Приложение 3 / Appendix 3

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MASTER'S THESIS

Articulation Points in Multiplex Networks

Master's Educational Program: Advanced Computational Sciences

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Приложение 3 / Appendix 3

Skolkovo Institute of Science and Technology

МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

Критические узлы в многослойных сетях

Магистерская образовательная программа: Современные вычислительные методы

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Articulation Points in Multiplex Networks

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ABSTRACT

In mathematical modeling, a concept of a network can represent many of the real-life complex systems. Many of the physical, technological, social, biological etc. phenomena can be described simulated in terms of networks. Studying networks as a general mathematical object and coming up with tools of analyzing them is crucial. One of the notions that come up while studying networks is a notion of an articulation point – a node whose removal disconnects the network.

Furthermore, some phenomena today are better described by multiplex networks, rather than usual networks. A multiplex network is a collection of monoplex networks, or layers, where all the nodes exist simultaneously in all the layers but the links within each layer can differ. An obvious example of such system is a network of Facebook as one layer and a network of Twitter as the other. Nodes represent people participating in these networks, and links represent connections between them in each of the networks. It is obvious that a classical notion of a network would not be enough to describe such system.

Despite their fundamental importance, a general framework of studying articulation points in complex networks is lacking. While articulation points distribution in simple monoplex networks has been studied and understood, this phenomenon in multiplex networks has not been addressed as thoroughly yet. The purpose of this work is to advance the existing theoretical results in the field and provide verification by simulation.

Keywords: complex networks, multiplex networks, articulation points, numerical simulation of complex networks

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INTRODUCTION

Many of the natural and artificial systems are represented efficiently by a notion of graphs, or networks. Thus, studying network science is very important for understanding the underlying principles of behavior of complex systems. Particularly, it is important to study the articulation points (AP) distribution in graphs. An articulation point in a network is a node whose removal disconnects the network. Being able to find articulation points and understanding the patterns of their behavior can help both prevent crucial systems from destruction (for example, designing more resilient infrastructure networks) or find weaknesses in such systems [1].

There has been some research on articulation points behavior in monoplex networks. However, some phenomena are more naturally described by multiplex networks. A multiplex network is a collection of monoplex networks, or layers, where all the nodes exist simultaneously in all the layers but the links within each layer can differ [2]. An example of such system is a network of Facebook as one layer and a network of Twitter as the other. Nodes represent people participating in these networks, and links represent connections between them in each of the networks.

Many effects found in multiplex networks are not observed in monoplex networks. This also concerns articulation points distribution. For example, removal of one such point from a usual network simply disconnects it leaving two independent clusters. However, removal of an articulation point from a multiplex network can produce an iterative cascade of failures in several interdependent networks. Example of this was observed in 2003 in Italy when the shutdown of power stations directly led to the failure of nodes in the Internet communication network, which in turn caused further breakdown of power stations resulting in electrical blackout [3].

This area is relatively young and there is a necessity and a possibility for research of articulation points’ behavior in multiplex networks.

LITERATURE REVIEW

Reproducing results from papers on AP distribution in mono- and multiplex networks was helpful for understanding the terminology, common approaches, and best practices in these fields. The following is the review of the works considering various network science scenarios in which articulation points are involved.

Articulation points in monoplex networks

In [1], authors develop equations governing the dependency of APs proportion in a random network on its parameters. For example, the analytical dependency of APs proportion on the mean degree parameter c in Erdős–Rényi network (ER network) was derived in the paper. For the purposes of this review, these results were tested numerically. One can see the comparison plot in the Figure 1. Authors also come up with strategies of a network attack and network decomposition in which AP distributions play the major role. These results are of great importance for budget-limited network construction/destruction but are only applicable for monoplex networks. Generalization of these results to the multiplex networks case is one the goals of this work.

[4][5][6] TO BE DONE.

Multiplex networks

In [2], TO BE DONE.

