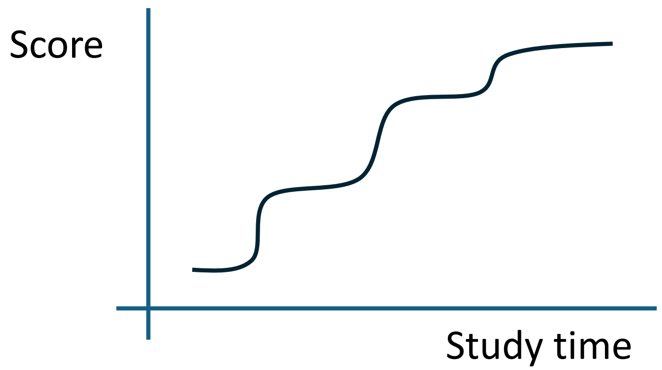
SNE3002 Linear Algebra – 2025 Spring

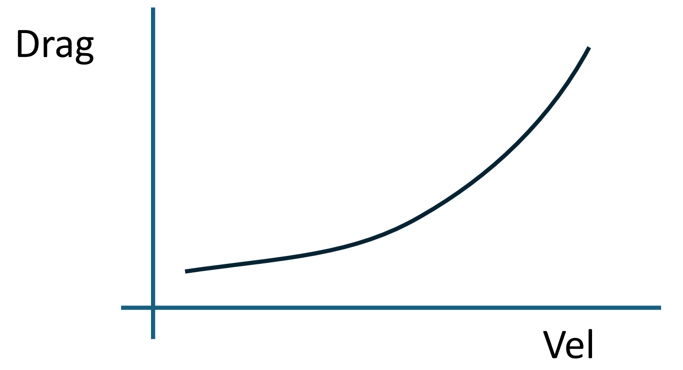
Introduction to Linear Algebra

INHA UNIVERSITY

Why Linear Algebra?

- Many things in real-world is nonlinear
  - Ex. 1) Study time vs Grade   2) Velocity vs Drag force



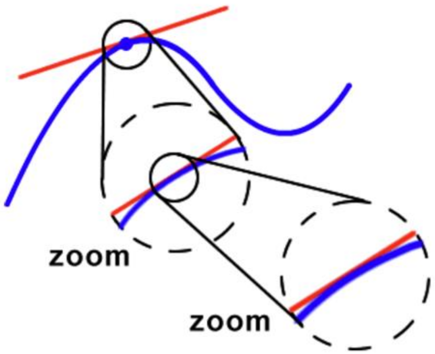


- How to deal with nonlinear system?
  - There is no universal rules

- The simplest and easiest method would be linearization!

Why Linear Algebra?

> Everything is approximately linear.



> The most fundamental (understandable) equations of nature appear to be linear.

$$F = ma$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}.$$

Why Linear Algebra?

- Linear Algebra – mathematical base
  - Machine Learning, Linear System, Dynamics, Control, Perception

- Theoretically solid
  - Safe and reliable

- Computationally efficient
  - Affordable and practical

- When our problems are reduced to linear algebra, we can actually solve them (problem formulation is half of the job!)

Why Linear Algebra?

> Tangible applications

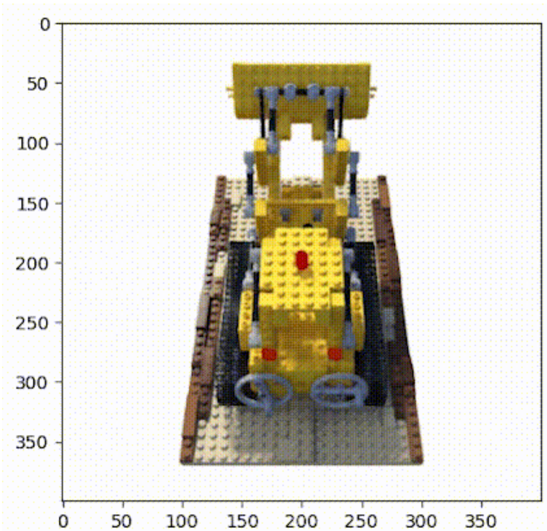
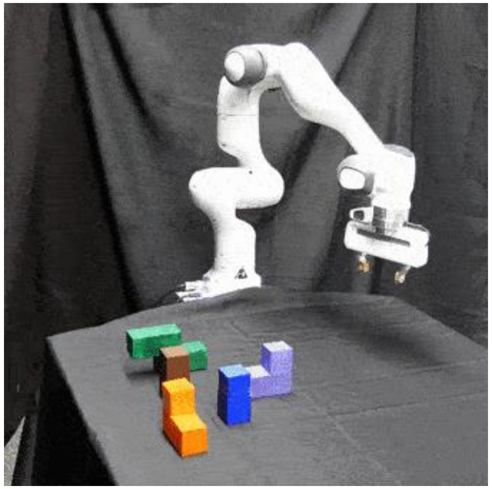
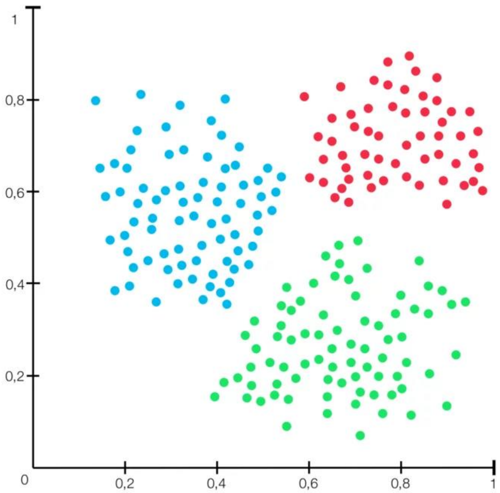


