

# ENV 710

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mixed models



# roadmap

- recap

## where we are

multivariate linear models



interactions

centering/scaling explanatory  
variables

random effects and mixed models



generalized linear models

# I – height & education

Do earnings increase with the height and education or interaction between height and education for a worker?

```
summary(c6)
```

```
Call:
lm(formula = earn ~ ht.c * ed.c, data = ht.dat)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	23020.45	518.59	44.390	< 2e-16 ***
ht.c	1121.57	135.08	8.303	2.74e-16 ***
ed.c	2546.81	215.48	11.819	< 2e-16 ***
ht.c:ed.c	138.16	53.36	2.589	0.00974 **

---

```
summary(c2)
```

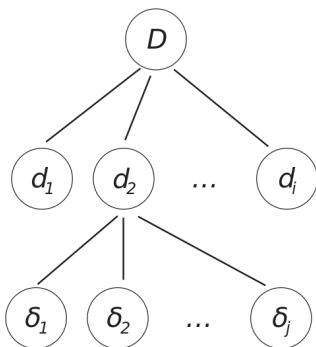
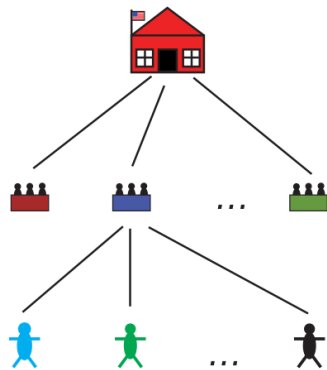
```
Call:
lm(formula = earn ~ height * ed, data = ht.dat)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	38433.60	48668.60	0.790	0.42986
height	-744.22	726.70	-1.024	0.30599
ed	-6699.50	3581.35	-1.871	0.06164 .
height:ed	138.16	53.36	2.589	0.00974 **

---

# mixed or multilevel models



**mixed model:** statistical model containing both fixed effects and random effects

accounts for (1) variation that is explained by the independent variables of interest – fixed effects, and (2) variation that is not explained by the independent variables of interest – random effects.

**fixed effect factor** if data has been collected from all levels of interest (e.g., gender: male/female, condition: easy/medium/hard)

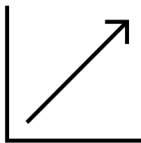
**random effect factor** if the variable has a bunch of possible levels but you only sample a random collection (e.g., subjects, stimuli, classrooms) and though these samples will have some idiosyncrasies, you generally won't care about them, with the goal of generalizing to the broader population (e.g., all people, all scenarios, all classrooms)

# fixed and random effects

am I interested in the variable (vs. just want to account for the correlation)?

- **yes** = fixed effect
- **no** = random effect

e.g., block effect



do I want an effect size for every level of the variable (vs. just want a single effect size)?

- **yes** = fixed effect
- **no** = random effect

e.g., 195 countries



do I want to make inference to an entire population (vs. interested in specific subjects)?

- **no** = fixed effect
- **yes** = random effect

e.g., random selection of tree species to study phenology



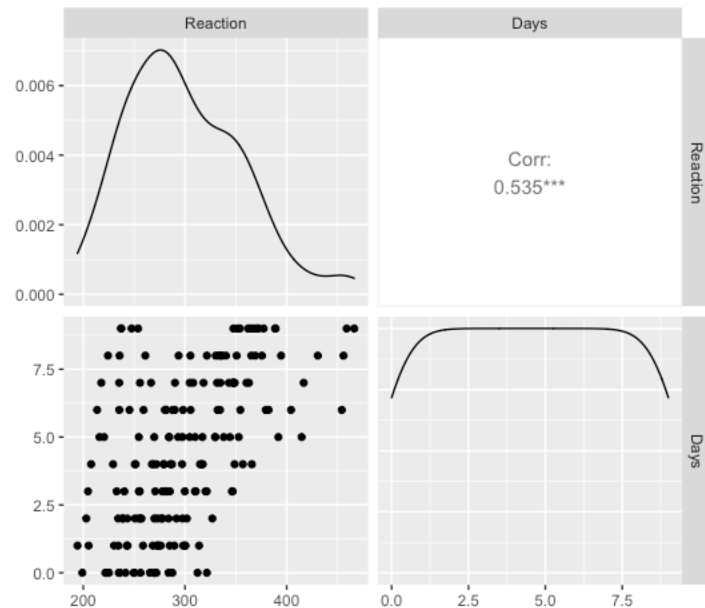
## example

## sleep study

The sleep study measures the average reaction time per day for subjects in a sleep deprivation study. On day 0-2 the subjects had their normal amount of sleep. Starting night 3, they were restricted to 3 hours of sleep per night. Observations represent the average reaction time (ms) on a series of tests given each day to each subject.

$n = 18$  subjects over 10 days (e.g., 10 measurements for each subject)

- run 4 models & discuss meaning of each model
- assess the variance explained by each model
- assess whether random effects for both the intercept (subjects) and slope (days) are necessary



# example

## sleep study

```
require(lme4)
data(sleepstudy)
ss <- sleepstudy[sleepstudy$Days > 2,]
```

Run the “pooled” model – no effect of subject

```
mod1 <- lm(Reaction ~ Days, ss)
summary(mod1)
```

```
lm(formula = Reaction ~ Days, data = ss)
```

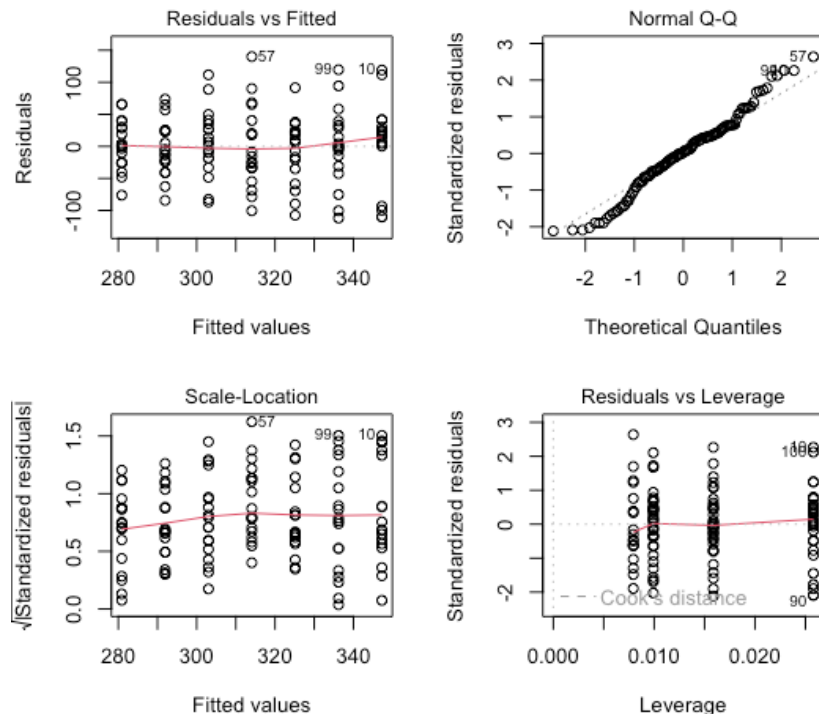
Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	247.702	15.015	16.50	< 2e-16 ***
Days	11.063	2.374	4.66	8.03e-06 ***

---

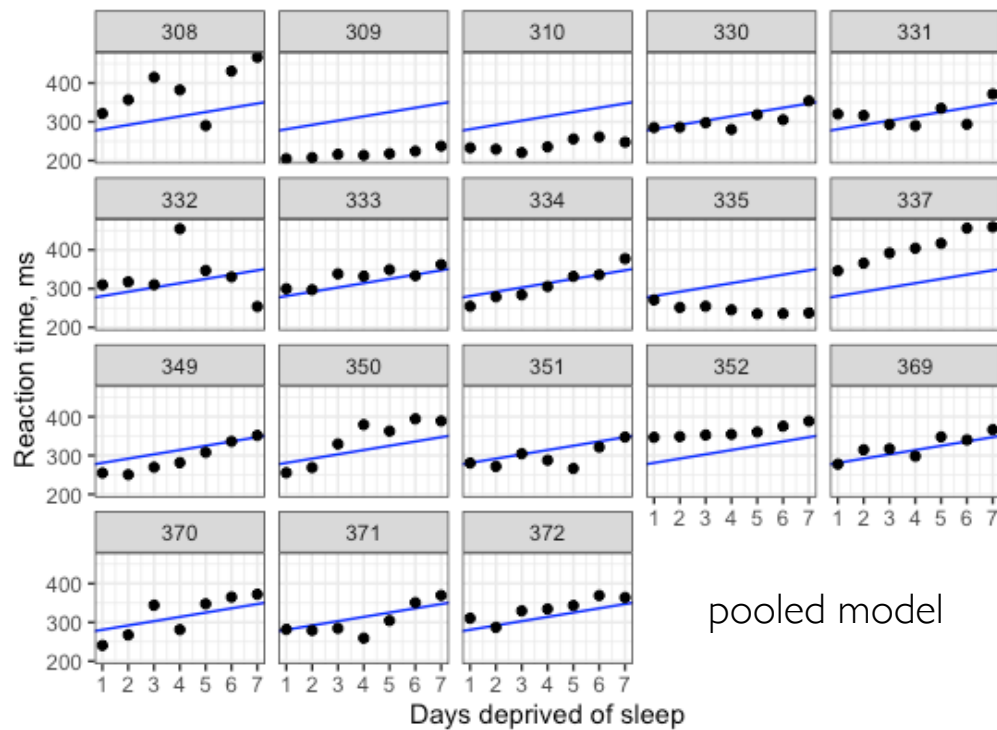
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 53.3 on 124 degrees of freedom  
Multiple R-squared: 0.149, Adjusted R-squared: 0.1422  
F-statistic: 21.71 on 1 and 124 DF, p-value: 8.032e-06

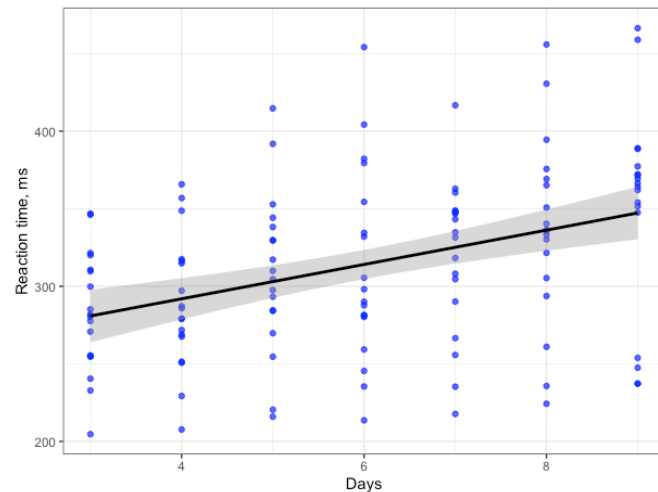


example

sleep study



pooled model





# example

## sleep study

Run the “unpooled model” as mod2

```
mod2 <- lm(Reaction ~ Days + Subject, ss)
summary(mod2)
```

```
lm(formula = Reaction ~ Days + Subject, data = ss)
```

