989194
                              2.992518
                                             3.203084
                                                                  3.061599
    16
              3.248507
                              3.253194
                                             3.453704
                                                                  3.318468
    17
              3.469337
                              3.475132
                                            3.663762
                                                                  3.536077
    18
              3.663300
                              3.669397
                                             3.846273
                                                                  3.726323
    19
              3.835344
                              3.840982
                                             4.006635
                                                                  3.894320
    20
              3.948987
                              3.955858
                                             4.109219
                                                                  4.004688
    21
              4.045738
                              4.052542
                                             4.197060
                                                                  4.098446
    22
              4.119317
                              4.126641
                                             4.262957
                                                                  4.169638
    23
              4.146219
                              4.156830
                                             4.283431
                                                                  4.195493
    24
              4.093022
                              4.112257
                                            4.225563
                                                                  4.143614
    25
              4.028049
                              4.055340
                                             4.157753
                                                                  4.080381
    26
              3.978958
                              4.011707
                                             4.107177
                                                                  4.032614
    27
              3.942882
                              3.978914
                                            4.070634
                                                                  3.997477
    28
              3.908357
                              3.946501
                                             4.036530
                                                                  3.963796
    29
              3.875349
                              3.914528
                                            4.004820
                                                                  3.931566
    30
              3.842630
                              3.882115
                                            3.974246
                                                                  3.899664
    Average Prediction for November 2024 (t+1): 2.268301248550415
    Average Prediction for December 2024 (t+2): 2.2517552375793457
    Average Prediction for January 2025 (t+3): 2.3554482460021973
    Root Mean Squared Error for November 2024 (t+1): 2.6393624413216643
    Root Mean Squared Error for December 2024 (t+2): 2.6537827374118343
    Root Mean Squared Error for January 2025 (t+3): 2.771034934148278
    Root Mean Squared Error for Average Prediction: 2.687817033564091
[2]: #Gated Recurrent Units Model
     import os
     import random
     import numpy as np
     import pandas as pd
```

January 2025

Average Prediction

November 2024

December 2024

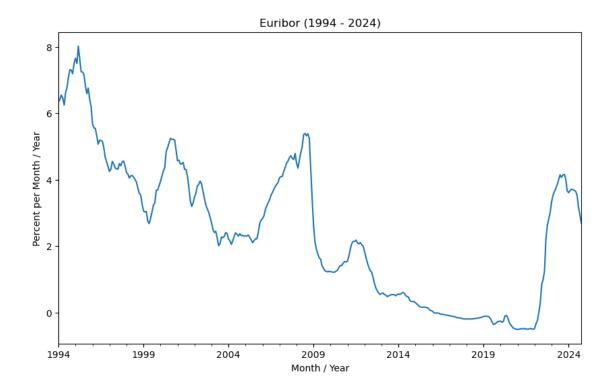
import matplotlib.pyplot as plt

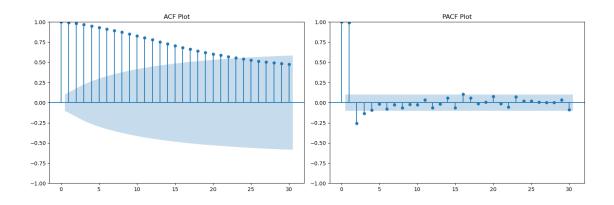
```
import statsmodels.api as sm
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from sklearn.preprocessing import MinMaxScaler
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import GRU, Dense, Input
from tensorflow.keras.callbacks import EarlyStopping
from sklearn.metrics import mean_squared_error
random.seed(42)
new_directory = 'C:/Users/artem/Desktop'
os.chdir(new_directory)
euribor_data = pd.read_csv('ECB Data Portal_20241114135109.csv')
euribor_data.rename(columns={'Euribor 1-year - Historical close, average of_
 ⇔observations through period (FM.M.U2.EUR.RT.MM.EURIBOR1YD_.HSTA)':⊔
 columns to remove = ['DATE', 'TIME PERIOD']
euribor_data = euribor_data.drop(columns=columns_to_remove)
dates = pd.date_range(start='1994', periods=len(euribor_data), freq='M')
euribor_data.index = dates
plt.figure(figsize=(10, 6))
euribor_data['Euribor'].plot()
plt.title('Euribor (1994 - 2024)')
plt.xlabel('Month / Year')
plt.ylabel('Percent per Month / Year')
plt.show()
def acf_pacf_fig(series, both=True, lag=30):
   fig, axes = plt.subplots(1, 2 if both else 1, figsize=(15, 5))
    if both:
       plot_acf(series, lags=lag, ax=axes[0])
       plot_pacf(series, lags=lag, ax=axes[1])
       axes[0].set_title('ACF Plot')
       axes[1].set_title('PACF Plot')
    else:
       plot_acf(series, lags=lag, ax=axes[0])
       axes[0].set_title('ACF Plot')
```

