Advanced Computational Science

Articulation Points in Multiplex Networks

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Research Advisor: *Vladimir Palyulin*

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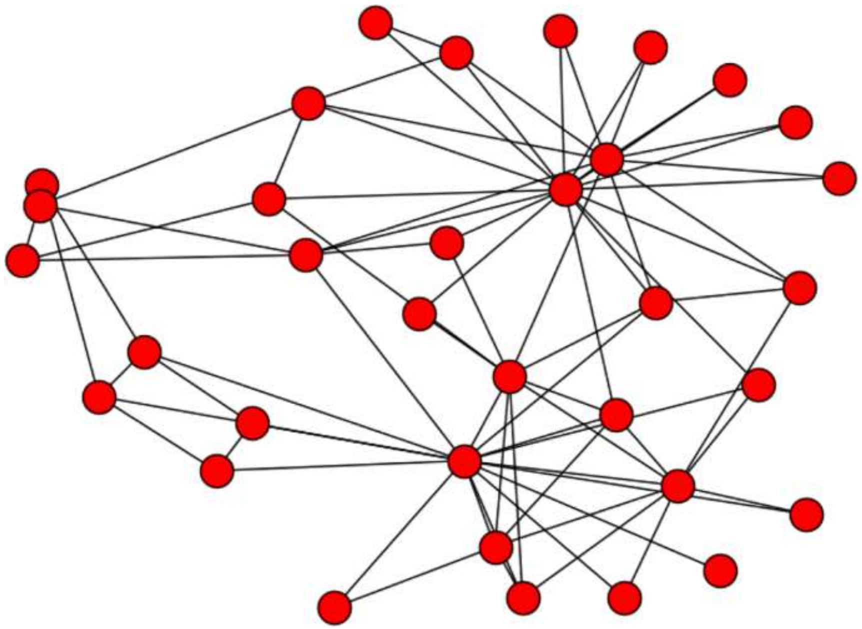
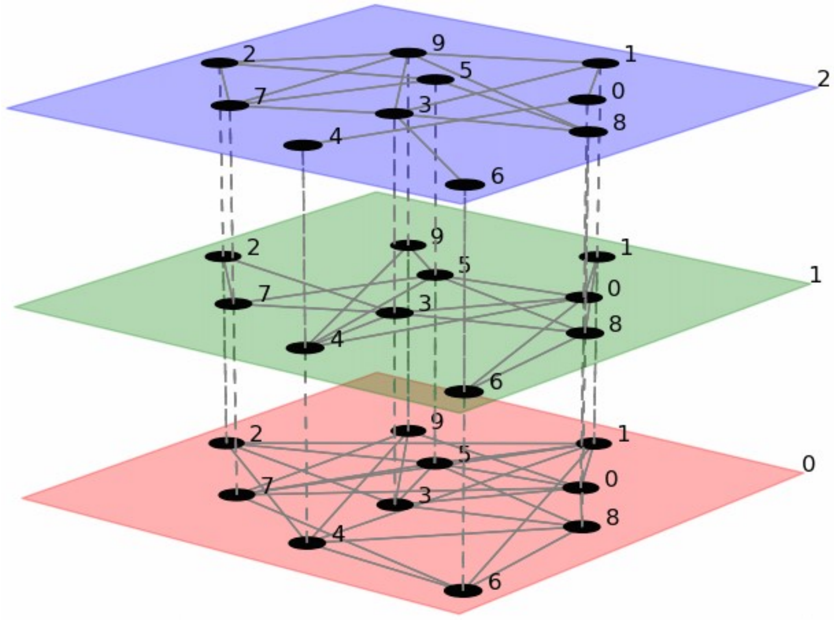
Introduction. Networks



Simple network



Multiplex network

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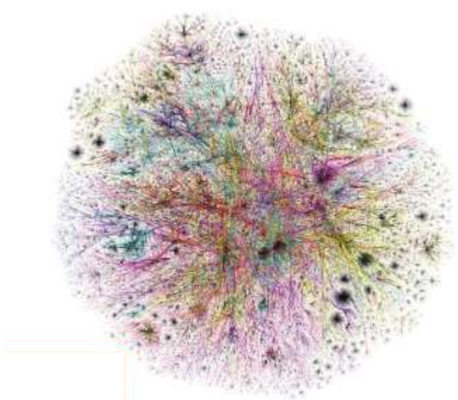
Introduction. Networks

Social Network

The topology of the Internet



Multiplex network examples

Transport
- Underground layer
- Bus layer

Social
- Twitter layer
- LinkedIn layer

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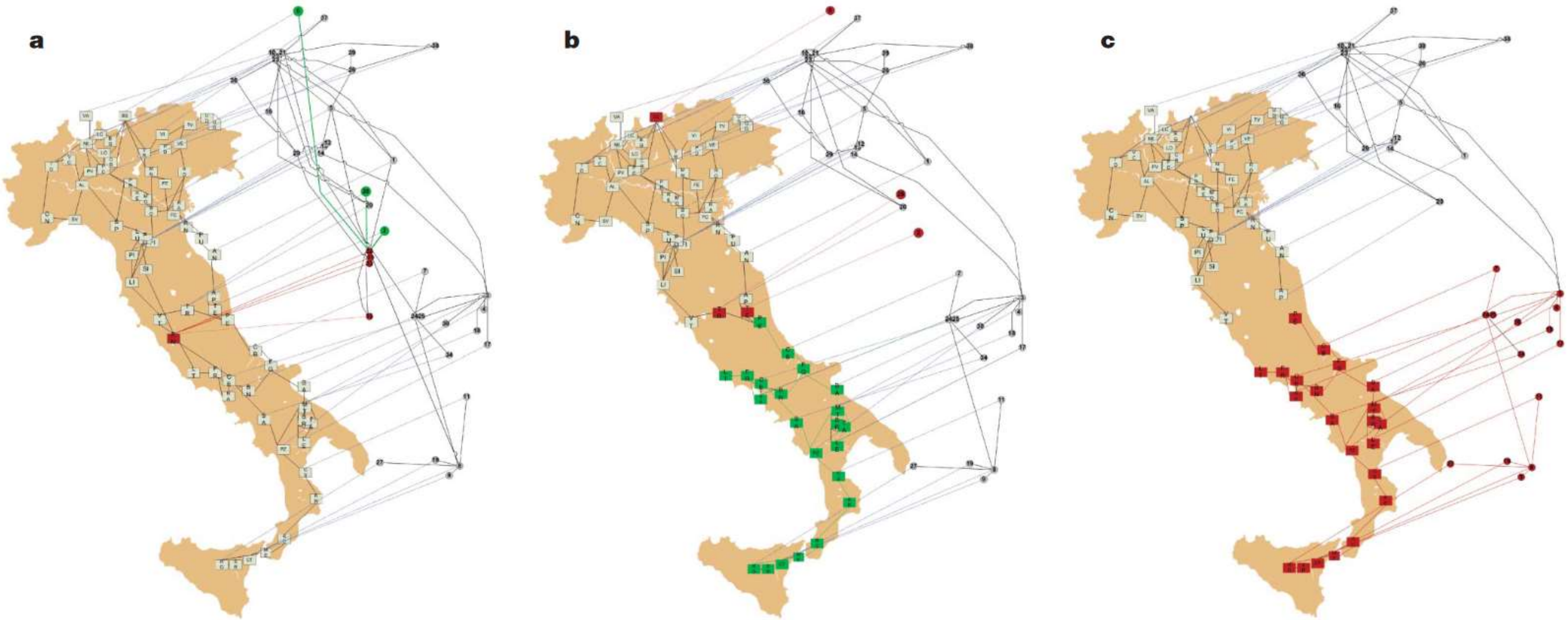
Introduction. Motivation

What defines stability of the network?