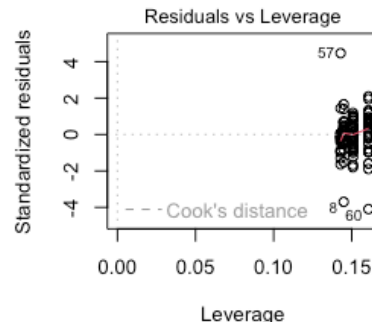
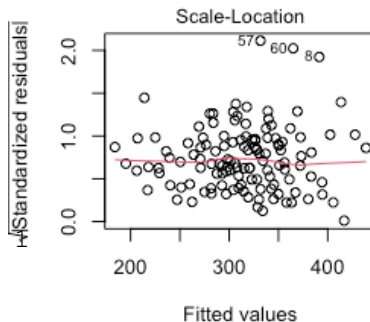
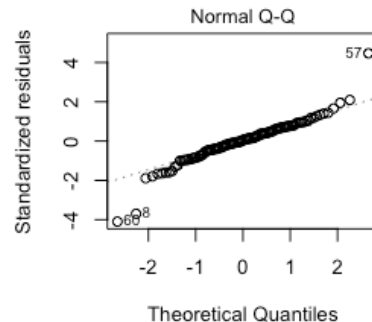
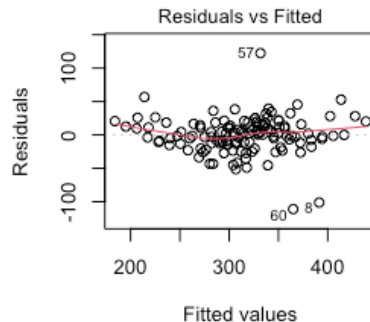
Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	313.945	13.703	22.912	< 2e-16 ***
Days	11.063	1.319	8.391	2.16e-13 ***
Subject309	-162.989	15.822	-10.301	< 2e-16 ***
Subject310	-139.995	15.822	-8.848	2.04e-14 ***
...				
Subject371	-76.094	15.822	-4.809	4.98e-06 ***
Subject372	-46.271	15.822	-2.924	0.004214 **

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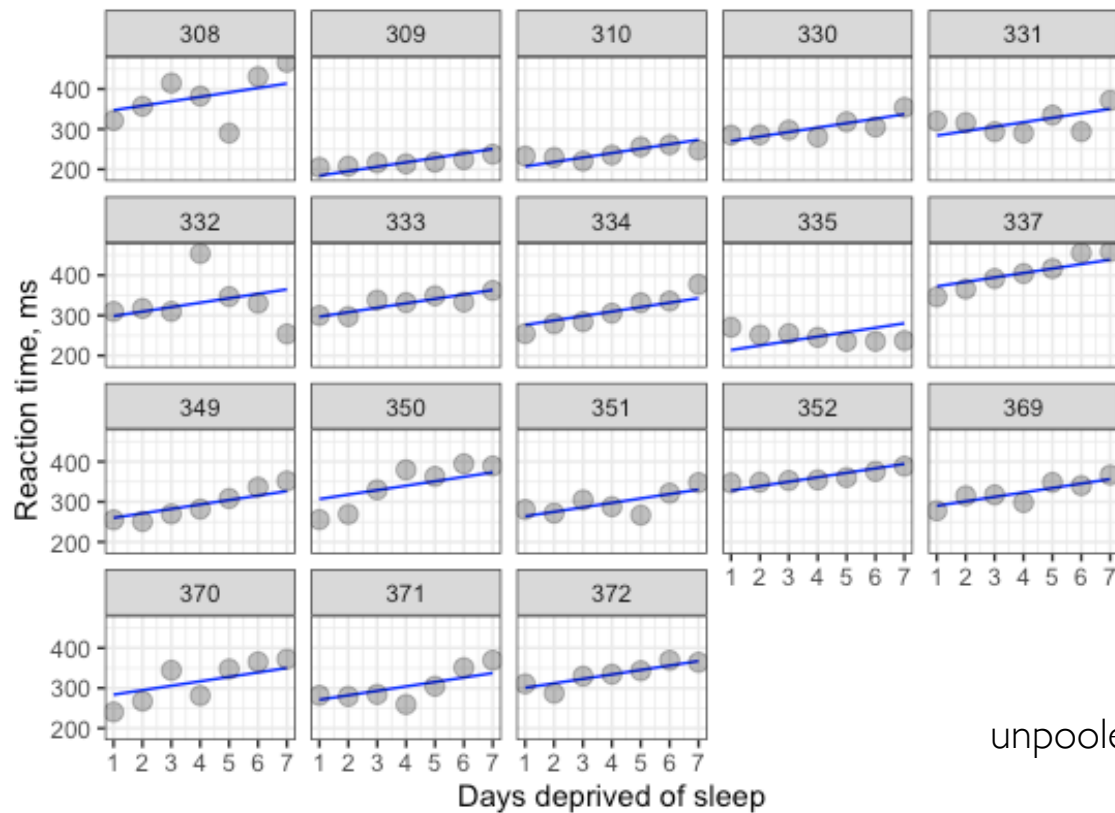
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29.6 on 107 degrees of freedom  
Multiple R-squared: 0.7735, Adjusted R-squared: 0.7354  
F-statistic: 20.3 on 18 and 107 DF, p-value: < 2.2e-16



## example

## sleep study



unpooled model – subject treated as a factor

# example

## sleep study

Run the “unpooled model” as mod2a

```
mod2a <- lm(Reaction ~ Days * Subject, ss)
summary(mod2a)
```

Call:

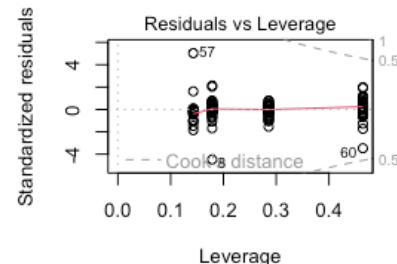
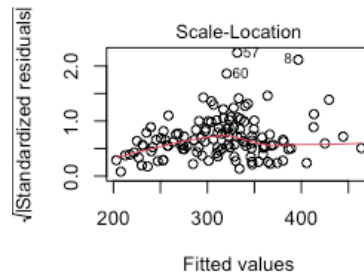
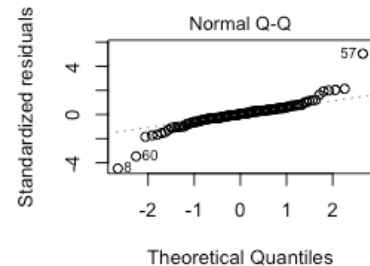
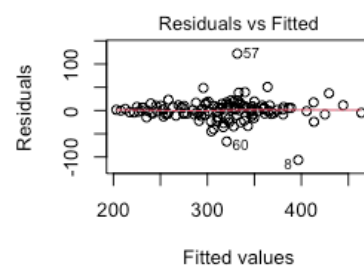
```
lm(formula = Reaction ~ Days * Subject, data = ss)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	282.253	31.380	8.995	3.53e-14	***
Days	16.345	4.962	3.294	0.00141	**
Subject370	-92.616	44.378	-2.087	0.03972	*
Subject371	-69.389	44.378	-1.564	0.12142	
Subject372	-20.633	44.378	-0.465	0.64309	
...					
Days:Subject370	4.866	7.017	0.693	0.48980	
Days:Subject371	-1.117	7.017	-0.159	0.87383	
Days:Subject372	-4.273	7.017	-0.609	0.54409	

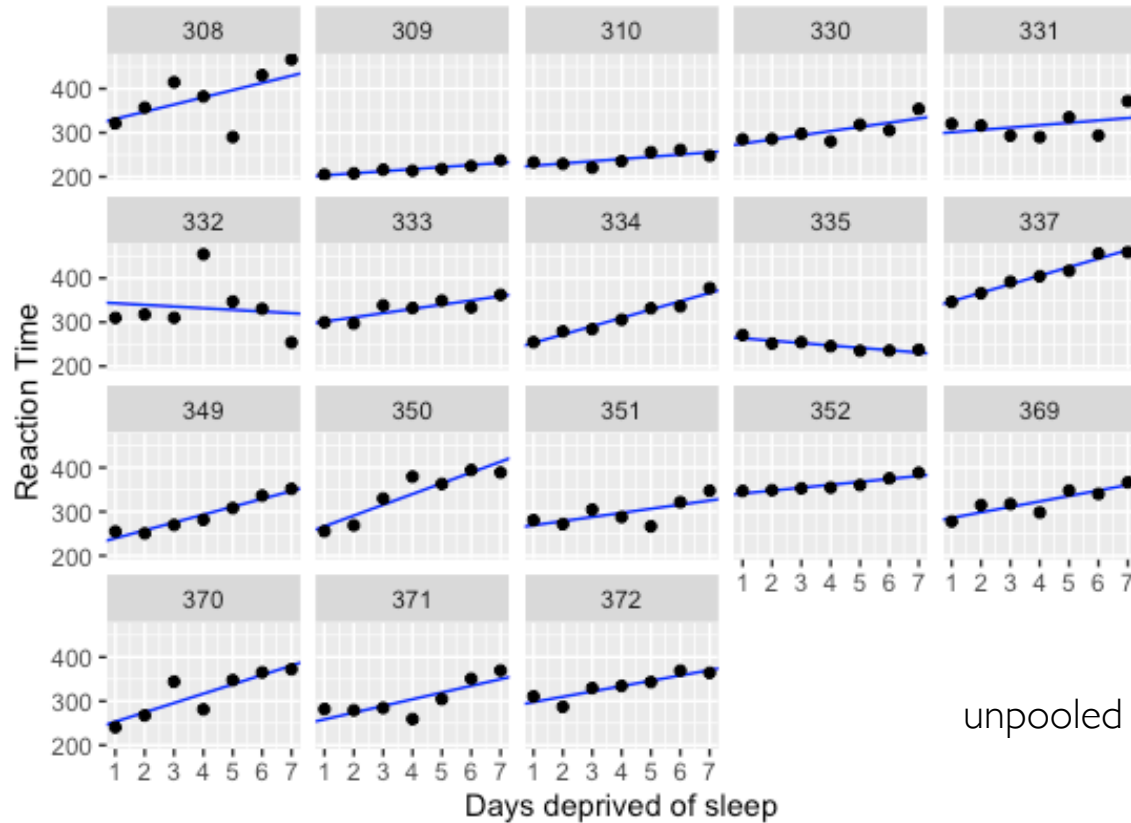
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 26.25 on 90 degrees of freedom  
Multiple R-squared: 0.8501, Adjusted R-squared: 0.7919  
F-statistic: 14.59 on 35 and 90 DF, p-value: < 2.2e-16



## example

## sleep study



unpooled model – interaction

# example

## sleep study

Run the “varying intercept model” as mod3

