Advanced Solvers for Numerical PDEs

Homework 1

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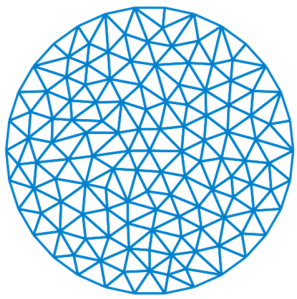
Consider a function of two variables in the unit disk, Ω,

$$u(x,y) = \ln(0.1 + x^2 + y^2)$$

In FEniCS, generate an unstructured triangular grid.



Solve the following boundary value problem,

$$-\Delta u = f \text{ in } \Omega,$$
$$u = u_0 \text{ on } \Gamma.$$

- Use five grids with different resolution, from roughly 100 to 1'000'000 vertices.
- Use Lagrange finite elements of the 1st and 2nd order.
- Study the error of the solution and its gradient,

$$\|u - u_h\|^2 = \int_\Omega (u(x,y) - u_h(x,y)^2 dxdy$$

$$\|\nabla u - \nabla u_h\|^2 = \int_\Omega (\nabla u(x,y) - \nabla u_h(x,y)^2 dxdy$$

- Record solution time.

Make a table:

Grid resolution	P1 Element	P1 Element	P1 Element	P2 Element	P2 Element	P2 Element
	$\|\|u - u_h\|\|$	$\|\|\nabla u - \nabla u_h\|\|$	CPU time	$\|\|u - u_h\|\|$	$\|\|\nabla u - \nabla u_h\|\|$	CPU time

For one of the grids, plot exact solution, u, numerical solution, u_h, solution error distribution, $u - u_h$, gradient error distribution, $|\nabla u - \nabla u_h|$. Comment the results. What would you suggest to improve modeling accuracy?