

Homework assignment 4

Due Date: 12th of December EOD

Course: Stochastic Methods in Mathematical Modelling

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Problem 1 Direct Monte-Carlo simulation by rejection and the curse of dimensionality (4 points)

1. Write a code which samples by rejection the volume of an ellipsoid in 2D, 3D, 4D and 5D cases. The length of the longest principal semi-axis of the ellipsoid is $a = 2$, the rest of the semi-axis have the length equal to $b = 1$.
2. Obtain the convergence towards an analytical result as a function of the number of samples for each case. Recommended number of samples for this task is about 100K or more.
3. Calculate the sampling efficiency for each case.
4. Compare the convergence, its standard deviation and the sampling efficiency in different dimensions. Do you observe any trend in performance?

Problem 2 Importance sampling (4 points)

Let X be a standard normal distribution ($\mu = 0, \sigma = 1$). Compute $P(X \geq 4)$ by using importance sampling Monte-Carlo. (Hint: use a sampling distribution with a peak of the density close to $x = 4$). Then try to get this value by generating the values by Box-Mueller transform and rejecting the values for $x \leq 4$. Compare the efficiency of two approaches.

Problem 3 Metropolis-Hastings algorithm (2 points)

By using the Metropolis-Hastings algorithm sample the exponential distribution $P(x) = 2 \exp(-2x)$, $x \in [0, \infty)$. Plot the histogram of sampled variables and superimpose the analytical formula.