```
mod3 <- lmer(Reaction ~ Days + (1|Subject), ss)
summary(mod3)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method
lmerModLmerTest]
Formula: Reaction ~ Days + (1 | Subject)
Data: ss
```

REML criterion at convergence: 1251.6

Random effects:

Groups	Name	Variance	Std.Dev.
Subject	(Intercept)	2047.1	45.25
Residual		876.2	29.60

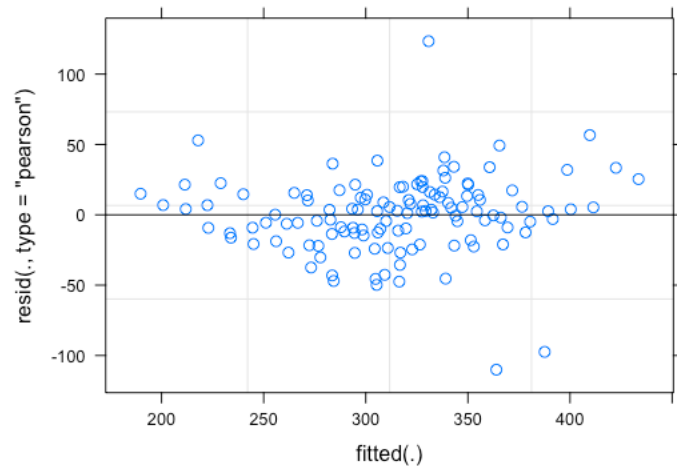
Number of obs: 126, groups: Subject, 18

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	247.702	13.538	37.598	18.297	< 2e-16 ***
Days	11.063	1.319	107.000	8.391	2.16e-13 ***

---

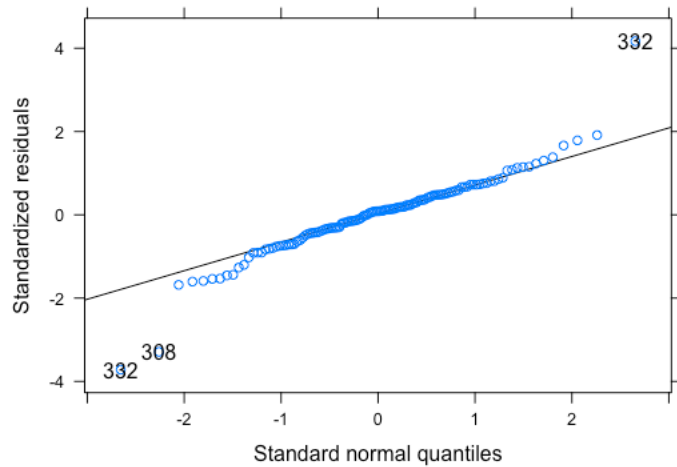
```
plot(mod3)
```



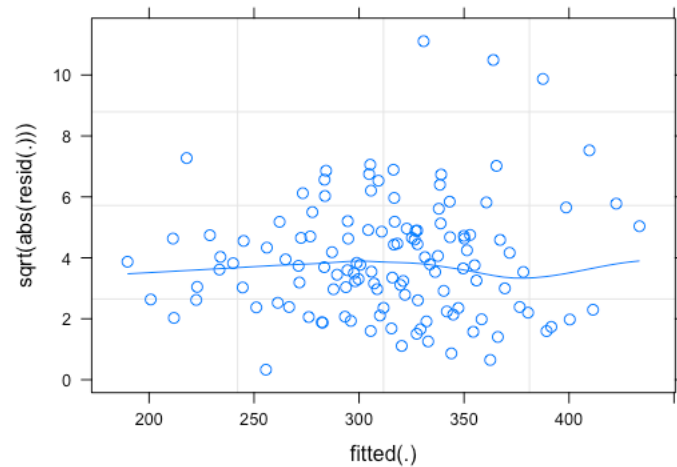
# example

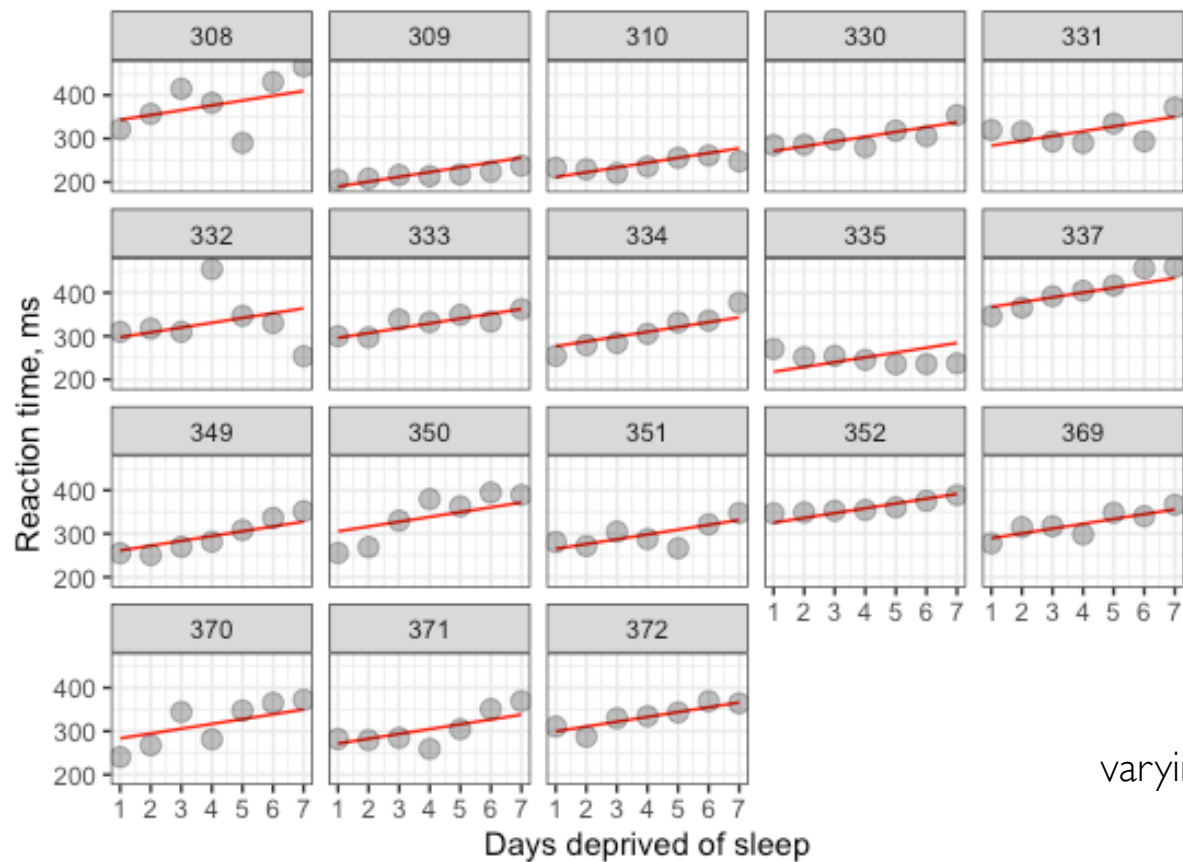
## sleep study

```
require(lattice)  
qqmath(mod3, id = 0.05)
```



```
plot(mod3, sqrt(abs(resid(.))) ~ fitted(.),  
     type = c("p", "smooth"))
```

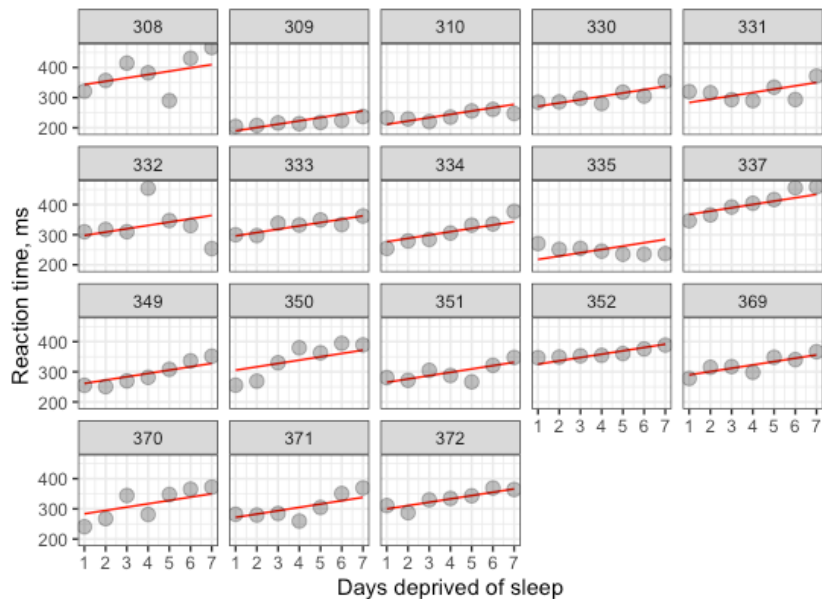




varying intercept model

# example

## sleep study



```
> re3 <- ranef(mod3)
```

```
> int <- fixef(mod3)[1] + re3$Subject
```

\$Subject	(Intercept)
308	310.1284
309	156.5314
310	178.2000
330	237.9882
331	250.5780
332	264.3711
333	262.8942
334	243.5842
335	184.7088
337	333.9777
349	228.1046
350	272.1368
351	231.8069
352	291.9799
369	256.3416
370	250.3623
371	238.4195
372	266.5240



# example

## sleep study

Run the “varying intercept/slope model” as mod4

```
mod4 <- lmer(Reaction ~ Days + (1 + Days|Subject), ss)
summary(mod4)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]  
Formula: Reaction ~ Days + (1 + Days | Subject)  
Data: ss

REML criterion at convergence: 1238.8

Random effects:

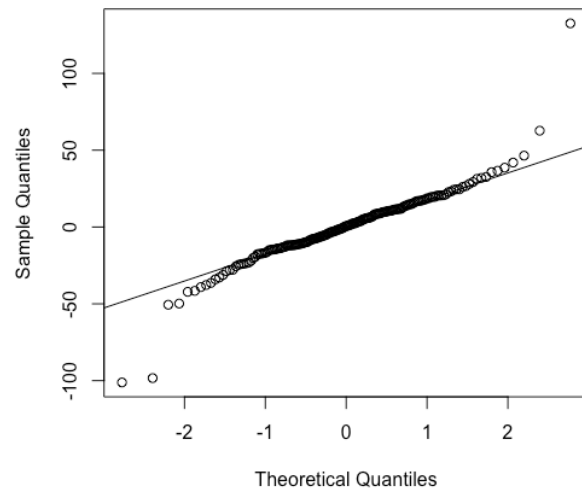
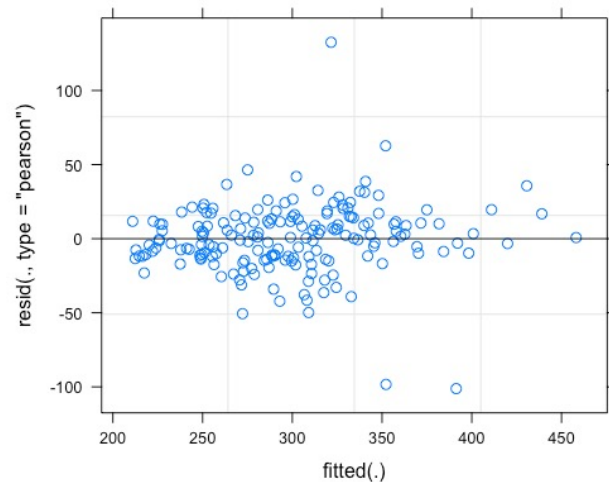
Groups	Name	Variance	Std.Dev.	Corr
Subject	(Intercept)	1473.60	38.387	
	Days	42.03	6.483	-0.31
Residual		689.25	26.254	

