PowerExample

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Calculating Type I and II error and Power

This document demonstrates the ideas behind statistical error and power. In simple cases, the below techniques could be used to quantify Type II error (β) or power ($1 - \beta$).

Bone Mineral Content

Can a 6-month exercise program increase the total body bone mineral content (TBBMC) of young women? A team of researchers is planning a study to examine this question. Based on a previous study, they are willing to assume $\sigma = 2$ for the percent change in TBBMC over the 6-month period. A change in TBBMC of 1% would be considered important, and the researchers would like to have a reasonable chance of detecting a change this large or larger. Is 25 subjects large enough to sample for this project?

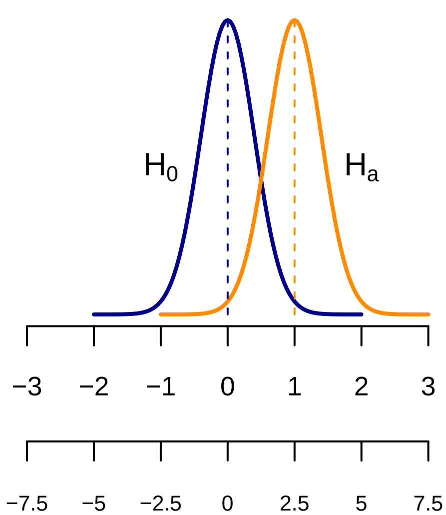
Step 1: State the hypotheses.

$$H_0 : \mu_0 = 0$$

$$H_a : \mu_0 > 0$$

In stating the hypotheses, you are choosing betweeen a one- or two-tailed test. Also, choose the significance level, α, which we will set at 5% for this example.

Step 2: Sketch the sampling distributions of the null and alternative hypotheses.
Note that the figure has two x axes. The top axis plots the x-values, and the bottom axis plots the corresponding z-values.



Step 3: Determine the critical values (x or z-values) to reject the null hypothesis.

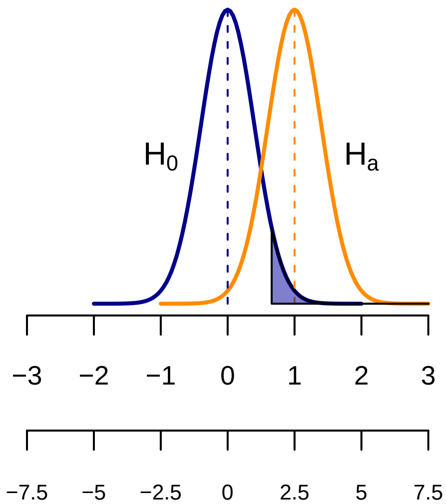
What is the critical value of X or Z at which the null hypothesis, $\mu_0 = 0$, would be rejected in favor of the alternative hypothesis? Since we know $\alpha = 0.05$, we can use the standard normal distribution to figure out our critical statistic, Z_c, in R (or using a statistical table):

```
qnorm(0.95, mean = 0, sd = 1)
```

Now calculate the x-value that corresponds to Z_c. Recall that generally $Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$, so we can solve for X and replace Z with Z_c:

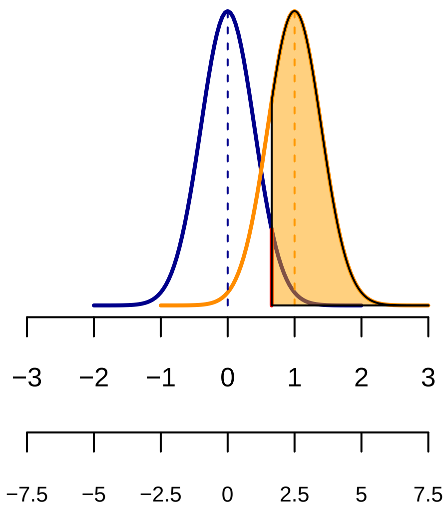
$$X = \mu + Z_c \frac{\sigma}{\sqrt{n}}$$

$$X = 0 + 1.645(\frac{2}{\sqrt{25}}) = 0.658$$



Our calculated X_c and Z_c mark the start of the rejection region. In other words, the shaded region starts at $X = 0.658$ and at $Z = 1.645$.

Step 4: Sketch the area that you are interested in, assuming the alternative hypothesis is true.



The question asks that we calculate the statistical power. Therefore, we want to find the probability that $X \geq X_c$, conditional upon our H_a: $P\big(X \geq 0.658 \mid \mu = 1\big)$. Because X and Z are exchangeable, we could equivalently find $P\big(Z \geq 1.645 \mid \mu = 1\big)$

Step 5a: *Calculate the probability of obtaining values under a specified H_1, using X_c.*

Since X is 0.658, then we can find the area to the right of this number by calculating the z-value of 0.658 under the alternative hypothesis (orange distribution) and finding its probability. Let's do this...

$$P(Z \geq \frac{x - \mu_1}{\frac{\sigma}{\sqrt{n}}}) = P(Z \geq \frac{0.658 - 1}{\frac{2}{\sqrt{25}}}) = P(Z \geq -0.855)$$

$$P(Z \geq -0.855) = 0.80$$

Again, we can get the probability of Z from statistical tables, or:

```
1-pnorm(-0.855, mean = 0, sd = 1)
```

```
## [1] 0.8037244
```

We estimate the right hand side of the alternative distribution because we are calculating power. Therefore, we have a power of 80%.

Step 5b: *Calculate the probability of obtaining values under a specified H_1, using Z_c.*

Another way of calculating this area without first calculating X_c is to first calculate β as:

$$\beta = P(Z \geq \frac{\mu_0 - \mu_1}{\frac{\sigma}{\sqrt{n}}} + Z_c) =$$

$$P(Z \geq \frac{0 - 1}{\frac{2}{\sqrt{25}}} + 1.645) =$$

$$P(Z \geq -0.855) = 0.196$$

```
pnorm(-0.855, mean = 0, sd = 1)
```

```
## [1] 0.1962756
```

And, power is equal to $1 - \beta = 0.80$.

What is our conclusion regarding the proposed sample size of 25 subjects for the experiment? Because power is 80%, we would conclude that 25 subjects is sufficient.