```
plt.tight_layout()
    plt.show()
acf_pacf_fig(euribor_data['Euribor'], both=True, lag=30)
train = euribor_data.iloc[:300]
valid_start = euribor_data.index[300]
test_start = euribor_data.index[324]
valid = euribor_data[(euribor_data.index >= valid_start) & (euribor_data.index_
 << test_start)]</pre>
test = euribor_data[test_start:]
scaler = MinMaxScaler()
train.loc[:, 'Euribor'] = scaler.fit_transform(train[['Euribor']])
valid.loc[:, 'Euribor'] = scaler.transform(valid[['Euribor']])
test.loc[:, 'Euribor'] = scaler.transform(test[['Euribor']])
T = 12
HORIZON = 3
def prepare_shifted_data(data, T, horizon):
    data_shifted = data.copy()
    y_columns = []
    for h in range(1, horizon + 1):
        data_shifted[f'y_t+{h}'] = data_shifted['Euribor'].shift(-h)
        y_columns.append(f'y_t+{h}')
    for t in range(1, T + 1):
        data_shifted[f'Euribor_t-{t}'] = data_shifted['Euribor'].shift(t)
    data_shifted.dropna(inplace=True)
    y = np.array(data_shifted[y_columns])
    X = np.array(data_shifted[[f'Euribor_t-{t}' for t in range(1, T + 1)]])
    return X.reshape(X.shape[0], T, 1), y
X_train, y_train = prepare_shifted_data(train, T, HORIZON)
X_valid, y_valid = prepare_shifted_data(valid, T, HORIZON)
X_test, y_test = prepare_shifted_data(test, T, HORIZON)
print(f"X_train shape: {X_train.shape}, y_train shape: {y_train.shape}")
latent_dim = 6
batch_size = 16
```

```
epochs = 50
model = Sequential()
model.add(Input(shape=(X_train.shape[1], X_train.shape[2])))
model.add(GRU(latent_dim, return_sequences=True))
model.add(GRU(units=64, return sequences=False))
model.add(Dense(HORIZON))
model.compile(optimizer='adam', loss='mean_squared_error')
GRU_earlystop = EarlyStopping(monitor='val_loss', patience=5)
model_fit = model.fit(
   X_train, y_train,
   batch_size=batch_size,
   epochs=epochs,
   validation_data=(X_valid, y_valid),
   callbacks=[GRU_earlystop],
   verbose=1
)
preds = model.predict(X_test)
evdta = pd.DataFrame(preds, columns=[f't+{t}' for t in range(1, 4)])
predicted_values = evdta[['t+1', 't+2', 't+3']].values
scaled_predicted_values = scaler.inverse_transform(predicted_values)
evdta[['t+1', 't+2', 't+3']] = scaled_predicted_values
evdta['Average Prediction'] = evdta[['t+1', 't+2', 't+3']].mean(axis=1)
evdta.columns = [f'November 2024' if col == 't+1' else
                 f'December 2024' if col == 't+2' else
                 f'January 2025' if col == 't+3' else
                 'Average Prediction' if col == 'Average Prediction' else col
                 for col in evdta.columns]
evdta = evdta.drop(columns=['actual'], errors='ignore')
print(evdta[['November 2024', 'December 2024', 'January 2025', 'Average_
 ⇔Prediction']])
```

```
y_test_flattened = y_test.reshape(-1, 1)
avg_nov_2024 = evdta['November 2024'].mean()
avg_dec_2024 = evdta['December 2024'].mean()
avg_jan_2025 = evdta['January 2025'].mean()
print(f"Average Prediction for November 2024 (t+1): {avg_nov_2024}")
print(f"Average Prediction for December 2024 (t+2): {avg_dec_2024}")
print(f"Average Prediction for January 2025 (t+3): {avg_jan_2025}")
mse_avg = mean_squared_error(y_test_flattened[:len(evdta)], evdta['Average_u
 →Prediction'])
mse_t1 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['November_L
 →2024¹])
mse_t2 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['December_
mse_t3 = mean_squared_error(y_test_flattened[:len(evdta)], evdta['January__
 →2025'])
rmse_avg = np.sqrt(mse_avg)
rmse_t1 = np.sqrt(mse_t1)
rmse_t2 = np.sqrt(mse_t2)
rmse_t3 = np.sqrt(mse_t3)
print(f"Root Mean Squared Error for November 2024 (t+1): {rmse_t1}")
print(f"Root Mean Squared Error for December 2024 (t+2): {rmse t2}")
print(f"Root Mean Squared Error for January 2025 (t+3): {rmse_t3}")
print(f"Root Mean Squared Error for Average Prediction: {rmse_avg}")
```