Number of obs: 126, groups: Subject, 18

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	247.702	11.686	16.994	21.20	1.16e-13 ***
Days	11.063	1.924	16.995	5.75	2.36e-05 ***

---





```
{merTools}  
plotREsim(Resim(mod4))
```

```
> AIC(mod1, mod2, mod2a, mod3, mod4)
```

	df	AIC
mod1	3	1363.488
mod2	20	1230.703
mod2a	37	1212.670
mod3	4	1259.646
mod4	6	1250.799

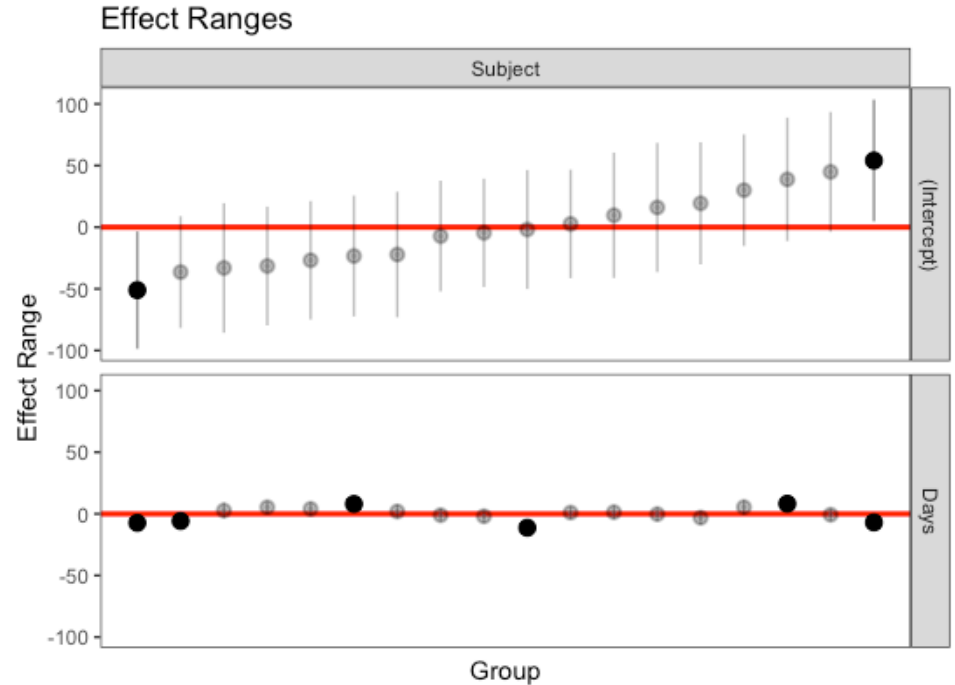
```
{MuMIn}
```

```
r.squaredGLMM(mod3)
```

R2m	R2c
0.1444303	0.743562

```
r.squaredGLMM(mod4)
```

R2m	R2c
0.1441022	0.7987363



## example

## sleep study

Run the “varying intercept/slope model” as mod4

```
{car}  
outlierTest(mod4)
```

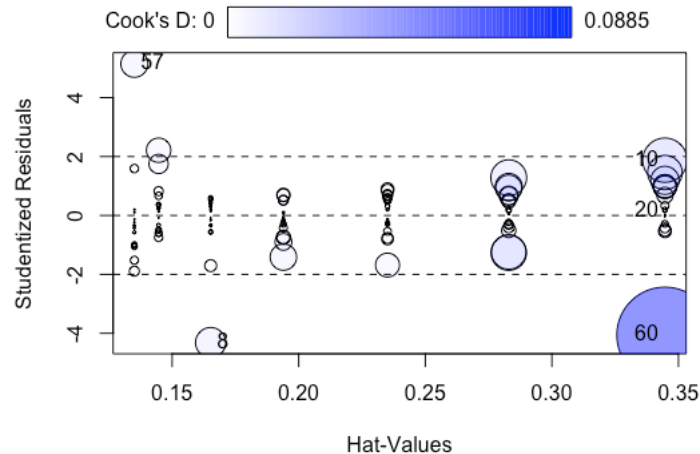
	rstudent	unadj p-value	Bonferroni p
57	5.147059	1.0576e-06	0.00013326
8	-4.316780	3.2938e-05	0.00415020
60	-4.059410	8.8413e-05	0.01114000

check out extreme residuals

```
sort(abs(resid(mod4)))
```

```
influencePlot(mod4)
```

	StudRes	Hat	CookD
8	-4.3167800	0.1652161	0.008768417
60	-4.0594101	0.3446554	0.088474558
80	0.9892771	0.3446554	0.005254465
10	1.8766544	0.3446554	0.018908677
57	5.1470591	0.1350741	0.007011464



# example

# sleep study

## Remove extreme observations

```
ss1 <- ss[ss$resid4 > -80 & ss$resid4 < 50,]  
mod5 <- lmer(Reaction ~ Days + (1 + Days|Subject), ss1)
```

Formula: Reaction ~ Days + (1 + Days | Subject)  
Data: ss1

REML criterion at convergence: 1121.2

### Random effects:

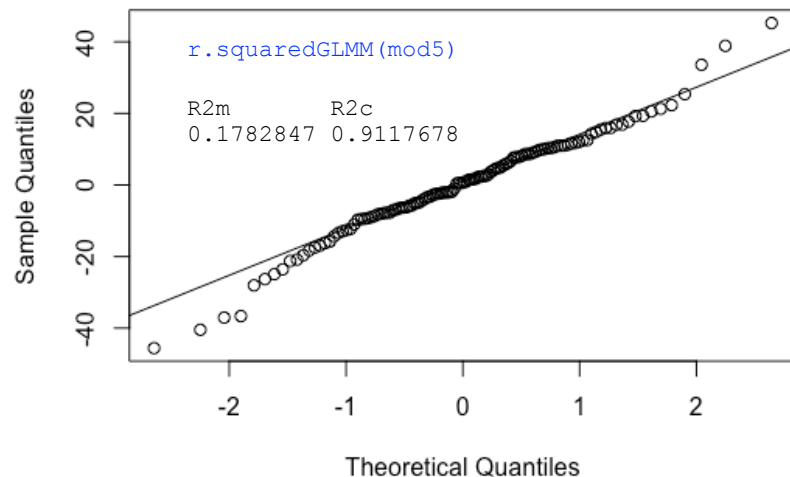
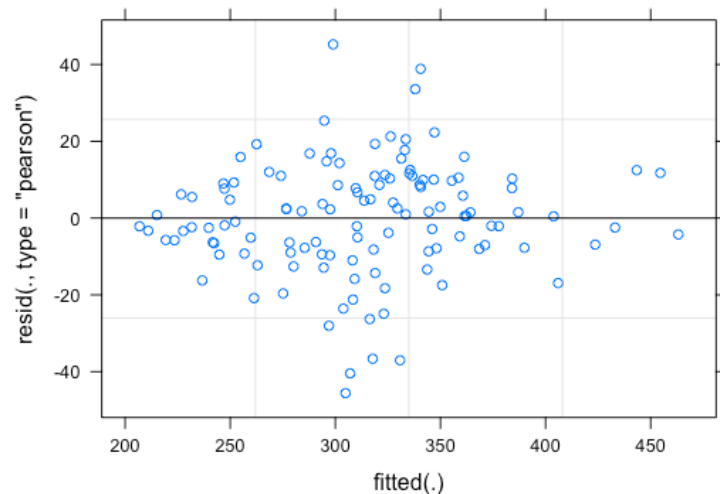
Groups	Name	Variance	Std.Dev.	Corr
Subject	(Intercept)	1382.06	37.176	
	Days	49.76	7.054	-0.30
Residual		289.41	17.012	

Number of obs: 122, groups: Subject, 18

### Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	242.559	10.013	17.005	24.224	1.27e-14 ***
Days	11.981	1.833	17.047	6.537	5.00e-06 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



a. pooled model

$$y \sim x$$

b. no pooling model

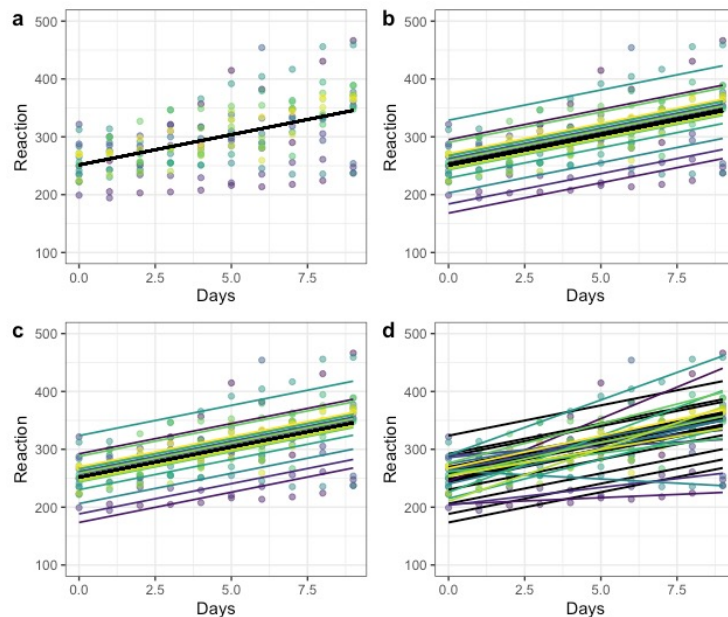
$$y \sim x + \text{subject}$$

c. varying intercept model

$$y \sim x + (1 | \text{subject})$$

d. varying intercept, varying slope model

$$y \sim x + (1 + x | \text{subject})$$



“Shrinkage”

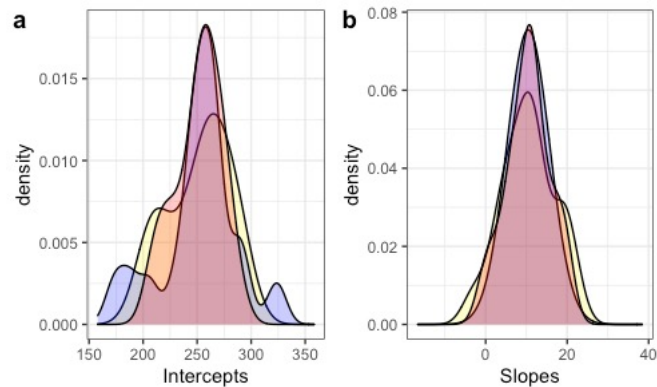
a. intercept estimates

b. slope estimates

yellow = no pooling,

blue = varying intercept

red = varying intercept & slope



# example

## chick weight

```
chl <- lmer(weight ~ Diet + (1|Chick), data = ChickWeight)
```

```
{emmmeans}
```

```
> emmmeans(chl, list(pairwise ~ Diet), adjust = "tukey")
```

```
$`emmmeans of Diet`
```