Sep 2003: cascade node failure

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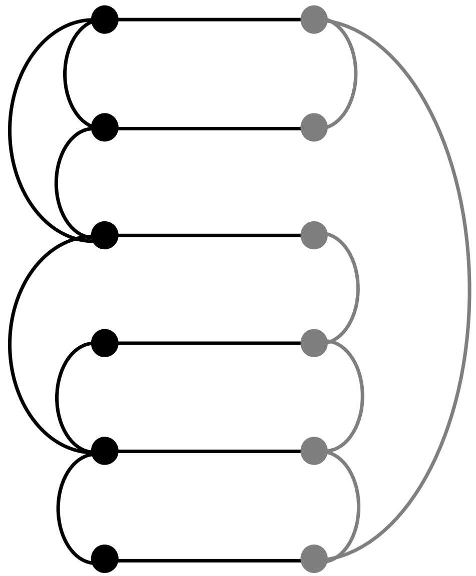
Introduction. Cascade failure of nodes



Power grids layer Computer servers layer

Review: Buldyrev, S., Parshani, R., Paul, G. et al. Catastrophic cascade of failures in interdependent networks. Nature 464, 1025–1028 (2010). https://doi.org/10.1038/nature08932

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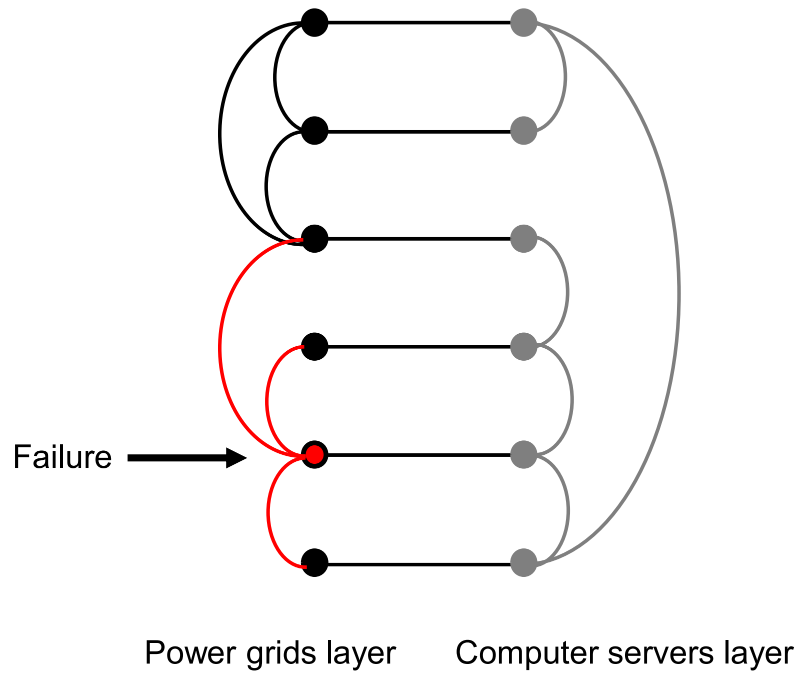

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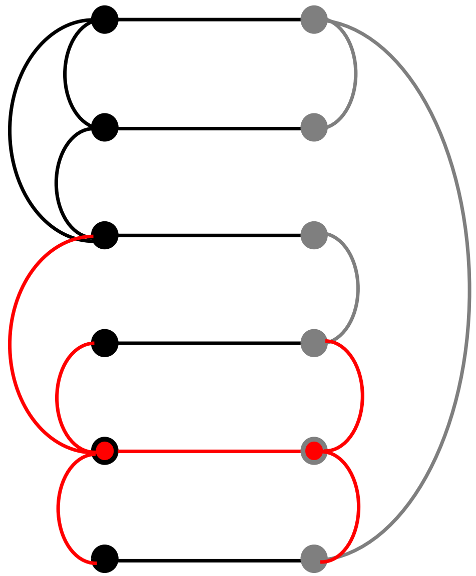

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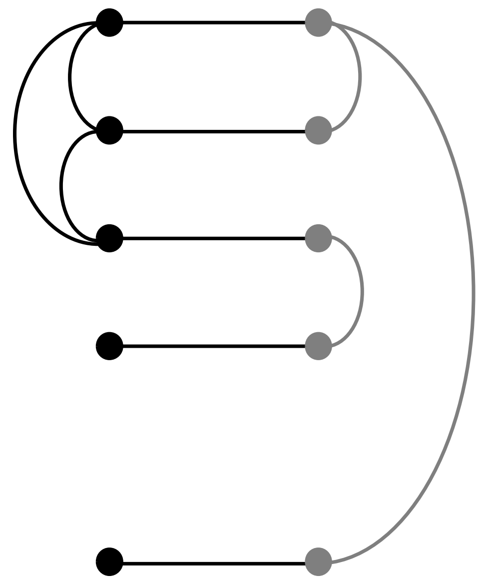

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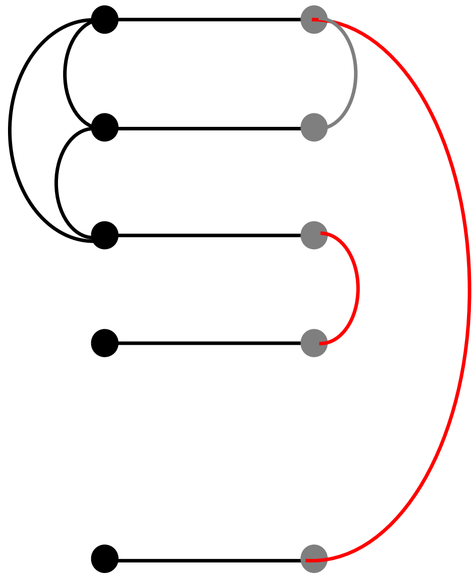

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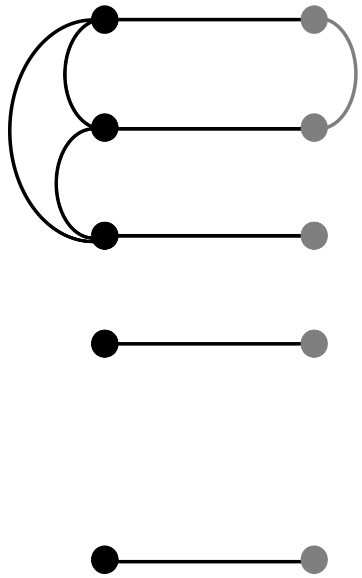

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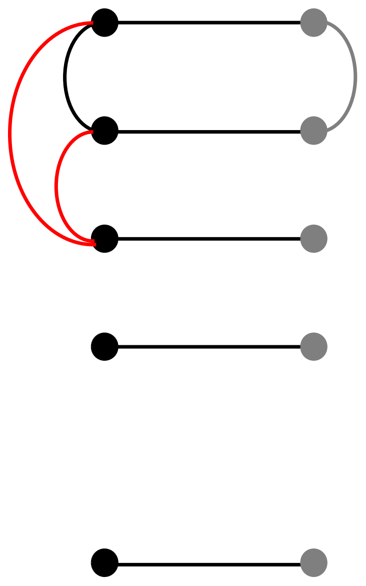

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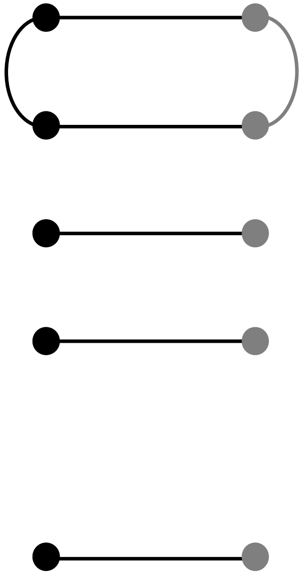

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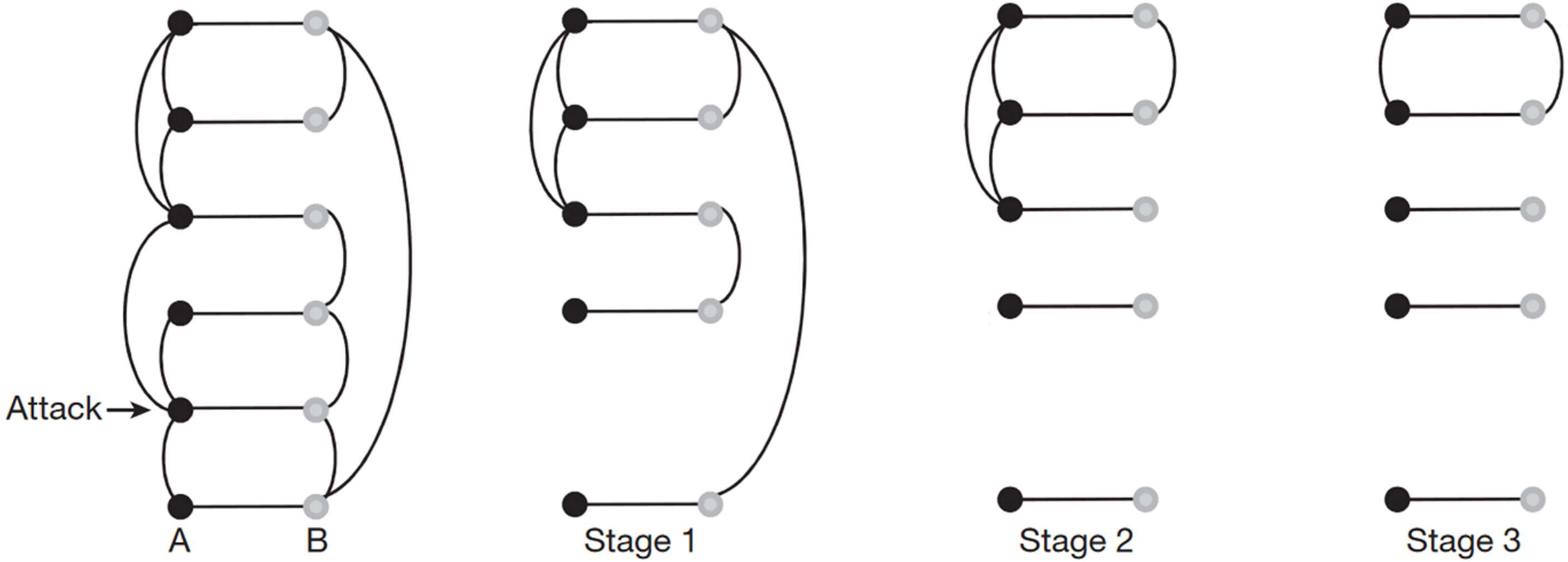