```
X_train shape: (285, 12, 1), y_train shape: (285, 3)
Epoch 1/50
18/18 6s 41ms/step -
```

18/18 6s 41ms/step - loss: 0.1222 - val\_loss: 0.0391

Epoch 2/50

Epoch 3/50

Epoch 4/50

```
18/18
                  Os 10ms/step -
loss: 0.0106 - val_loss: 9.0887e-04
Epoch 5/50
                  Os 9ms/step - loss:
18/18
0.0119 - val_loss: 3.6561e-04
Epoch 6/50
18/18
                  Os 16ms/step -
loss: 0.0115 - val_loss: 3.0521e-04
Epoch 7/50
18/18
                  Os 9ms/step - loss:
0.0088 - val_loss: 3.4999e-04
Epoch 8/50
18/18
                  Os 11ms/step -
loss: 0.0085 - val_loss: 3.3910e-04
Epoch 9/50
18/18
                  Os 9ms/step - loss:
0.0085 - val_loss: 5.0409e-04
Epoch 10/50
18/18
                  Os 12ms/step -
loss: 0.0078 - val_loss: 4.3261e-04
Epoch 11/50
                  Os 16ms/step -
18/18
loss: 0.0081 - val_loss: 7.7090e-04
                1s 551ms/step
    November 2024 December 2024
                                   January 2025 Average Prediction
0
        -0.745658
                        -0.730921
                                       -0.721642
                                                            -0.732740
1
        -0.744282
                        -0.729317
                                       -0.719936
                                                            -0.731178
2
        -0.734205
                        -0.719783
                                       -0.709674
                                                            -0.721221
3
        -0.716583
                        -0.703471
                                       -0.692342
                                                            -0.704132
4
        -0.679347
                        -0.668983
                                       -0.656007
                                                            -0.668112
5
        -0.617557
                        -0.611570
                                       -0.596003
                                                            -0.608377
6
        -0.506603
                        -0.507993
                                       -0.488355
                                                            -0.500984
7
        -0.369560
                        -0.379378
                                       -0.355968
                                                            -0.368302
8
                                       -0.189010
        -0.195975
                        -0.215553
                                                            -0.200180
9
                                        0.068481
         0.072771
                         0.039125
                                                             0.060126
10
         0.401500
                         0.353028
                                        0.381571
                                                             0.378700
11
         0.771339
                         0.709771
                                        0.732881
                                                             0.737997
12
         1.169519
                         1.097743
                                        1.111009
                                                             1.126091
13
         1.591260
                         1.511859
                                        1.511304
                                                             1.538141
14
         2.011276
                         1.927144
                                        1.909200
                                                             1.949207
15
                                                             2.337449
         2.406398
                         2.320913
                                        2.285037
16
         2.762033
                         2.677352
                                        2.624202
                                                             2.687862
17
         3.072652
                         2.990239
                                        2.921999
                                                             2.994963
18
         3.339995
                         3.260576
                                        3.178306
                                                             3.259625
19
         3.560955
                         3.486130
                                        3.397637
                                                             3.481574
20
         3.720880
                         3.649873
                                        3.561971
                                                             3.644241
21
         3.850365
                         3.779558
                                        3.687975
                                                             3.772633
```

3.789582

3.876668

3.884954

22

3.955470

23	4.026760	3.957835	3.861346	3.948647
24	4.048717	3.983481	3.888854	3.973684
25	4.047154	3.986150	3.893045	3.975450
26	4.034515	3.978023	3.886957	3.966499
27	4.011814	3.959972	3.872726	3.948170
28	3.977286	3.929852	3.848495	3.918545
29	3.934641	3.890621	3.816166	3.880476
30	3.888546	3.846554	3.778188	3.837763
Average	Prediction	for November 2024	(t+1): 1.914	43894910812378

Average Prediction for November 2024 (t+1): 1.9143894910812378

Average Prediction for December 2024 (t+2): 1.8694769144058228

Average Prediction for January 2025 (t+3): 1.8347738981246948

Root Mean Squared Error for November 2024 (t+1): 2.492480994575092 Root Mean Squared Error for December 2024 (t+2): 2.442061149004987 Root Mean Squared Error for January 2025 (t+3): 2.38390893399653 Root Mean Squared Error for Average Prediction: 2.439454710689772

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