Diet	emmmean	SE	df	lower.CL	upper.CL
1	102	6.04	47.0	89.5	114
2	123	8.26	44.0	106.0	139
3	143	8.26	44.0	126.3	160
4	135	8.31	44.8	118.3	152

Degrees-of-freedom method: kenward-roger

Confidence level used: 0.95

```
$`pairwise differences of Diet`
```

	estimate	SE	df	t.ratio	p.value
1					
1 - 2	-20.98	10.2	45.0	-2.049	0.1856
1 - 3	-41.32	10.2	45.0	-4.035	0.0012
1 - 4	-33.40	10.3	45.5	-3.252	0.0112
2 - 3	-20.33	11.7	44.0	-1.740	0.3159
2 - 4	-12.42	11.7	44.4	-1.060	0.7152
3 - 4	7.91	11.7	44.4	0.675	0.9059

Degrees-of-freedom method: kenward-roger

P value adjustment: tukey method for comparing a family of 4 estimates

```
> data(ChickWeight)
```

```
> head(ChickWeight)
```

Grouped Data: weight ~ Time |  
Chick

	weight	Time	Chick	Diet
1	42	0	1	1
2	51	2	1	1
3	59	4	1	1
4	64	6	1	1
5	76	8	1	1
6	93	10	1	1

# example

## chick weight

```
chl <- lmer(weight ~ Diet + (1|Chick), data = ChickWeight)
```

```
{multcomp}
```

```
> summary(glht(chl, linfct = mcp(Diet = "Tukey")), test = adjusted("holm"))
```

Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

Fit: lmer(formula = weight ~ Diet + (1 | Chick), data = ChickWeight)

Linear Hypotheses:

	Estimate	Std. Error	z value	Pr(> z )	
2 - 1 == 0	20.983	10.236	2.050	0.161521	
3 - 1 == 0	41.316	10.236	4.036	0.000326	***
4 - 1 == 0	33.404	10.270	3.253	0.005718	**
3 - 2 == 0	20.333	11.688	1.740	0.245709	
4 - 2 == 0	12.421	11.717	1.060	0.578201	
4 - 3 == 0	-7.912	11.717	-0.675	0.578201	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Adjusted p values reported -- holm method)

```
> head(ChickWeight)
```

Grouped Data: weight ~ Time |  
Chick

	weight	Time	Chick	Diet
1	42	0	1	1
2	51	2	1	1
3	59	4	1	1
4	64	6	1	1
5	76	8	1	1
6	93	10	1	1



# example

## tree mass

```
chl <- lmer(weight ~ Diet + (1|Chick), data = ChickWeight)
```

```
{multcomp}
```

```
> summary(glht(chl, linfct = mcp(Diet = "Tukey")), test = adjusted("holm"))
```

Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts

Fit: lmer(formula = weight ~ Diet + (1 | Chick), data = ChickWeight)

Linear Hypotheses:

	Estimate	Std. Error	z value	Pr(> z )	
2 - 1 == 0	20.983	10.236	2.050	0.161521	
3 - 1 == 0	41.316	10.236	4.036	0.000326	***
4 - 1 == 0	33.404	10.270	3.253	0.005718	**
3 - 2 == 0	20.333	11.688	1.740	0.245709	
4 - 2 == 0	12.421	11.717	1.060	0.578201	
4 - 3 == 0	-7.912	11.717	-0.675	0.578201	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Adjusted p values reported -- holm method)

```
> head(ChickWeight)
```

Grouped Data: weight ~ Time |  
Chick

	weight	Time	Chick	Diet
1	42	0	1	1
2	51	2	1	1
3	59	4	1	1
4	64	6	1	1
5	76	8	1	1
6	93	10	1	1

## example

## tree mass

The sleep study examines the relationship between tree biomass (kg) and its height (m), dbh (mm), and disturbance (logged+hunted, logged, none).

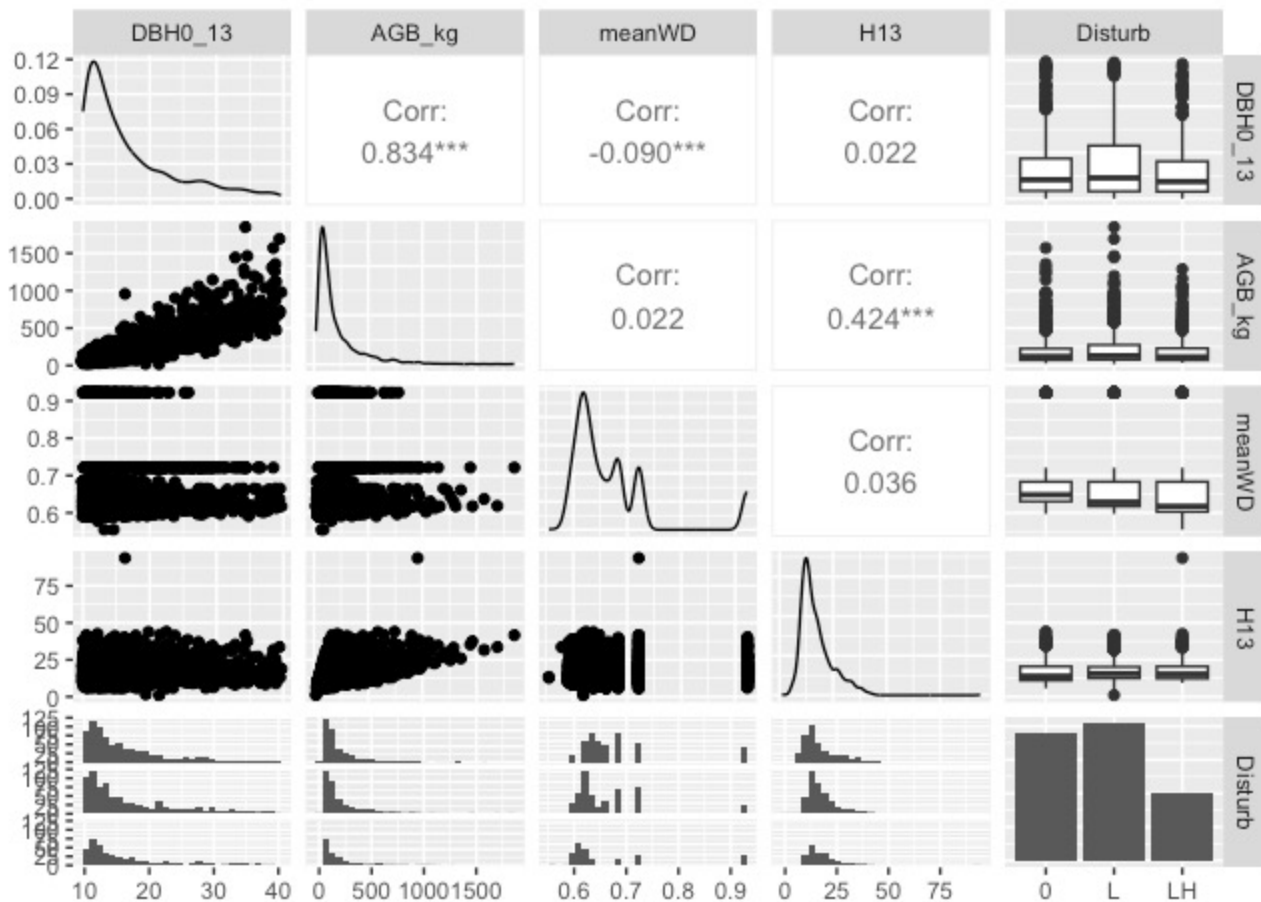
There are 1980 trees from 7 species and 30 plots.

- what type of model? why?
- model structure?

example

tree mass

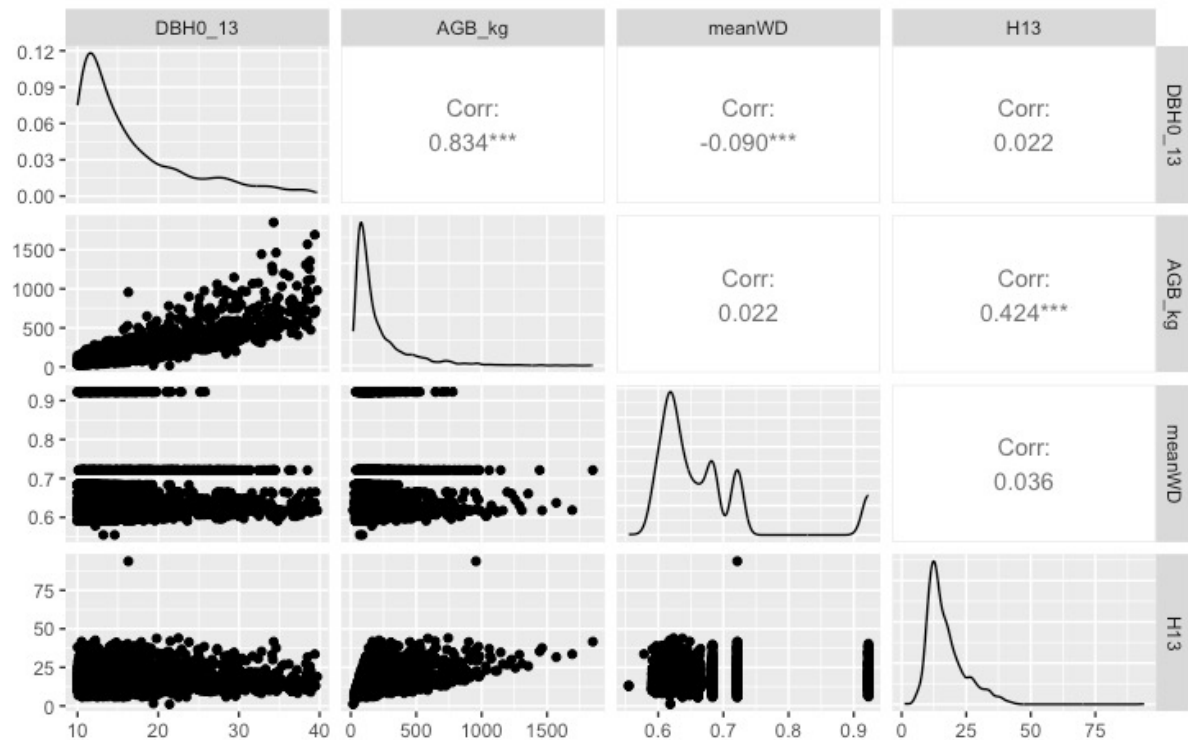
```
ggpairs(ss1, columns = c("DBH0_13",  
"AGB_kg", "meanWD", "H13", "Disturb"))
```