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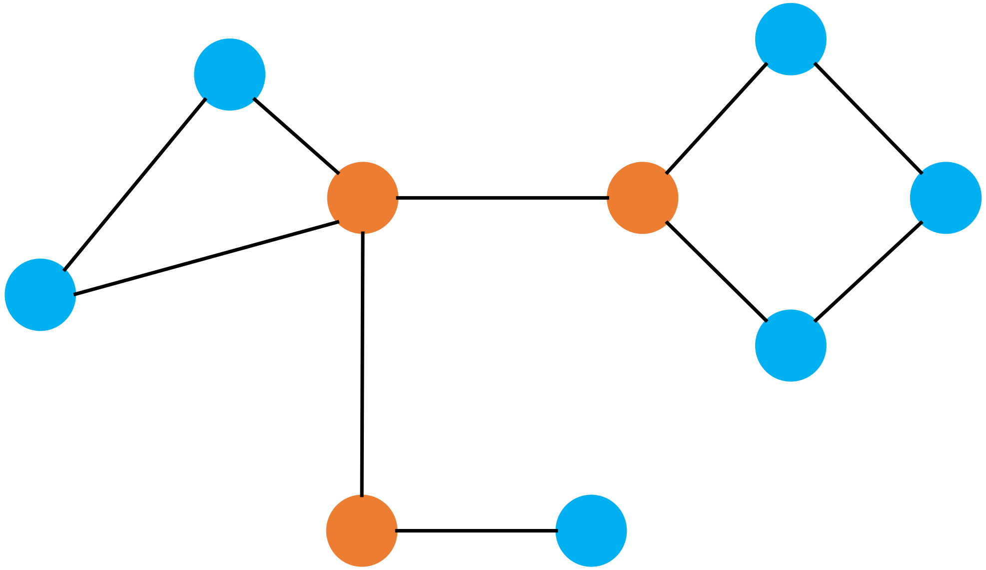
Introduction. Articulation points



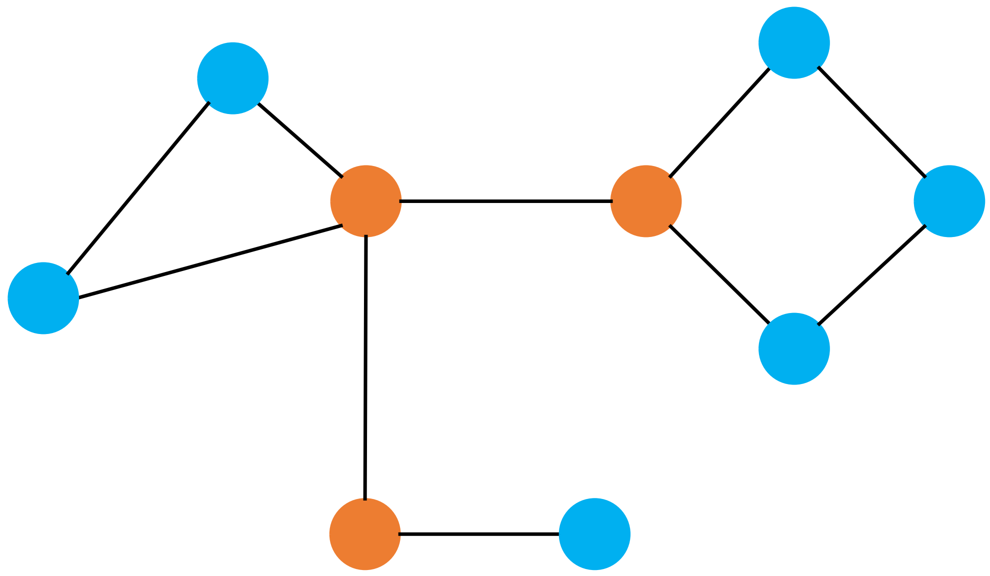
Articulation points (AP)

An articulation point (AP) is a node whose removal disconnects the graph

Review: Tian, L. et al. Articulation points in complex networks. Nat. Commun. 8, 14223 doi: 10.1038/ncomms14223 (2017).

Introduction. Articulation points removal

Step 0



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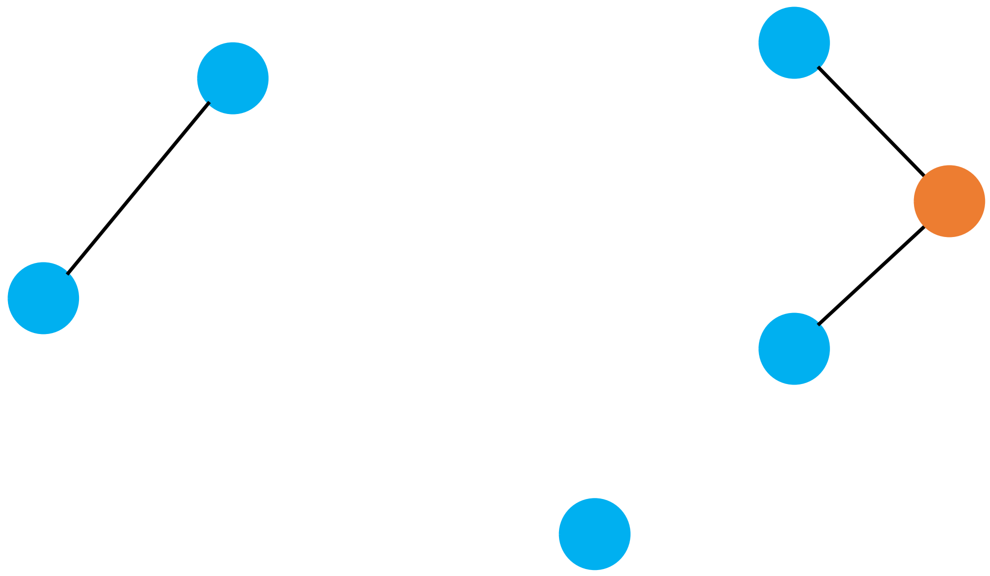
Introduction. Articulation points removal

Step 1



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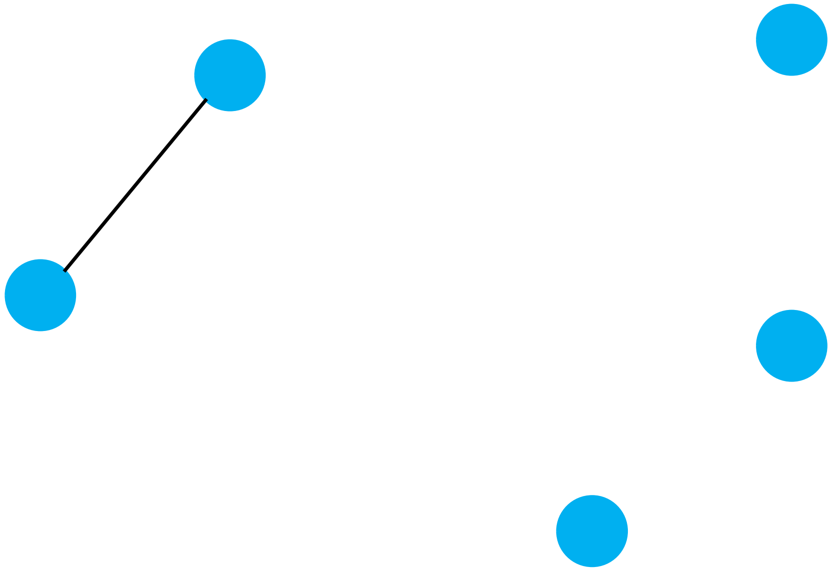
Introduction. Articulation points removal