Image Processing



Manipulator Control



Data Clustering

What is Linear Algebra?

- Algebra : equations with variables

$$3x^2 + 4y^5 = 1$$

$$\pi e^x + \sqrt{\log(2y)} = 2$$

$$\sin(x) + \cos(2y) = 3$$

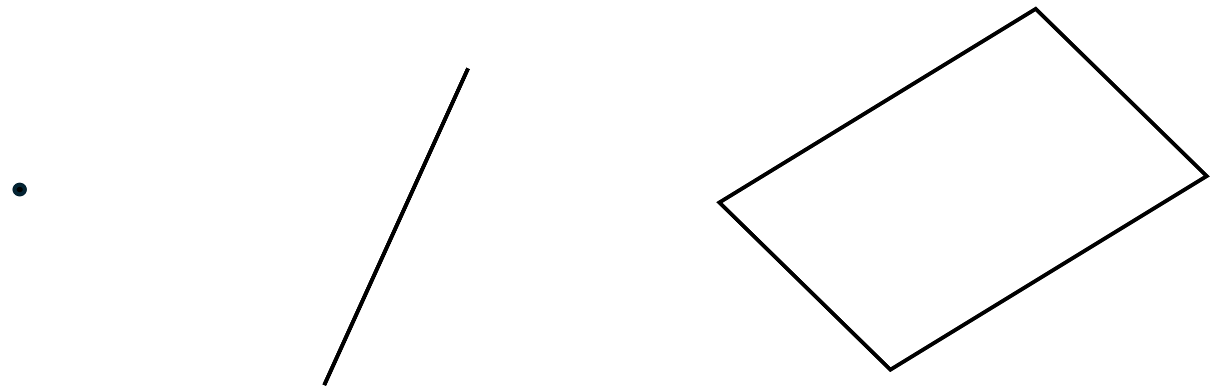
- Linear algebra : each variable appears by itself, not raised to a power, not multiplied by each others, not inside a function (log, sin)

$$3x + 4y = 5$$

$$2x - 6y + 8z = 15$$

What is Linear Algebra?

- Geometrically, the study of 'flat', 'linear', objects like points, lines, and planes.



$$x = \alpha \qquad y = \beta \qquad z = \gamma$$

$$ax + by + cz = \alpha \qquad cx + by + dz = \beta$$

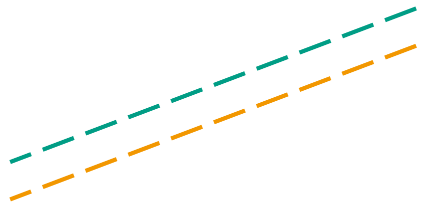
$$ax + by + cz = \alpha$$

$$where\ (x, y, z) \in \mathbb{R}^3$$

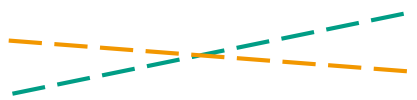
Linear Systems, Solutions

> A system of linear equations can have

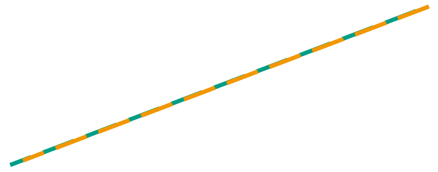
- No solution;



- Unique solution (one and only);



- Infinite number of solutions.



Linear Systems, Solutions

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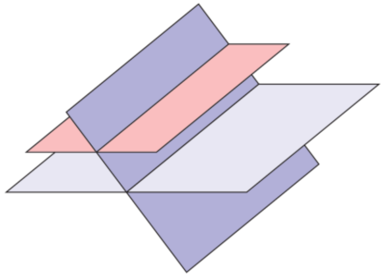
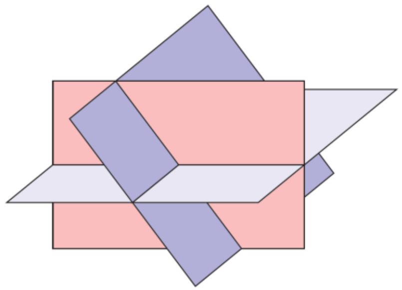
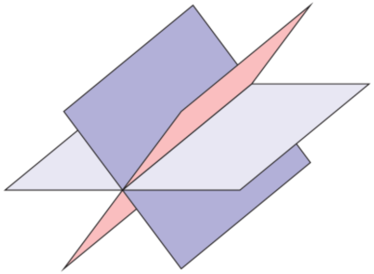


- Unique solution (one and only);



- Infinite number of solutions.



