Приложение 3 / Appendix 3

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MASTER'S THESIS

Articulation Points in Multiplex Networks

Master's Educational Program: Advanced Computational Sciences

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Moscow 2022

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МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

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Москва 2022

Articulation Points in Multiplex Networks

Artem Vergazov

Submitted to the Skolkovo Institute of Science and Technology on April 21, 2023

ABSTRACT

In mathematical modeling, a concept of a network can represent many of the real-life complex systems. Many of the physical, technological, social, biological etc. phenomena can be described and simulated in terms of networks. Studying networks as a general mathematical object and coming up with tools of analyzing them is crucial.

Furthermore, more complex phenomena are better described by multiplex networks, rather than simple monoplex networks. A multiplex network is a collection of monoplex networks, or layers, where all the nodes exist simultaneously in all the layers but the links within each layer can differ. An obvious example of such system is a network of Facebook as one layer and a network of Twitter as the other. Nodes represent people participating in these networks, and links represent connections between them in each of the networks. Obviously, classical notion of a network would not be sufficient in describing such system.

One of the notions that come up while studying networks is a notion of an articulation point – a node whose removal disconnects the network. Despite their fundamental importance, a general framework of studying articulation points in complex networks is lacking. While articulation points distribution in simple monoplex networks has been studied well enough, this phenomenon in multiplex networks has not been addressed as thoroughly yet. The purpose of this work is to advance the existing theoretical results in the field and provide verification by simulation.

Keywords: complex networks, multiplex networks, articulation points, numerical simulation of complex networks

Research Advisor:
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INTRODUCTION

Many of the natural and artificial systems are represented efficiently by a notion of graphs, or networks. Thus, studying network science is important for understanding the underlying principles of behavior of complex systems. Particularly, it is important to study the articulation points (AP) distribution in graphs. An articulation point in a network is a node whose removal disconnects the network. Being able to find articulation points and understanding the patterns of their behavior can help both prevent crucial systems from destruction (for example, designing more resilient infrastructure networks) or find weaknesses in such systems [1].

There has been some research on articulation points behavior in monoplex networks. However, some phenomena are more naturally described by multiplex networks. A multiplex network is a collection of monoplex networks, or layers, where all the nodes exist simultaneously in all the layers but the links within each layer can differ [2]. An example of such system is a network of Facebook as one layer and a network of Twitter as the other. Nodes represent people participating in these networks, and links represent connections between them in each of the networks.

Many effects found in multiplex networks are not observed in monoplex networks. This also concerns articulation points distribution. For example, removal of one such point from a usual network simply disconnects it leaving two independent clusters. However, removal of an articulation point from a multiplex network can produce an iterative cascade of failures in several interdependent networks. Example of this was observed in 2003 in Italy when the shutdown of power stations directly led to the failure of nodes in the Internet communication network, which in turn caused further breakdown of power stations resulting in electrical blackout [3].

This area is relatively young and there is a necessity and a possibility for research of articulation points behavior in multiplex networks.

LITERATURE REVIEW

Reproducing results from papers on AP distribution in mono- and multiplex networks was helpful for understanding the terminology, common approaches, and best practices in these fields. The following is the review of the works considering various network science scenarios which will be of help in this research.

Articulation points in monoplex networks

In [1], authors develop equations governing the dependency of APs proportion in a random network on its parameters. For example, the analytical dependency of APs proportion on the mean degree parameter c in Erdős–Rényi network (ER network) was derived in the paper. For the purposes of this review, these results were tested numerically. One can see the comparison plot in the Figure 1. Authors also come up with strategies of network attack and network decomposition in which AP distributions play the major role. These results are of great importance for budget-limited network construction/destruction but are only applicable for monoplex networks. Generalization of these results to the multiplex networks case is one the goals of this work.

Multiplex networks

In [3], authors consider a cascade of failures scenario which occurs when failure of nodes in one layer leads to failures of dependent nodes in other layers. This leads to iterative process of failures which stops when only the giant connected component (GCC) is left. For this review, the dependency of the size of this final GCC on the fraction of nodes that are left p is obtained numerically and can be found on the Figure 2.

Literature Review Conclusions

Results of the papers listed above have been numerically verified. Comparisons of the plots obtained numerically and the analytical curves are presented in the figures below.