# example

## tree mass

```
ggpairs(ss1, columns = c("DBH0_13",  
"AGB_kg", "meanWD", "H13"))
```



# example

## tree mass

```
m2 <- lmer(AGB_kg ~ H13 + DBH0_13 + Disturb + (1|spp) + (1|Plot), data = ss1)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]  
Formula: AGB\_kg ~ H13 + DBH0\_13 + Disturb + (1 | spp) + (1 | Plot)  
Data: ss1

REML criterion at convergence: 22805.6

Random effects:

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	36.73	6.06
spp	(Intercept)	547.04	23.39
Residual		5711.30	75.57

Number of obs: 1984, groups: Plot, 30; spp, 7

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	-424.3423	11.3617	14.4426	-37.349	8.93e-16 ***
H13	11.4182	0.2328	1921.3592	49.057	< 2e-16 ***
DBH0_13	26.0274	0.2957	1897.0509	88.012	< 2e-16 ***
DisturbL	1.7596	4.8046	24.5533	0.366	0.717
DisturbLH	4.1783	5.7263	35.0128	0.730	0.470

-----

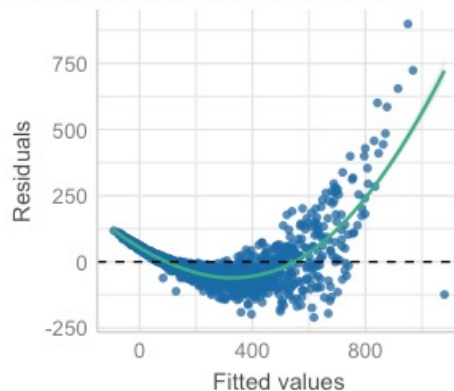
example

tree mass

```
performance::check_model(m2,  
  check = c("reqq", "normality", "linearity"))
```

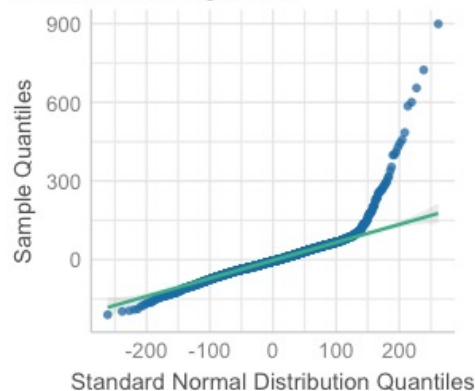
Linearity

Reference line should be flat and horizontal



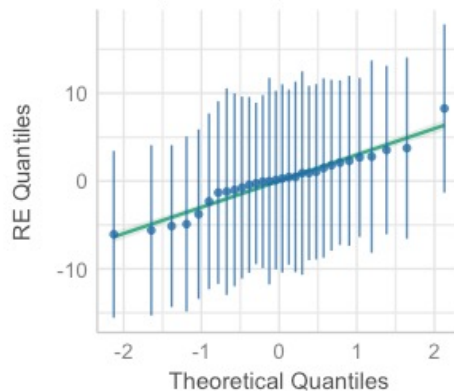
Normality of Residuals

Dots should fall along the line



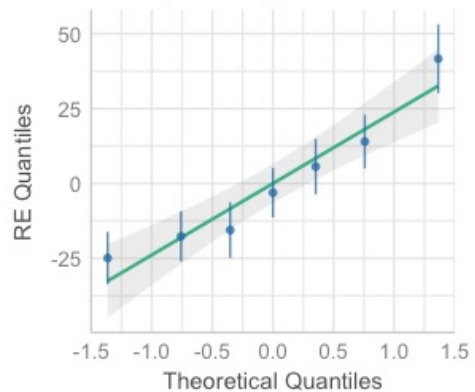
Normality of Random Effects (Plot)

Dots should be plotted along the line



Normality of Random Effects (spp)

Dots should be plotted along the line



## example

## tree mass

```
m3 <- lmer(AGB_kg ~ H13 + DBH0_13 + Disturb + (1 + Plot|spp), data = ss1)  
boundary (singular) fit: see help('isSingular')
```

the parameters are on the boundary of the feasible parameter space: variances of one or more linear combinations of effects are (close to) zero

when a random effect variance is estimated very near zero and (very loosely) the data is not sufficiently informative to drag the estimate away from the zero starting value

# example

## tree mass

```
m4 <- lmer(log(AGB_kg) ~ H13 + DBH0_13 + meanWD + Disturb + (1|spp) + (1|Plot), data = ssl)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]

Formula: log(AGB\_kg) ~ H13 + DBH0\_13 + Disturb + (1 | spp) + (1 | Plot)

Data: ssl

REML criterion at convergence: -1529.5

Random effects:

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	0.001714	0.0414
spp	(Intercept)	0.022916	0.1514
Residual		0.025483	0.1596

Number of obs: 1984, groups: Plot, 30; spp, 7

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )	
(Intercept)	2.459e+00	6.053e-02	7.413e+00	40.632	5.55e-10	***
H13	5.040e-02	4.979e-04	1.972e+03	101.216	< 2e-16	***
DBH0_13	9.925e-02	6.316e-04	1.966e+03	157.159	< 2e-16	***
DisturbL	1.051e-02	2.043e-02	2.559e+01	0.514	0.611	
DisturbLH	-2.347e-03	2.236e-02	3.099e+01	-0.105	0.917	

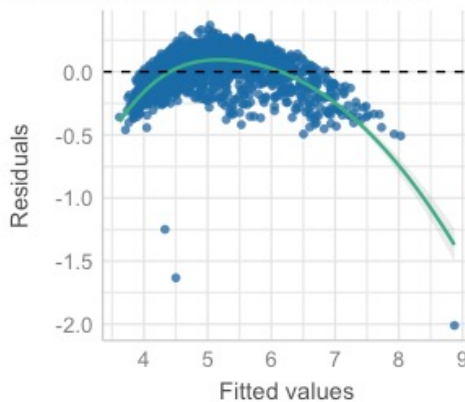
-----



```
performance::check_model(m4,
  check = c("reqq", "qq", "linearity"))
```

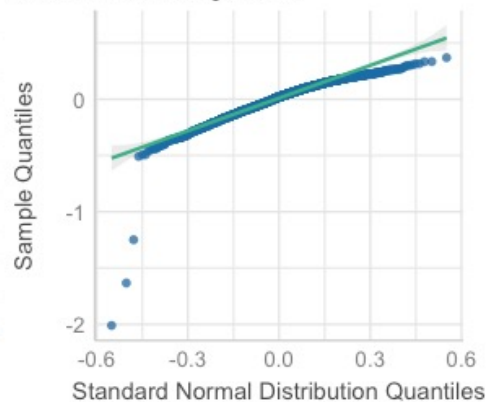
### Linearity

Reference line should be flat and horizontal



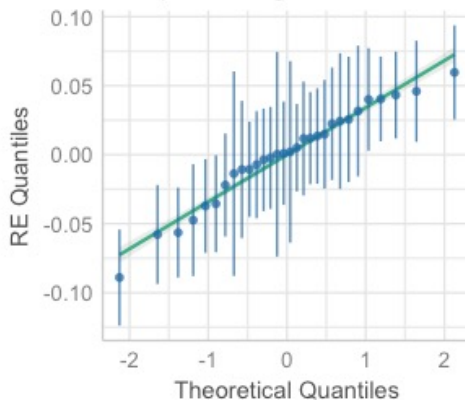
### Normality of Residuals

Dots should fall along the line



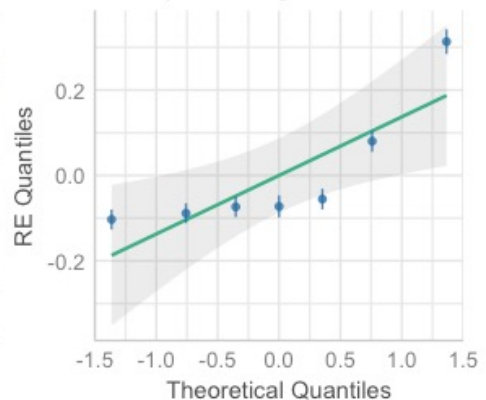
### Normality of Random Effects (Plot)

Dots should be plotted along the line



### Normality of Random Effects (spp)

Dots should be plotted along the line



# example

## tree mass

```
m5 <- lmer(log(AGB_kg) ~ H13 + log(DBH0_13) + Disturb + (1|spp) + (1|Plot), data = ss1)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest  
]
```

```
Formula: log(AGB_kg) ~ H13 + DBH0_13 + Disturb + (1 | spp) + (1 | Plot)
```

```
Data: ss1
```

```
REML criterion at convergence: -1529.5
```

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	0.001714	0.0414
spp	(Intercept)	0.022916	0.1514
Residual		0.025483	0.1596

```
Number of obs: 1984, groups: Plot, 30; spp, 7
```

```
Fixed effects:
```

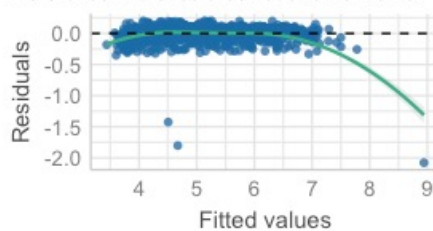
	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	2.459e+00	6.053e-02	7.413e+00	40.632	5.55e-10 ***
H13	5.040e-02	4.979e-04	1.972e+03	101.216	< 2e-16 ***
DBH0_13	9.925e-02	6.316e-04	1.966e+03	157.159	< 2e-16 ***
DisturbL	1.051e-02	2.043e-02	2.559e+01	0.514	0.611
DisturbLH	-2.347e-03	2.236e-02	3.099e+01	-0.105	0.917

```
-----
```

```
performance::check_model(m5,
  check = c("vif", "reqq", "qq", "linearity"))
```

### Linearity

Reference line should be flat and horizontal



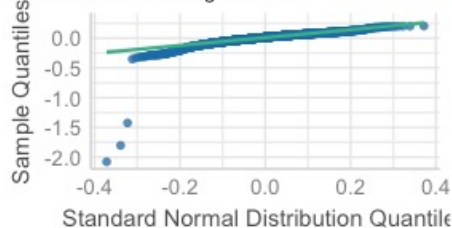
### Collinearity

High collinearity (VIF) may inflate parameter uncertainty



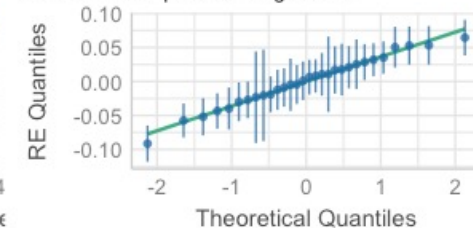
### Normality of Residuals

Dots should fall along the line



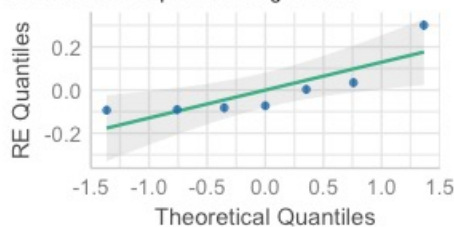
### Normality of Random Effects (Plot)

