Advanced Solvers for Numerical PDEs
Homework 2
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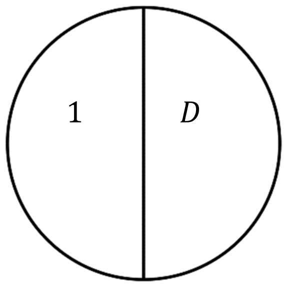
A dipole source is injecting direct current in the center of a heterogenous conductive unit disk. The electric potential $u(x, y)$ in this case satisfies the diffusion equation,

$$-div(\sigma(x, y)\nabla u) = f \quad in\ \Omega.$$

Further assume the disk is insulated. The conductivity is defined as follows,

$$\sigma(x, y) = \begin{cases} 1, & x < 0 \\ D, & x > 0 \end{cases}$$

$$D = 1 \text{ or } 100$$



$\sigma(x, y)$

Also use the following smooth representation for the dipole source in the right-hand side,

$$f(x, y) = exp(-100[(x + 0.1)^2 + y^2]) - exp(-100[(x - 0.1)^2 + y^2]).$$

Discretize this problem with P1 elements on a sequence of three appropriate triangular grids (100 to 100'000 vertices).

Assess the following solvers,
* The conjugate gradient method with SOR preconditioner
* The conjugate gradient method with Incomplete Cholesky factorization
* The conjugate gradient method with multigrid preconditioner
* Default solver of FEniCS.

Study the impact of D on the performance.

Present the results in a table (CPU time, iteration count, memory usage versus grid size and D). Comment the results. Plot the electric potential and induced currents, $j = -\sigma\nabla u$, for one of the grids.