Advanced Solvers for Numerical PDEs

Homework 3

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Consider the 2D Burgers' equation for $u(x, y, t)$ and $\mathrm{v}(x, y, t)$,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} - \nu\Delta u = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial \mathrm{v}}{\partial x} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial y} - \nu\Delta \mathrm{v} = 0$$

$$0 < x < 1;\ 0 < y < 1$$

$$0 < t < 1$$

completed by the initial conditions,

$$u(x, y, 0) = \mathrm{v}(x, y, 0) = e^{-3x^2-3y^2}$$

and zero Neumann boundary conditions. Assume the viscosity, ν, is 1e-2 (or other).

Generate a uniform triangular grid near 100x100 vertices in space and pick an appropriate time step, τ. Apply P1 finite-elements in space and the backward Euler scheme with the Newton's method in time (FEM-BE) and compute the FEM-BE solution within $0 < t < 0.5$. Let $N_t = 0.5/\tau$ be the number of time steps performed.

Next, compute the following two numerical solutions within the segment $0.5 < t < 1$:

- The FEM-BE solution, u_{FEM-BE}, v_{FEM-BE} , i.e. simply continue time-tepping.
- The DMD solution, u_{DMD-r}, v_{DMD-r}: use the data from previous N_t steps to predict later values. Check different SVD truncation ranks, r.

Make a table of L2 difference, $\|u_{FEM-BE}(\cdot,\cdot, t_k) - u_{DMD-r}(\cdot,\cdot, t_k)\|$ for $t_k = 0.6, 0.8, 1$ and appropriate ranks, r. Also record CPU time. Compare plots $u_{FEM-BE}(\cdot,\cdot, t_k)$ and $\mathrm{v}_{DMD-r}(\cdot,\cdot, t_k)$ Comment the results.