Dots should be plotted along the line



### Normality of Random Effects (spp)

Dots should be plotted along the line



## example

## tree mass

```
ssl$resid <- resid(m5)
ssl[ssl$resid < -1, ]
```

Plot	DBH0_13	meanWD	AGB13.Mg	H13	genusCorr	sppCorr
496	10	16.3	0.7213065	0.95674746	93.843623	Strombosia nigropunctata
2164	14	21.4	0.6185869	0.01760324	1.058761	Greenwayodendron suaveolens
2361	14	19.4	0.6185869	0.02178308	1.602595	Greenwayodendron suaveolens

	familyAPG	AGB_kg	spp	Disturb	resid
496	Olacaceae	956.74746	Strombosia nigropunctata	LH	-2.077444
2164	Annonaceae	17.60324	Greenwayodendron suaveolens	L	-1.803027
2361	Annonaceae	21.78308	Greenwayodendron suaveolens	L	-1.424433

```
ss2 <- ssl[ssl$resid > -1,]
```

```
m6 <- lmer(log(AGB_kg) ~ H13 + log(DBH0_13) +  
            Disturb + (1|spp) + (1|Plot), data = ss2)
```

# example

## tree mass

```
> summary(m6)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]
```

```
Formula: log(AGB_kg) ~ H13 + log(DBH0_13) + Disturb + (1 | spp) + (1 | Plot)
```

```
Data: ss2
```

```
REML criterion at convergence: -4194.3
```

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	0.001731	0.04160
spp	(Intercept)	0.019695	0.14034
Residual		0.006501	0.08063

```
Number of obs: 1981, groups: Plot, 30; spp, 7
```

```
MuMIn::r.squaredGLMM(m6)
```

```
R2m R2c
```

```
[1,] 0.9581771 0.9902642
```

```
Fixed effects:
```

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	-1.295e+00	5.750e-02	8.243e+00	-22.518	1.08e-08 ***
H13	5.121e-02	2.598e-04	1.955e+03	197.081	< 2e-16 ***
log(DBH0_13)	1.961e+00	6.199e-03	1.951e+03	316.409	< 2e-16 ***
DisturbL	3.095e-02	1.912e-02	2.451e+01	1.619	0.118
DisturbLH	1.447e-02	2.000e-02	2.767e+01	0.724	0.475

```
---
```

## example

## tree mass

```
> summary(m6)
```

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )	
(Intercept)	-1.295e+00	5.750e-02	8.243e+00	-22.518	1.08e-08	***
H13	5.121e-02	2.598e-04	1.955e+03	197.081	< 2e-16	***
log(DBH0_13)	1.961e+00	6.199e-03	1.951e+03	316.409	< 2e-16	***
DisturbL	3.095e-02	1.912e-02	2.451e+01	1.619	0.118	
DisturbLH	1.447e-02	2.000e-02	2.767e+01	0.724	0.475	
---						

```
> cfs <- fixef(m6)
> (exp(cfs[2]) - 1) * 100
```

```
H13
5.254165
```

**Only DV is log-transformed:** Exponentiate the coefficient, subtract one from this number, and multiply by 100.

For every one-unit increase in H13, AGB increases by about 5.3%.

## example

## tree mass

```
> summary(m6)
```

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )	
(Intercept)	-1.295e+00	5.750e-02	8.243e+00	-22.518	1.08e-08	***
H13	5.121e-02	2.598e-04	1.955e+03	197.081	< 2e-16	***
log(DBH0_13)	1.961e+00	6.199e-03	1.951e+03	316.409	< 2e-16	***
DisturbL	3.095e-02	1.912e-02	2.451e+01	1.619	0.118	
DisturbLH	1.447e-02	2.000e-02	2.767e+01	0.724	0.475	

---

For 5% percent increase,:

```
> (1.05^1.96-1)*100  
[1] 10.03505
```

AGB increases by about 10%

**Both DV & IV log-transformed:** Interpret the coefficient as the percent increase in the dependent variable for every 1% increase in the independent variable.

For every 1% increase in DBH, AGB increases by about 1.96%.

Only independent/predictor variable(s) is log-transformed. Divide the coefficient by 100. This tells us that a 1% increase in the independent variable increases (or decreases) the dependent variable by (coefficient/100) units.

## example

## tree mass

```
> summary(m6)
```

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )	
(Intercept)	-1.295e+00	5.750e-02	8.243e+00	-22.518	1.08e-08	***
H13	5.121e-02	2.598e-04	1.955e+03	197.081	< 2e-16	***
log(DBH0_13)	1.961e+00	6.199e-03	1.951e+03	316.409	< 2e-16	***
DisturbL	3.095e-02	1.912e-02	2.451e+01	1.619	0.118	
DisturbLH	1.447e-02	2.000e-02	2.767e+01	0.724	0.475	

---

Only independent/predictor variable(s) is log-transformed. Divide the coefficient by 100. This tells us that a 1% increase in the independent variable increases (or decreases) the dependent variable by (coefficient/100) units. Example: the coefficient is 0.198, then  $0.198/100 = 0.00198$ . For every 1% increase in the independent variable, our dependent variable increases by about 0.002. For x percent increase, multiply the coefficient by  $\log(1.x)$ . Example: For every 10% increase in the independent variable, our dependent variable increases by about  $0.198 * \log(1.10) = 0.02$ .

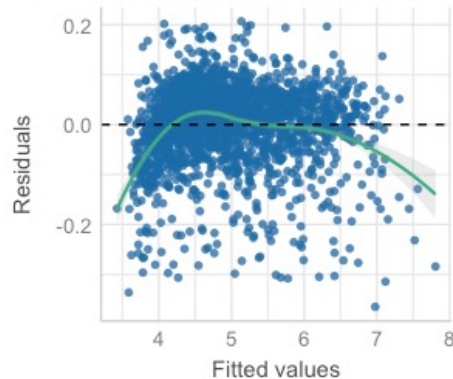
this situation not shown  
in this model!



```
performance::check_model(m6,
  check = c("reqq", "qq", "linearity"))
```

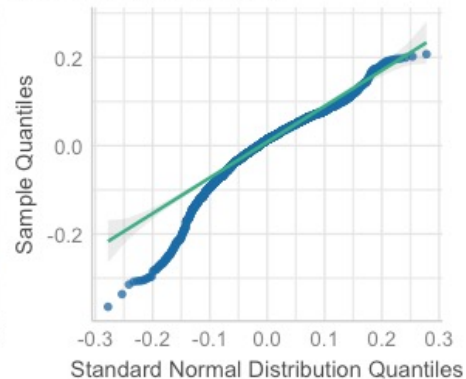
### Linearity

Reference line should be flat and horizontal



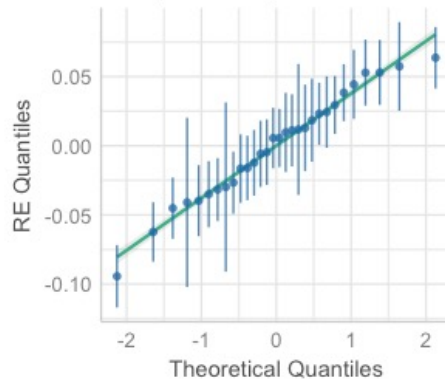
### Normality of Residuals

Dots should fall along the line



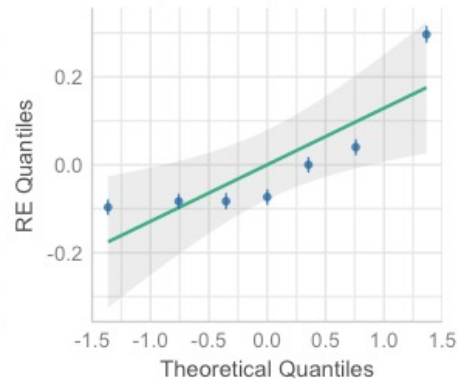
### Normality of Random Effects (Plot)

Dots should be plotted along the line



### Normality of Random Effects (spp)

Dots should be plotted along the line



## example

## tree mass

```
> summary(m6)
```

Random effects:

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	0.001731	0.04160
spp	(Intercept)	0.019695	0.14034
Residual		0.006501	0.08063

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )	
(Intercept)	-1.295e+00	5.750e-02	8.243e+00	-22.518	1.08e-08	***
H13	5.121e-02	2.598e-04	1.955e+03	197.081	< 2e-16	***
log(DBH0_13)	1.961e+00	6.199e-03	1.951e+03	316.409	< 2e-16	***
DisturbL	3.095e-02	1.912e-02	2.451e+01	1.619	0.118	
DisturbLH	1.447e-02	2.000e-02	2.767e+01	0.724	0.475	

---

Our model accounted for about 99% of the variance in AGB with main effects accounting for 95.8% of variance. Height (effect = 0.051,  $t = 197.1$ ,  $p < 0.001$ ) and tree diameter (log-effect = 1.96,  $t = 316.4$ ,  $p < 0.001$ ) strongly affect aboveground biomass. Tree plots (SD = 0.04) and species (SD = 0.14) accounted for a relatively small proportion of the variance in AGB. For every increase of 1-m in height, AGB increases by about 5.3%; whereas for every 1% increase in tree diameter, AGB increases by about 1.96%.

## example

## tree mass

```
ss2$Hz <- scale(ss2$H13, scale = T)
ss2$Dz <- scale(ss2$DBH0_13, scale = T)
m7 <- lmer(log(AGB_kg) ~ Hz + log(Dz) +
           Disturb + (1|spp) + (1|Plot), data = ss2)

> summary(m7)
Linear mixed model fit by REML. t-tests use Satterthwaite's method
['lmerModLmerTest']
Formula: log(AGB_kg) ~ Hz + log(Dz) + Disturb + (1 | spp) + (1 | Plot)
Data: ss2
```

Random effects:

Groups	Name	Variance	Std.Dev.
Plot	(Intercept)	0.001103	0.0332
spp	(Intercept)	0.020161	0.1420
Residual		0.065633	0.2562