Step 2



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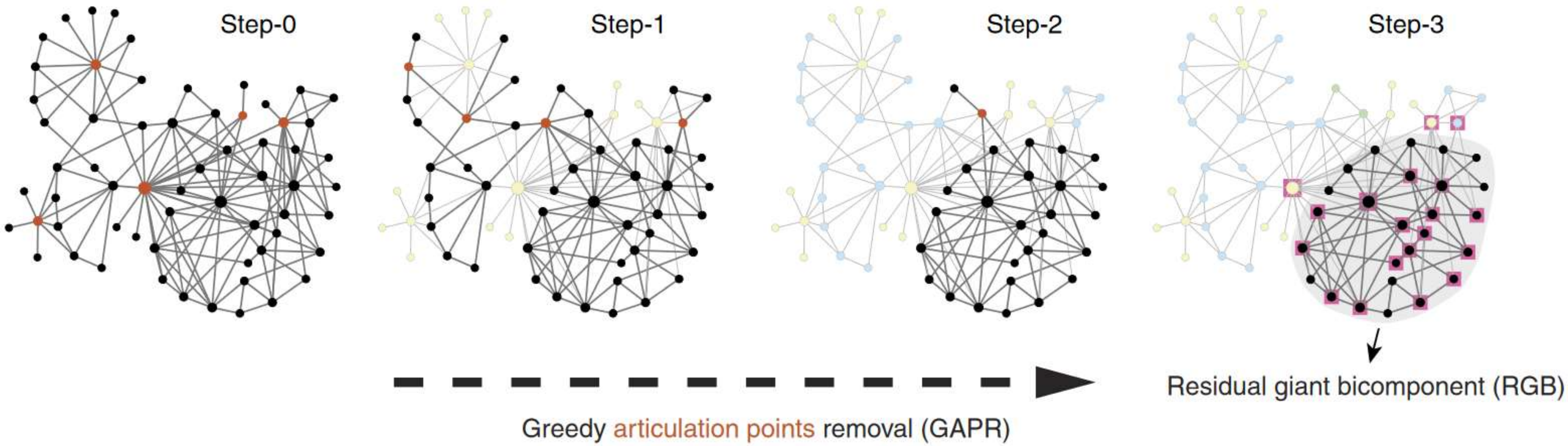
Introduction. Articulation points removal



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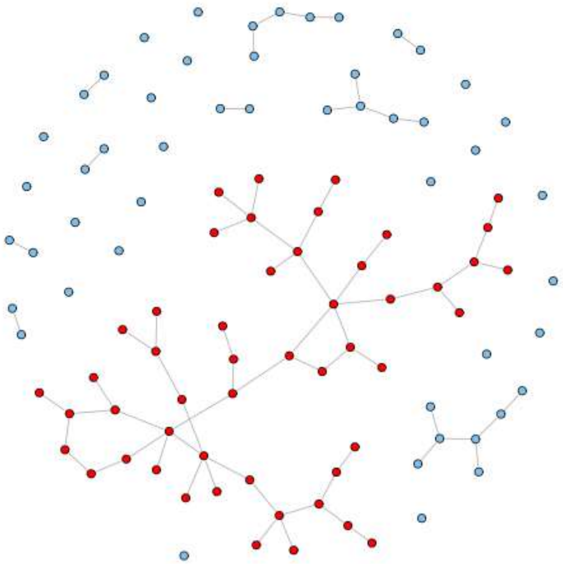
Introduction. Definitions



Giant Connected Component (GCC)

Giant connected component (GCC) is the largest connected component of the graph

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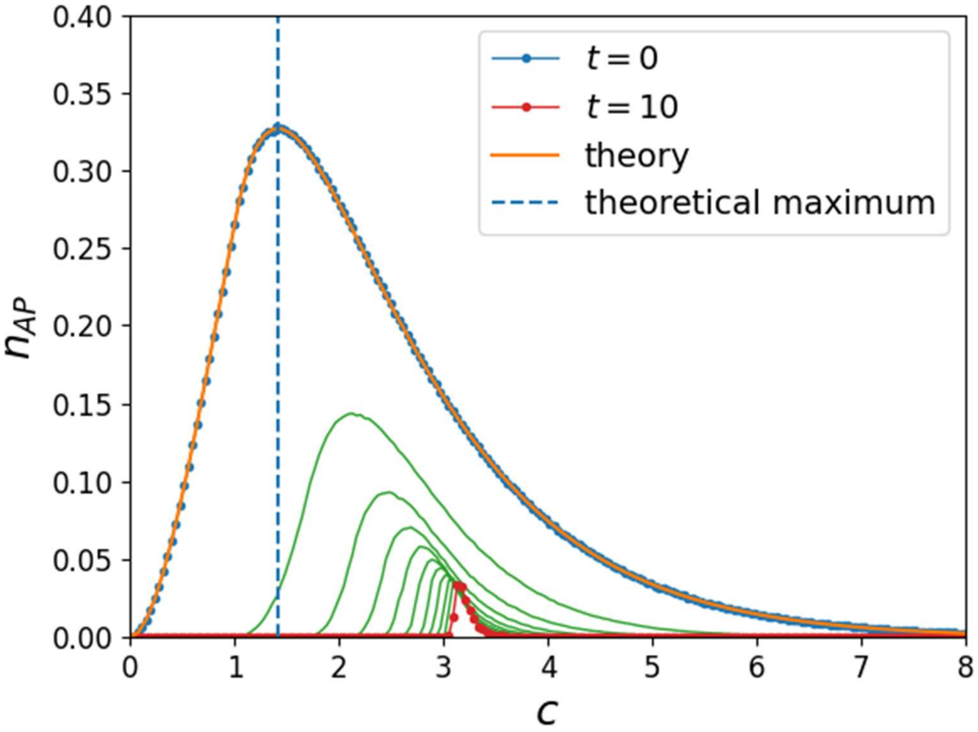
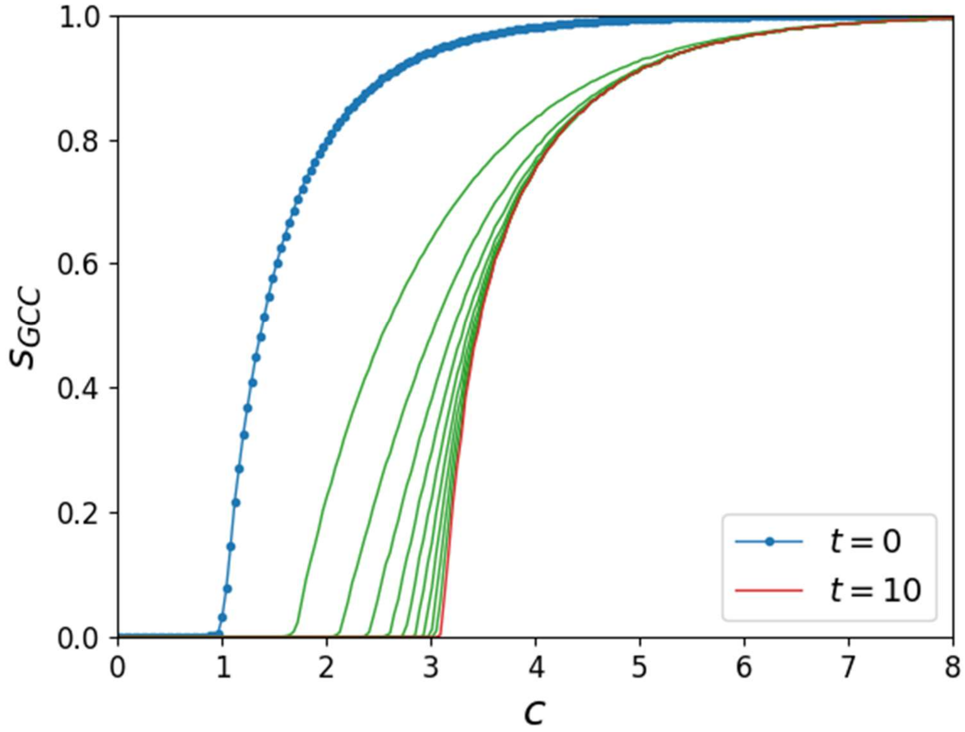
Introduction. Articulation points removal





Articulation points removal and GCC size in the case of monoplex Erdős–Rényi network ($N = 10^5$)

Review: Tian, L. et al. Articulation points in complex networks. Nat. Commun. 8, 14223 doi: 10.1038/ncomms14223 (2017).