In [3], authors consider a cascade of failures scenario which occurs when failure of nodes in one layer leads to failures of dependent nodes in other layers. This leads to iterative process of failures which stops when only the giant connected component (GCC) is left. For this review, the dependency of the size of this final GCC on the fraction of nodes that are left p is obtained numerically and can be found on the Figure 2.

Literature Review Conclusions

Results of the papers listed above have been numerically verified. Comparisons of the plots obtained numerically and the analytical curves are presented in the figures below.

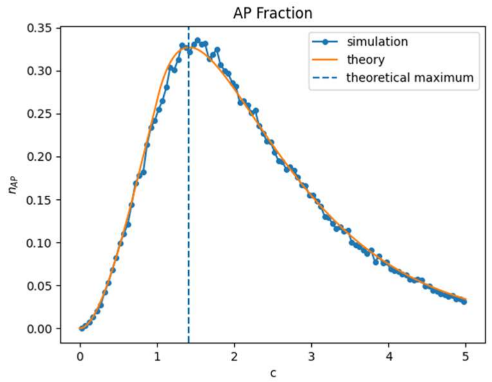

Figure 1. Verification of paper [1] results. Fraction of articulation points in ER network. c is the mean degree of the network nodes.

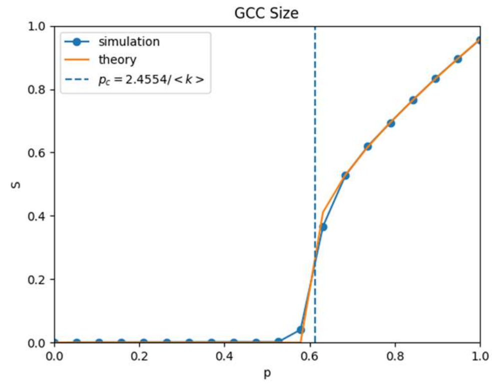

Figure 2. Verification of paper [3] results. Size of the GCC that is left after a cascade of node failures in a two-layer ER network. p is the fraction of nodes that are left after the first failure.

COMPUTATIONAL METHODOLOGY

The workflow can be described as obtaining analytical expressions for articulation points distribution and other parameters in different scenarios and verifying them experimentally via numerical simulations. Conducting analytical calculations and writing Python code for simulations are the main techniques.

Analytical derivations

TO BE DONE

Simulations in Python

For writing simulation code, NetworkX (https://networkx.org) Python library is used. After obtaining the analytical expression, it is verified via numerical simulations. According to the usual workflow, one needs to create an object of a network and calculate some of its properties that are of interest. For that purpose, it usually enough to use one of the function that NetworkX provides out of the box. For example, it has built-in ER network constructor. However, capabilities of NetworkX library are limited when it comes to multiplex networks simulations. According to the author's best knowledge, there are no well supported open-source Python libraries that would provide a framework for multilayer networks simulations.

For the purposes of this research, the following technique was developed to overcome this problem. A multiplex network is essentially a set of monoplex networks that are connected via the same nodes. To simulate such network, one can create multiple instances of ER networks with the same number of nodes via the built-in NetworkX ER constructor and add connections between the same nodes manually. This method is acceptable for the majority of task.

RESULTS AND DISCUSSION

TO BE DONE.

Subsection 1

TO BE DONE.

TO BE DONE.

Subsection 2

TO BE DONE.

TO BE DONE.

Subsection 3

TO BE DONE.

Discussion

TO BE DONE.

CONCLUSIONS

TO BE DONE.

INNOVATIONS

TO BE DONE

AUTHOR CONTRIBUTION

Author’s contribution is the Python code for simulations to verify the results of the papers discussed.

ACKNOWLEDGEMENTS

TO BE DONE.

ABBREVIATIONS

AP = Articulation Point

ER network/graph = Erdős–Rényi network/graph

GCC = Giant Connected Component

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