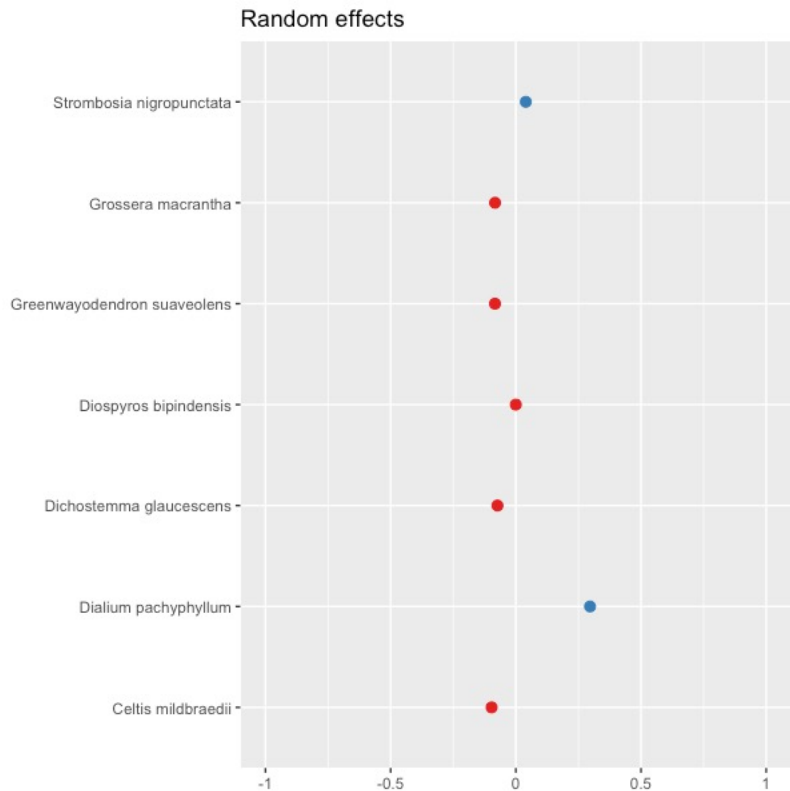
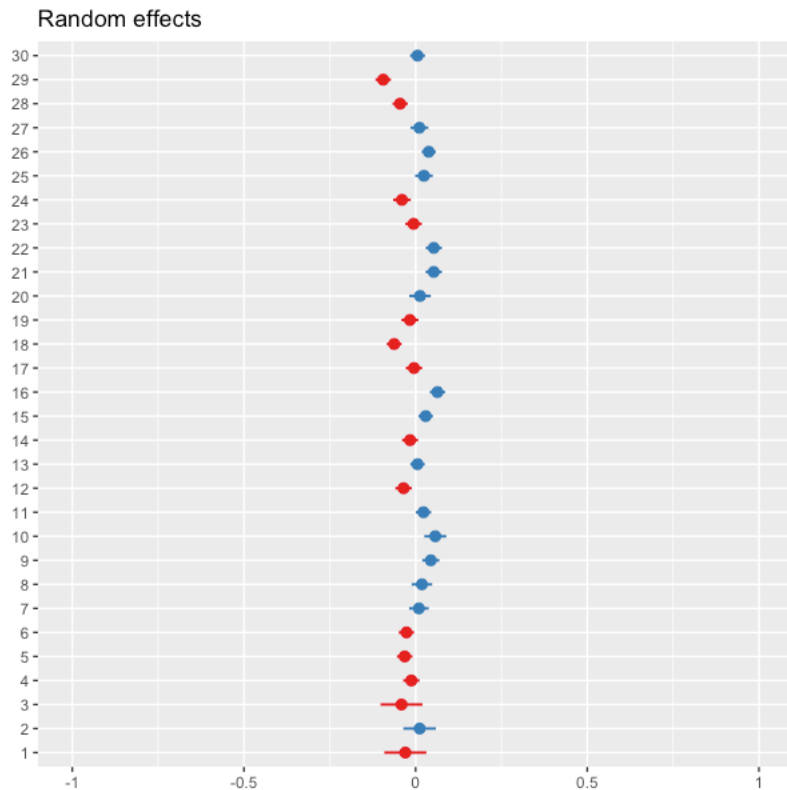
Number of obs: 696, groups: Plot, 28; spp, 7

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t )
(Intercept)	5.857e+00	5.808e-02	6.963e+00	100.830	2.8e-12 ***
Hz	3.625e-01	9.801e-03	6.775e+02	36.988	< 2e-16 ***
log(Dz)	2.925e-01	8.149e-03	6.854e+02	35.892	< 2e-16 ***
DisturbL	6.205e-02	2.728e-02	2.146e+01	2.275	0.0333 *
DisturbLH	2.668e-02	3.309e-02	3.425e+01	0.806	0.4256

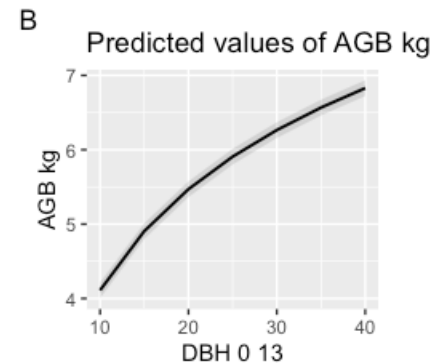
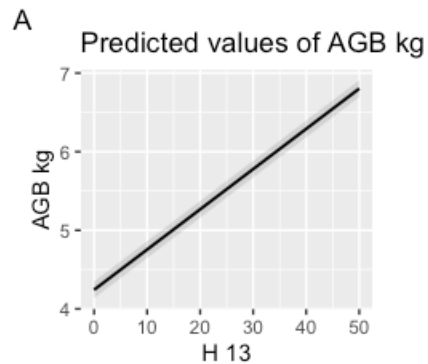
---

```
require(sjPlot)
re_gg <- plot_model(m6, type = "re", sort.est = TRUE)
re_gg[1]
re_gg[2]
```

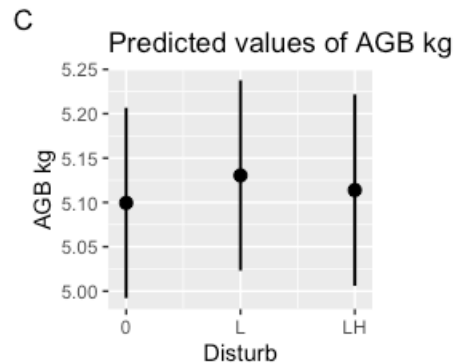


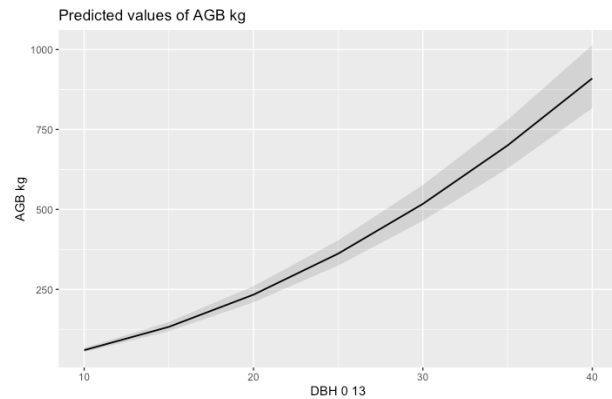
# example

## tree mass

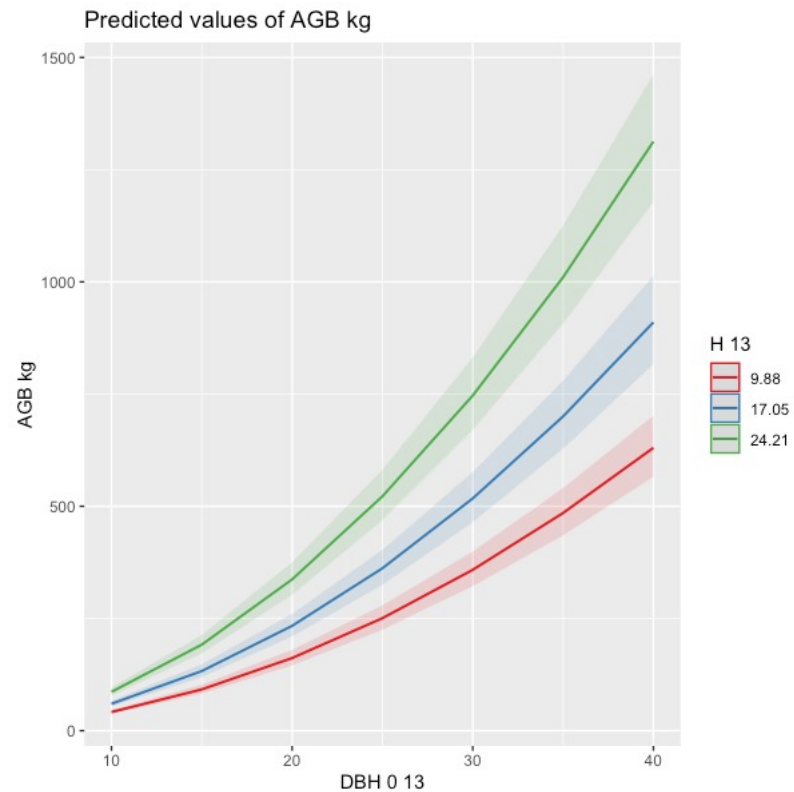


```
eff_gg <- plot_model(m6, type = "eff", sort.est = TRUE)  
plot_grid(eff_gg)
```



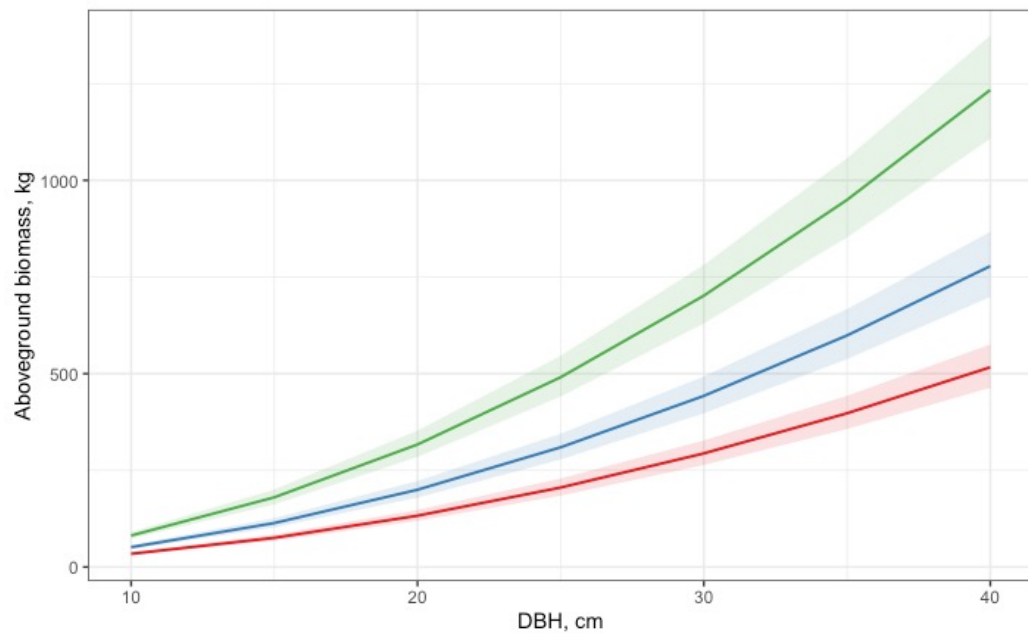


```
plot_model(m6, type = "pred", terms = "DBH0_13")
```



```
plot_model(m6, type = "pred", terms = c("DBH0_13", "H13"))
```

```
plot_model(m6, type = "pred", terms = c("DBH0_13", "H13 [6, 14, 23]"),  
           title = "") +  
  labs(y = "Aboveground biomass, kg", x = "DBH, cm") +  
  theme_bw() + theme(legend.position = "none")
```





Questions?