Aim & Objectives

Study AP removal process in multiplex networks applying cascade failure of nodes

1. in Erdős–Rényi multiplex networks
2. in scale-free networks

Study the difference in behavior between simple and multiplex cases

Compare different random graph models

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Algorithms and methodology

1. Generation of a set of networks from a certain distribution
2. Application of cascade of failures
3. Obtaining AP metrics (fraction)
4. Repeating steps 2-3 until no APs emerge

Stack:
vanilla C++ (final simulations on large graphs ($N \sim 10^5$))
Python: NetworkX, Matplotlib (prototyping and plotting)

https://github.com/ArtemVergazov/ap_in_multiplex_networks

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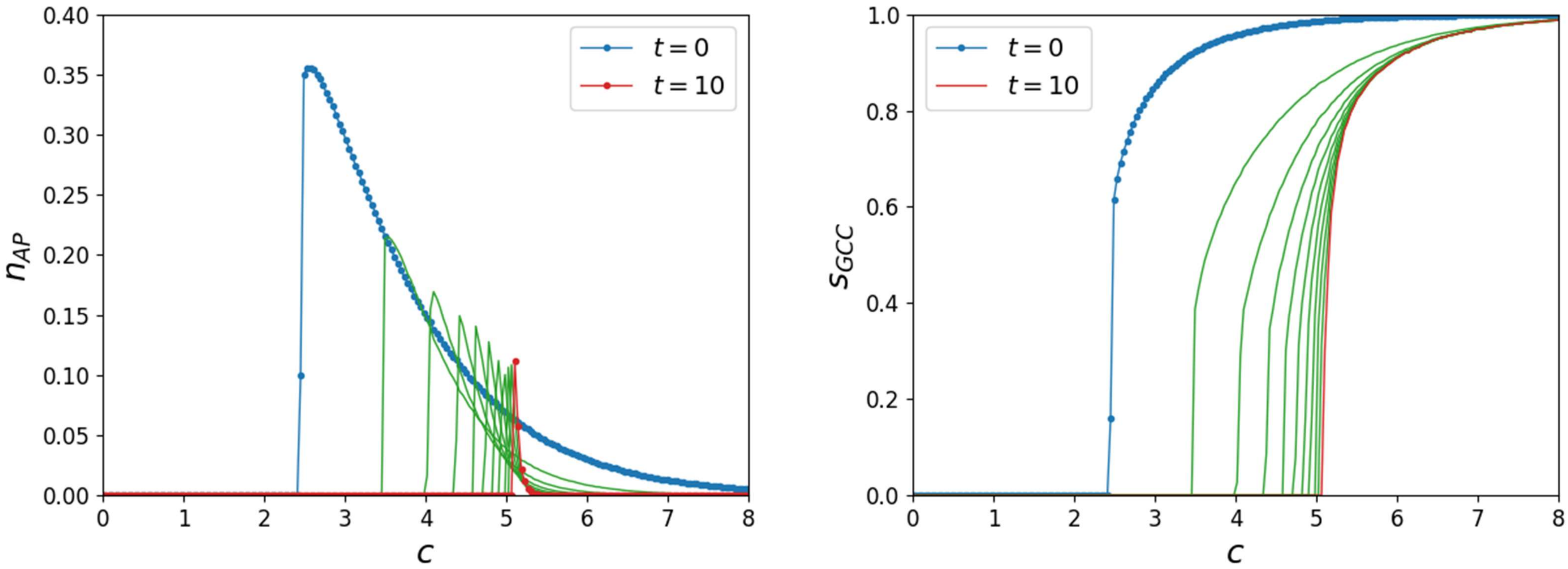
Results. ER multiplex network



Articulation points removal and GCC size in the case of two-layer multiplex Erdős–Rényi network ($N = 10^5$)

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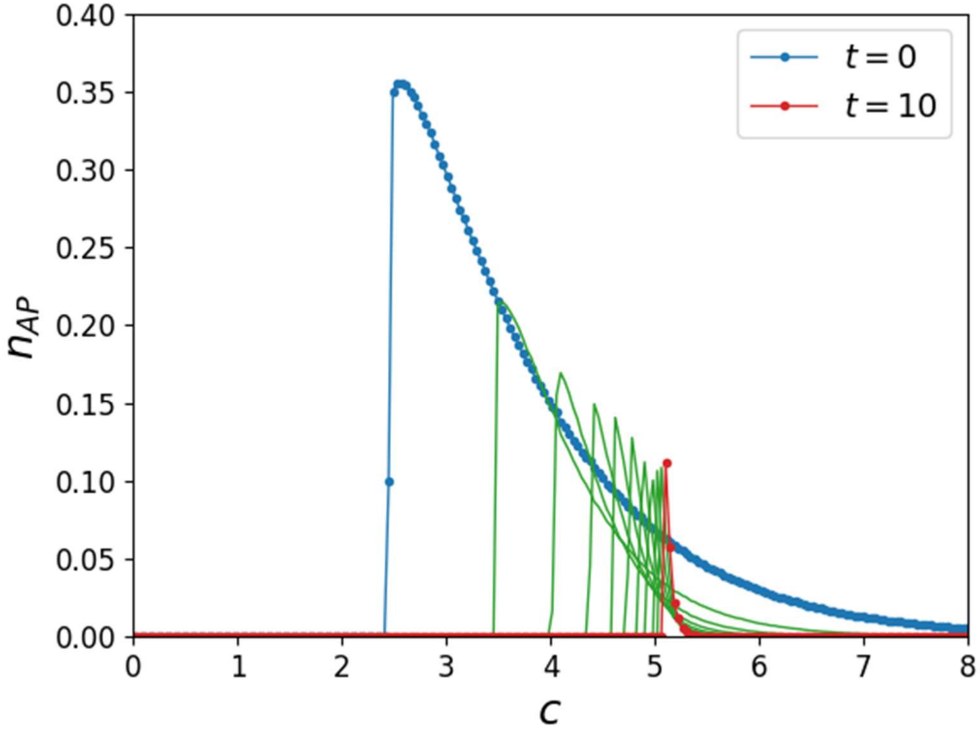
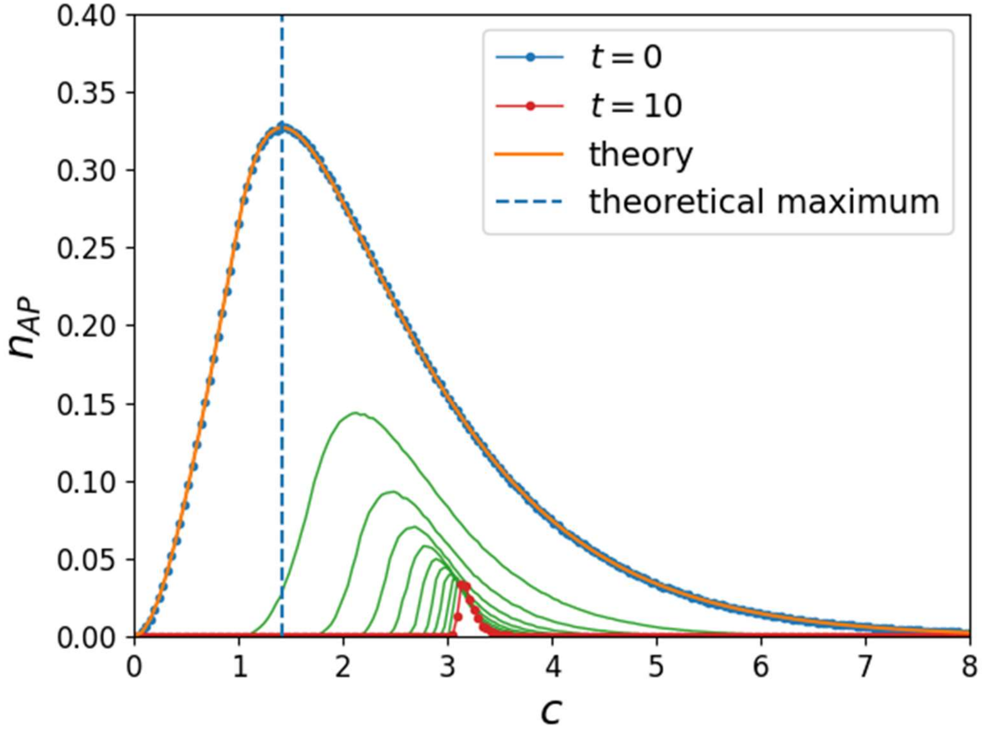
Results. ER multiplex and monoplex comparison