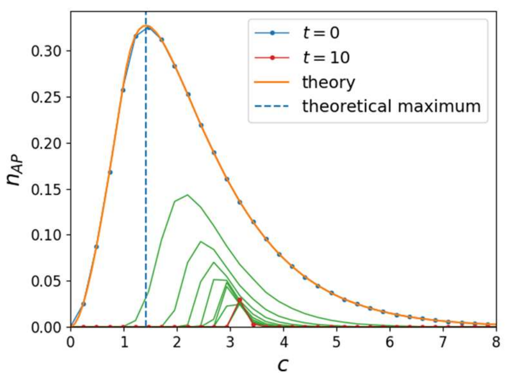

Figure 1. Verification of paper [1] results. Fraction of articulation points in ER network. c is the mean degree of the network nodes.

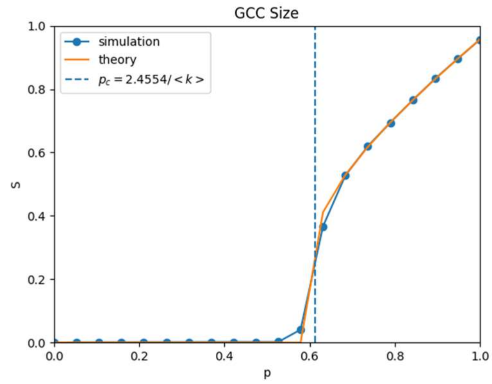

Figure 2. Verification of paper [3] results. Size of the GCC that is left after a cascade of node failures in a two-layer ER network. p is the fraction of nodes that are left after the first failure.

COMPUTATIONAL METHODOLOGY

The workflow can be described as

1. generation of a set of networks from a certain distribution
2. application of cascade of failures
3. obtaining AP metrics (fraction)

Simulations are done in vanilla C++ because of the extremely high computational cost of those simulations for large networks (millions of nodes). Python is used for prototyping and plotting.

TO BE DONE

RESULTS AND DISCUSSION

The following are the dependencies of the fraction of APs on the mean degree of the network for different random network models (ER and scale-free with different exponents) for different values of *t* which is the number of steps in the AP removal process. Essentially, these results are generalization of the results from [1] with application of cascade failure process from [3]. Scale-free networks are modeled as in [7-9].

ER monoplex and multiplex networks comparison

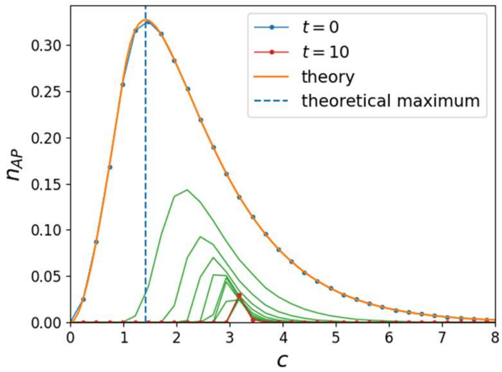

Figure 3. Verification of paper [1] results. Fraction of articulation points in ER network. c is the mean degree of the network nodes.

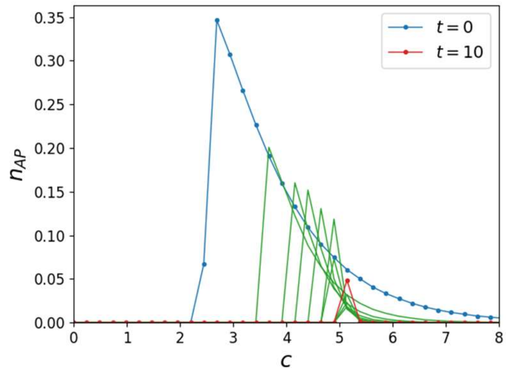

Figure 4. Fraction of APs in a two-layer ER network as function of the mean degree c.

Scale-free multiplex network with different exponents.

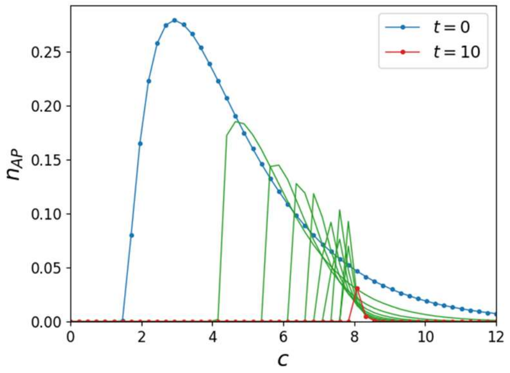

Figure 5. Fraction of APs in a two-layer scale-free network as function of the mean degree c.