Linear Systems, Solutions

- A system of linear equations can have
  - No solution;   Unique solution (one and only);   Inf. N. of Sols.

Linear Systems, Solutions

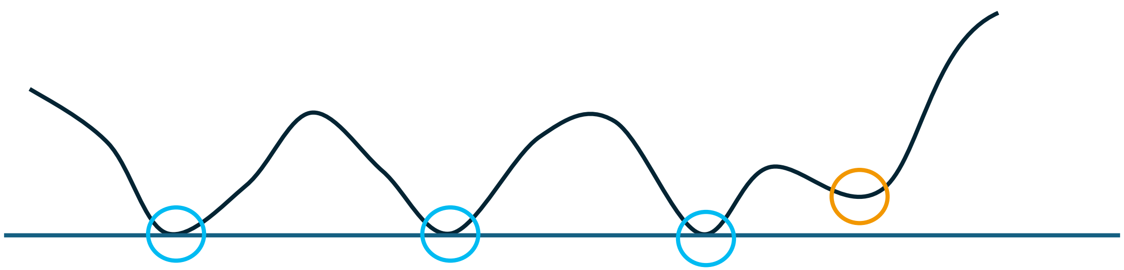
> We have considered a set of **2** equations with **2** variables (x, y).

> What if we have **n** equations with **m** variables?

- The same holds true: we could have a unique solution, no solution, or an infinitely many solutions.

- There is no case we have >2 (multiple) solutions.
(Very good property for obtaining a solution)

Cost



Convex optimization

Row Reduced Echelon Form (RREF)

> Q. Wanda the witch owns three types of pets – cats, ravens, and snakes. She has 10 pets in total, with a total of 16 legs and 4 wings. How many of each type of pet does she own?

Row Reduced Echelon Form (RREF)

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$$c + r + s = 10$$

$$4c + 2r = 16$$

$$2r = 4$$

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Def. Augmented Matrix of a System of Linear Equations

The **augmented matrix** of the system of linear equations

$$a_{11}x_1 + \ldots + a_{1n}x_n = b_1$$
$$\ldots$$
$$a_{m1}x_1 + \ldots + a_{mn}x_n = b_m$$

is

$$\left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{array}\right]$$

Def. Elementary Row Operations

1. Switch two rows.
2. Multiply a row by a non-zero number.
3. Add a multiple of one row to another row.

Row Reduced Echelon Form (RREF)

> Q. Wanda the witch owns three types of pets – cats, ravens, and snakes. She has 10 pets in total, with a total of 16 legs and 4 wings. How many of each type of pet does she own?

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 4 & 2 & 0 & 16 \\ 0 & 2 & 0 & 4 \end{array}\right]$$

Row Reduced Echelon Form (RREF)

Def. Row Echelon Form

An entry of an augmented matrix is called a **leading entry** or **pivot entry** if it is the leftmost non-zero entry of a row. An augmented matrix is in **row echelon form** if

1. All rows of zeros are below all non-zero rows.
2. Each leading entry of a row is in a column to the right of the leading entry of any row above it.

$$\left[\begin{array}{ccccc|c} 0 & 5 & 2 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccccc|c} 1 & 4 & 0 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right], \quad \left[\begin{array}{ccc|c} 3 & 0 & 6 & 2 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{cc|r} 1 & 2 & 3 \\ 2 & 4 & -6 \\ 4 & 0 & 7 \end{array}\right], \quad \left[\begin{array}{ccc|c} 0 & 2 & 3 & 3 \\ 1 & 5 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Row Reduced Echelon Form (RREF)

Def. Row Reduced Echelon Form

An augmented matrix is in **row reduced echelon form** if

1. Each leading entry is equal to 1.
2. All entries above a leading entry are zero.

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 5 & 0 & 3 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

> The number of pivots is called the **rank**. This will be very useful later

> The algorithm to put a matrix into RREF is Gauss-Jordan algorithm

> RREF is unique

Lecture Plan

> First half semester: mathematics

> Second half semester: mathematics + programming

> 5 Assignments (scheduled 2rd, 4th, 5th, 10th, 11th week)

- TA: 이민호 (mhlee00@inha.edu)

> 1 Project? 14th week

> Grade

- Attendance 5% (No grade if absences reach one-quarter of total class days.)
- Assignment 30%
- Mid-term 30%
- Final-term 35%

Reference

- Book
  - Matrix Theory and Linear Algebra, Peter Selinger
  - Introduction to Linear Algebra, Janis Lazovskis
  - Introduction to Applied Linear Algebra, Stephen Boyd

- Lecture
  - https://dept.math.lsa.umich.edu/~speyer/LinearAlgebraVideos/
  - https://github.com/michiganrobotics/rob101