Multiplex case



Monoplex case

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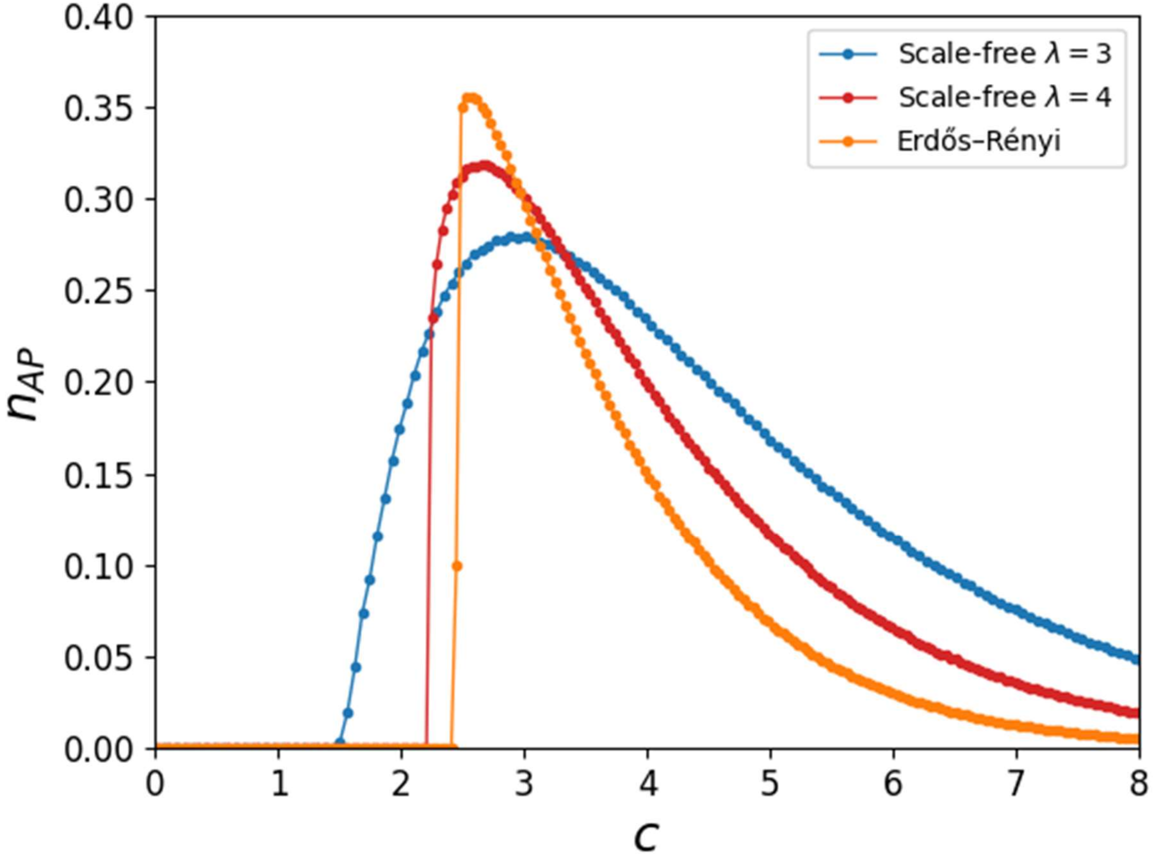
Results. Random graph models comparison



Erdős–Rényi: $P(k) = \frac{c^k}{k!} e^{-c}$

Scale-free: $P(k) \sim k^{-\lambda}$

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Discussion and conclusions

Critical value of mean degree c is larger for multiplex networks – more connections needed in general to resist multiple node failures

Network's degree distribution is related to the order of the phase transition – avenue for further research

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Current status and outlook

- APs removal strategy with cascade failure behavior studied on
  - Erdős–Rényi multiplex networks
  - scale-free multiplex networks
- comparison between different random graph models

Next step:
Preparation of the paper for submission

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Acknowledgements

Prof. Vladimir Palyulin

Dr. Saeed Osat

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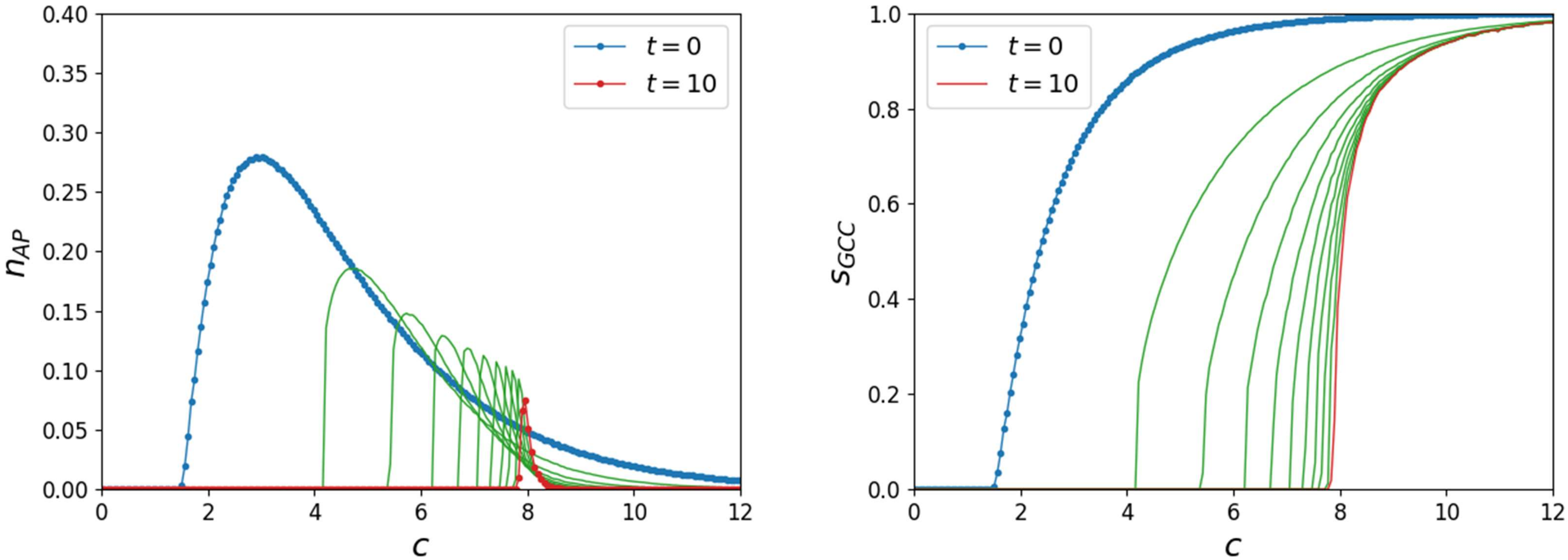
Thank you for attention

Results. Scale-free multiplex network



Articulation points removal and GCC size in scale-free two-layer multiplex network ($N \sim 10^5$) with the degree exponent $\lambda = 3$. c is the mean degree of the network nodes.