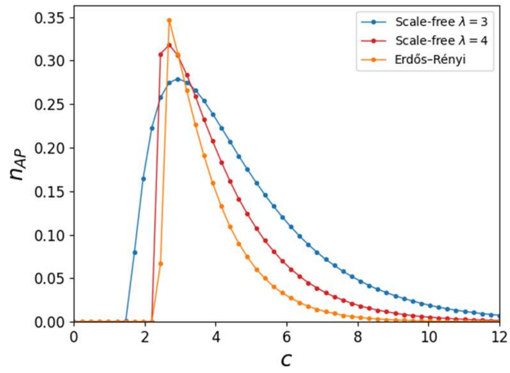

Figure 6. Scale-free networks with different degree exponents λ.

Subsection 3

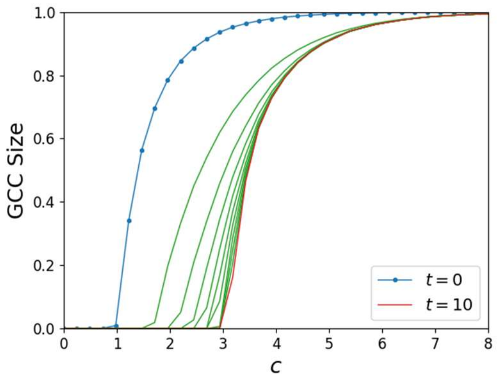

Figure 7. GCC size in a monoplex ER network as function of the mean degree c.

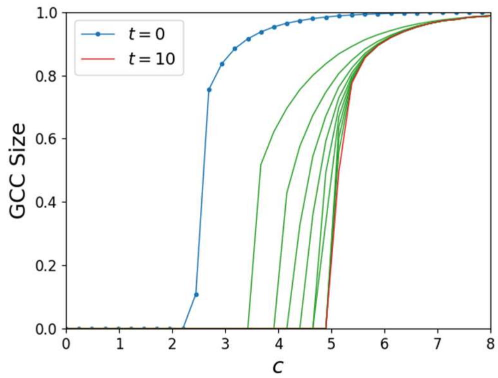

Figure 8. GCC size in a two-layer ER network as function of the mean degree c.

Discussion

From the differences in the peaks of the curves in Figures 3-5 and the differences in phase transitions coordinates in Figures 7-8, it follows that in multiplex networks phase transition is shifted to the higher values of c. This means multiplex networks require more connections to stay connected than monoplex networks do. This needs to be taken into account when building systems with complex underlying structure that can be described by a multiplex network.

CONCLUSIONS

TO BE DONE.

AUTHOR CONTRIBUTION

TBD

ACKNOWLEDGEMENTS

The author would like to thank Prof. Vladimir Palyulin and Dr. Saeed Osat for their invaluable help and advice.

ABBREVIATIONS

AP = Articulation Point

ER network/graph = Erdős–Rényi network/graph

GCC = Giant Connected Component

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APPENDIX

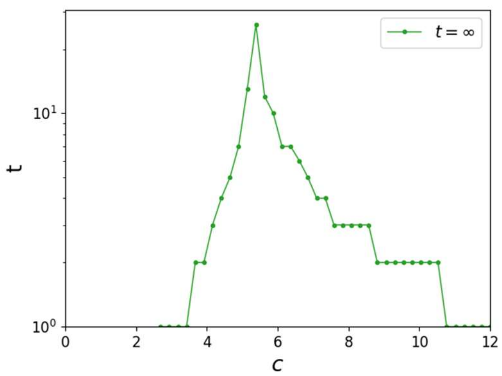

Figure 9. Total number of the AP removal steps as function of the mean degree *c* for ER two-layer multiplex network.

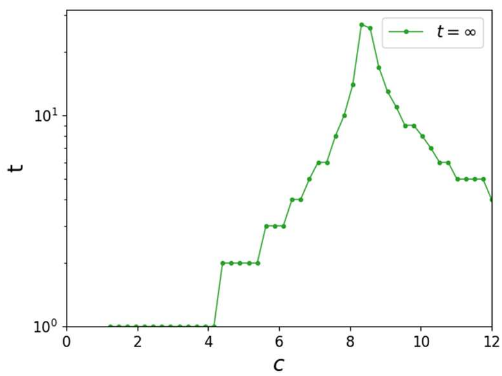

Figure 10. Total number of the AP removal steps as function of the mean degree *c* for scale-free two-layer multiplex network with the degree of the distribution 3.