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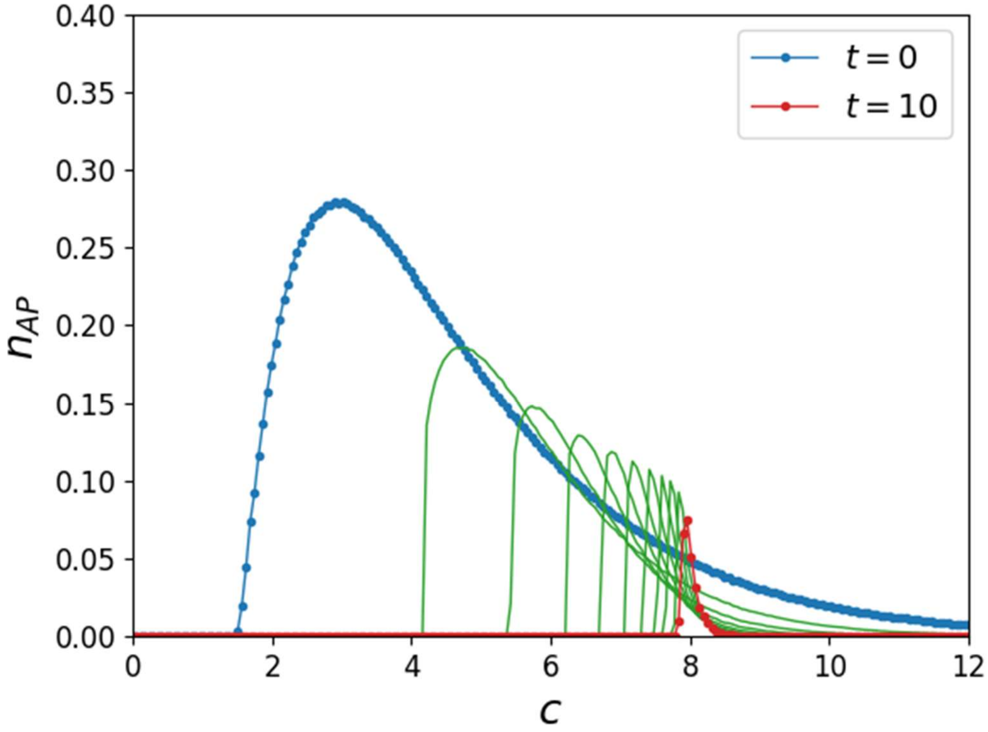
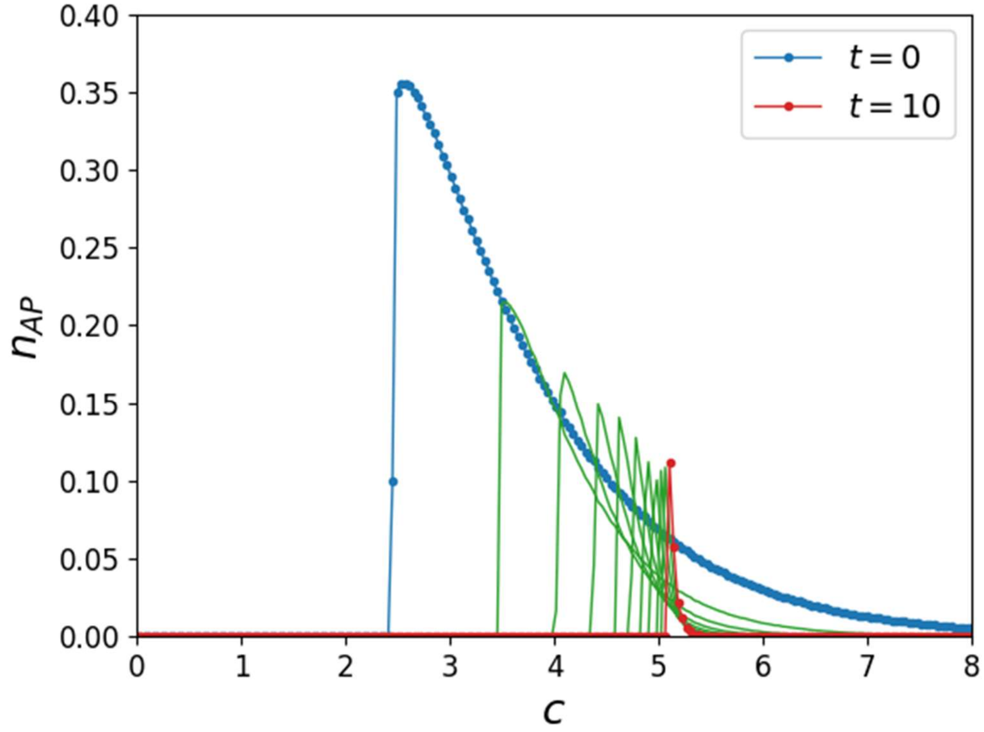
Results. Scale-free and ER comparison



Scale-free network, $\lambda = 3$



ER network

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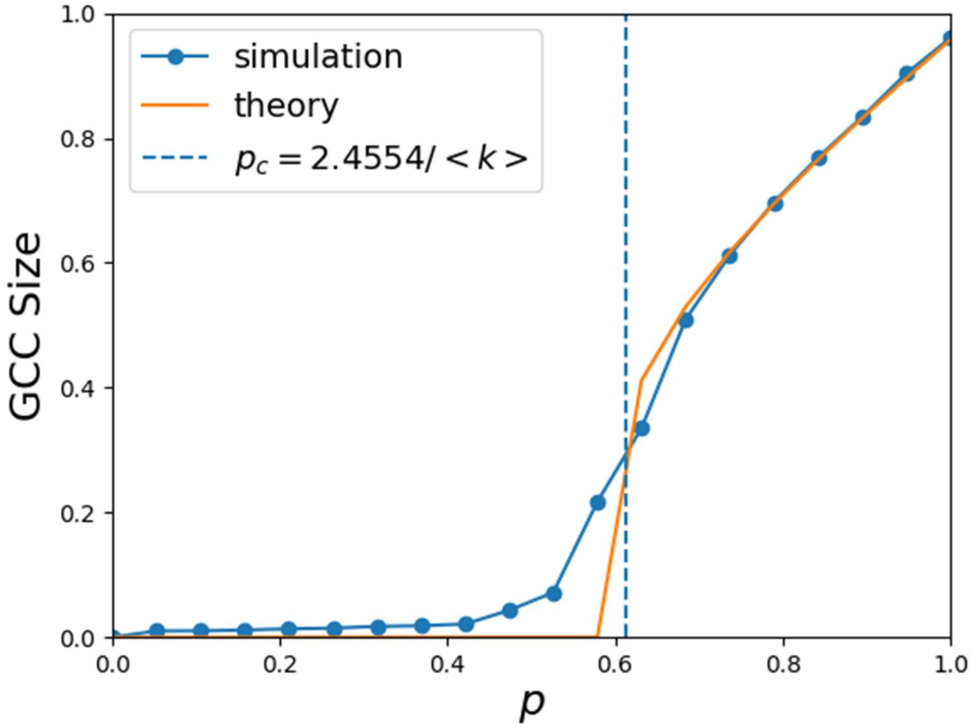
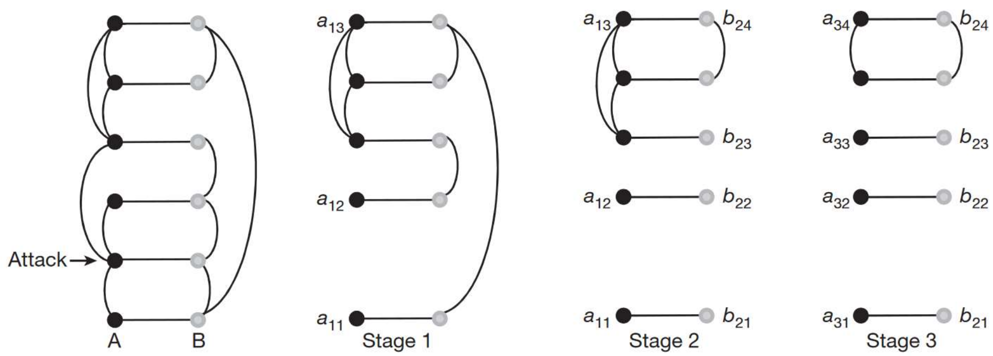
Introduction. Cascade failure of nodes



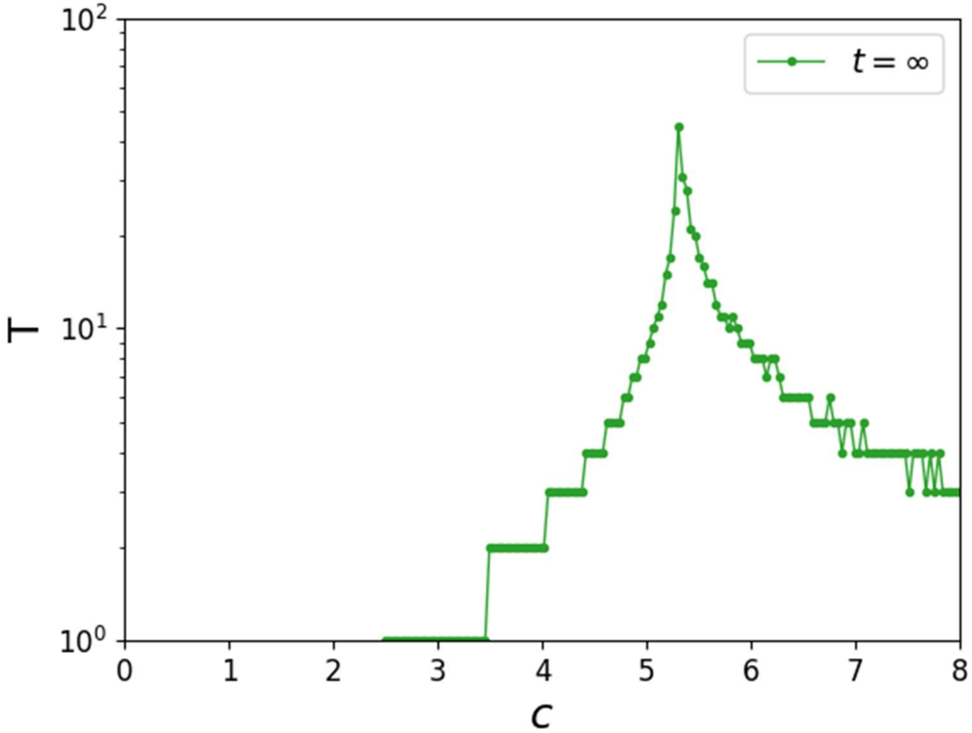


Size of the Giant Connected Component (GCC) that is left after a cascade of node failures in a two-layer Erdős–Rényi network ($N \sim 10^5$). p is the fraction of nodes that are left after the failure

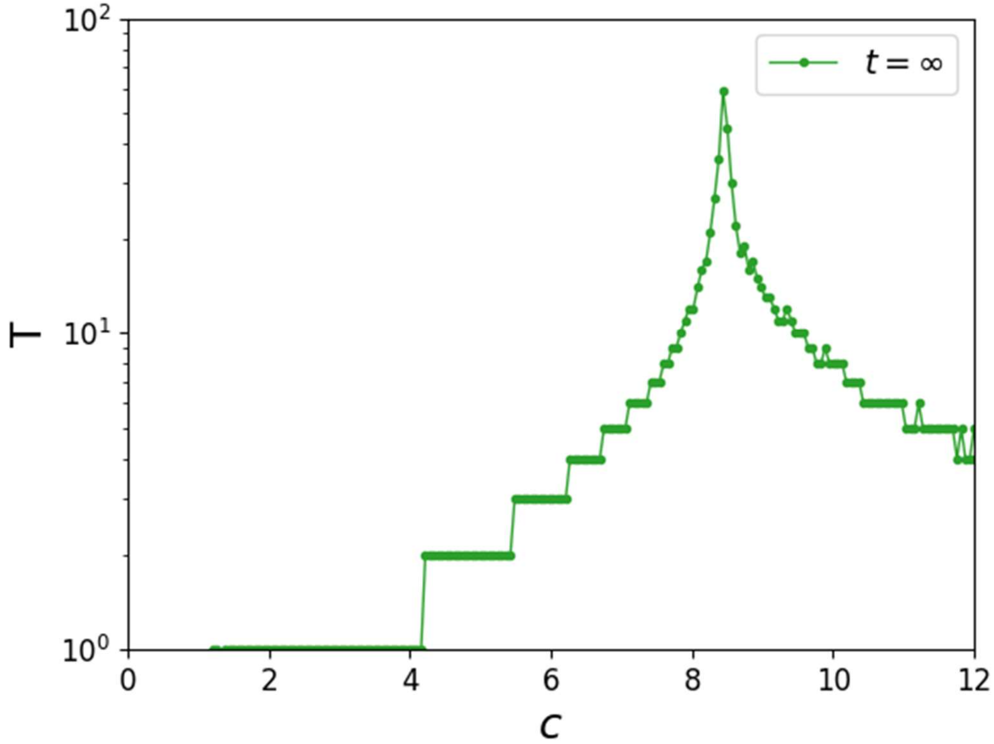
Review: Buldyrev, S., Parshani, R., Paul, G. et al. Catastrophic cascade of failures in interdependent networks. Nature 464, 1025–1028 (2010). https://doi.org/10.1038/nature08932

Appendix. Number of AP removal steps

Total number of AP removal steps as a function of mean degree *c* in multiplex ER and scale-free networks ($N \sim 10^5$)



ER network



Scale-free network

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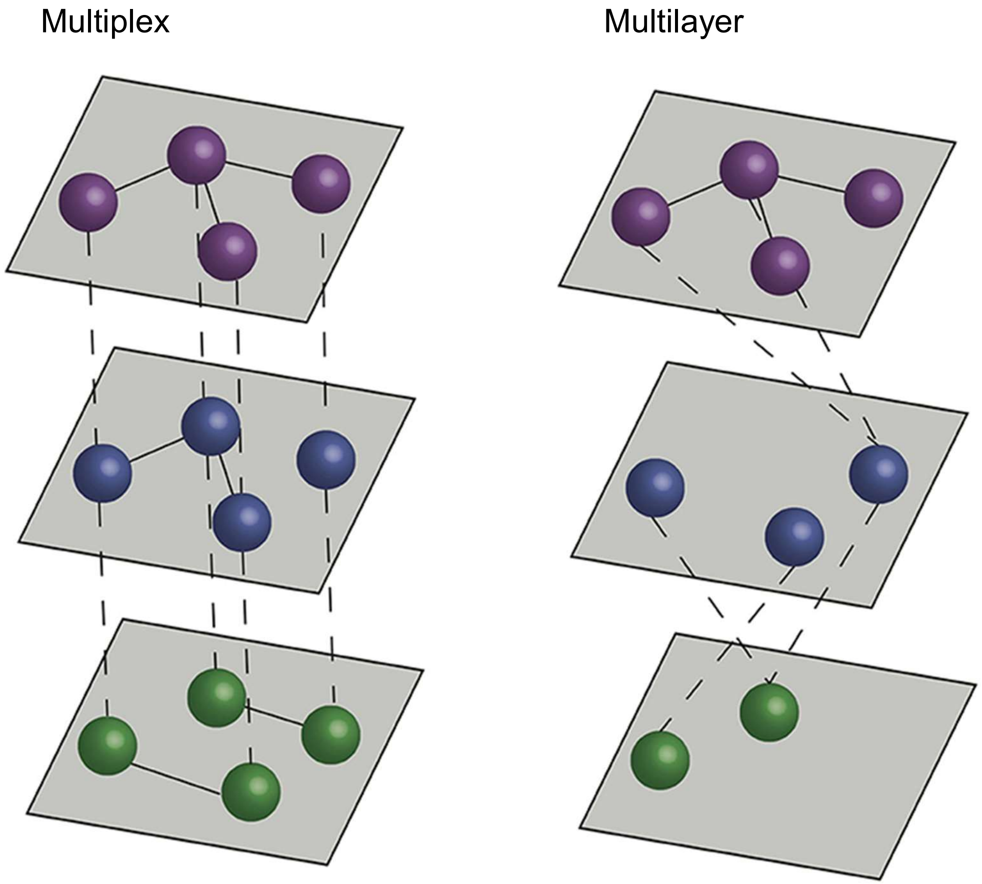
Multiplex vs Multilayer Networks



AP Identification Algorithm

Tarjan's algorithm based on DFS

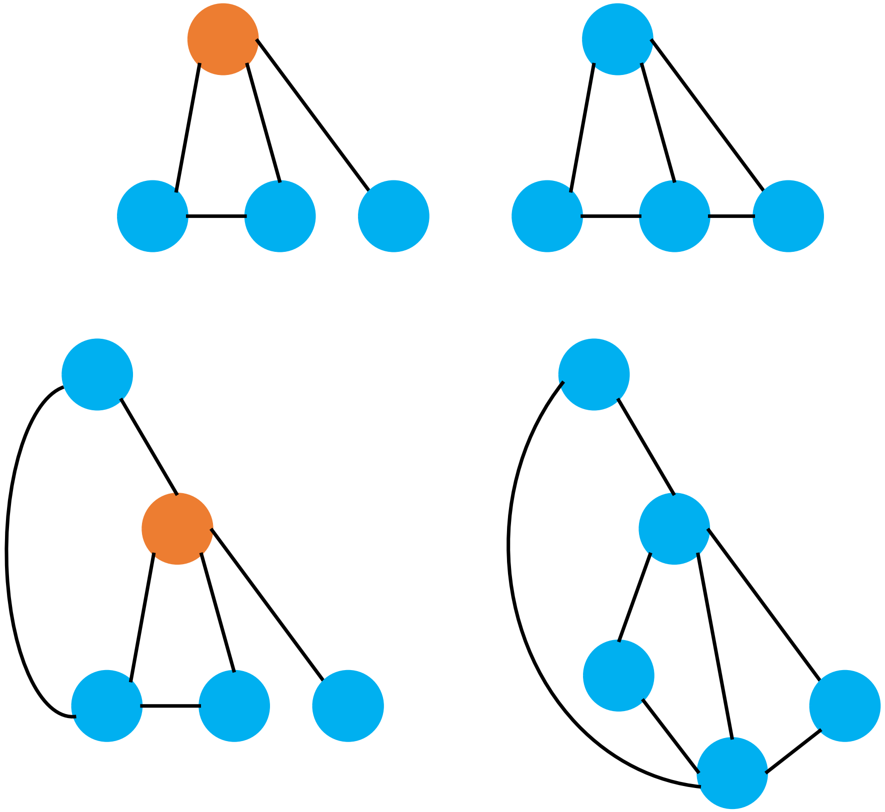
Node u is an AP iff:

1. u is the root of DFS tree and has at least 2 child subgraphs

OR

2. u is not the root of the DFS tree and it has a child v such that no vertex in the subtree rooted with v has a back edge to one of the ancestors in DFS tree of u

Complexity: $O(V+E)$



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