ENV 710: Lecture 4

inference

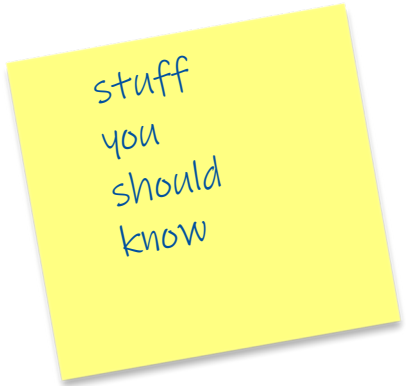
Duke | NICHOLAS SCHOOL OF THE ENVIRONMENT

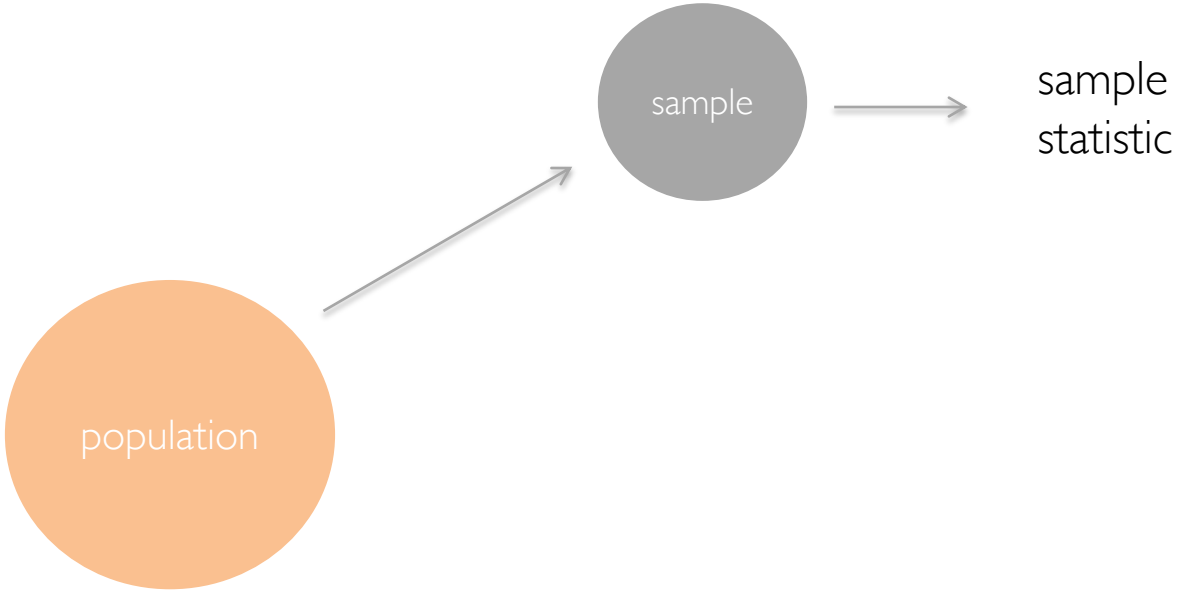
statistical inference

sampling

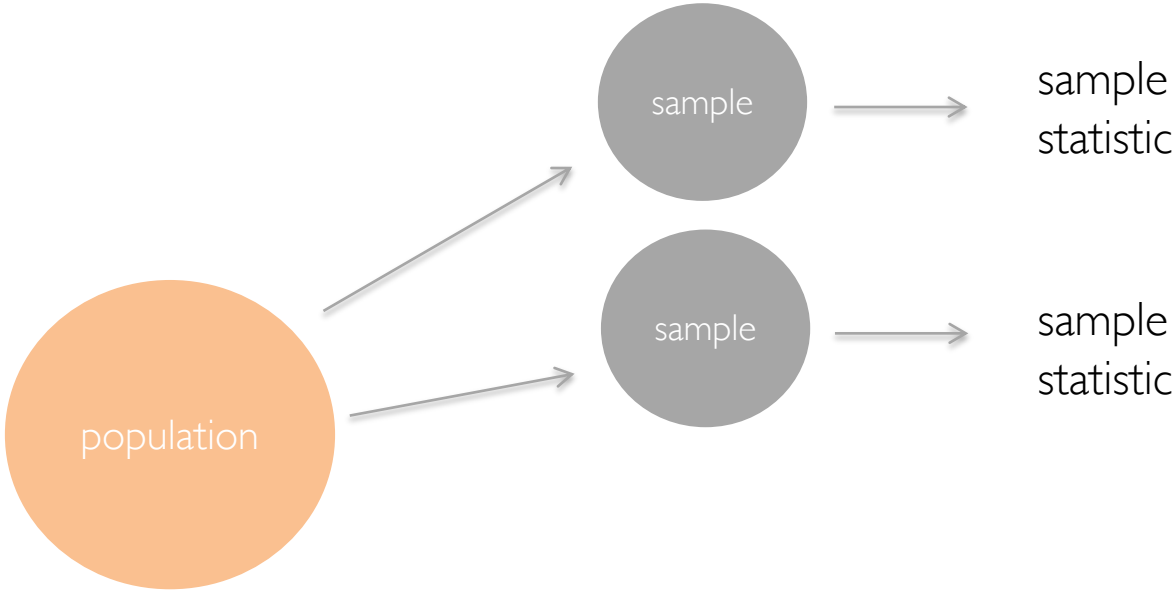
learning goals

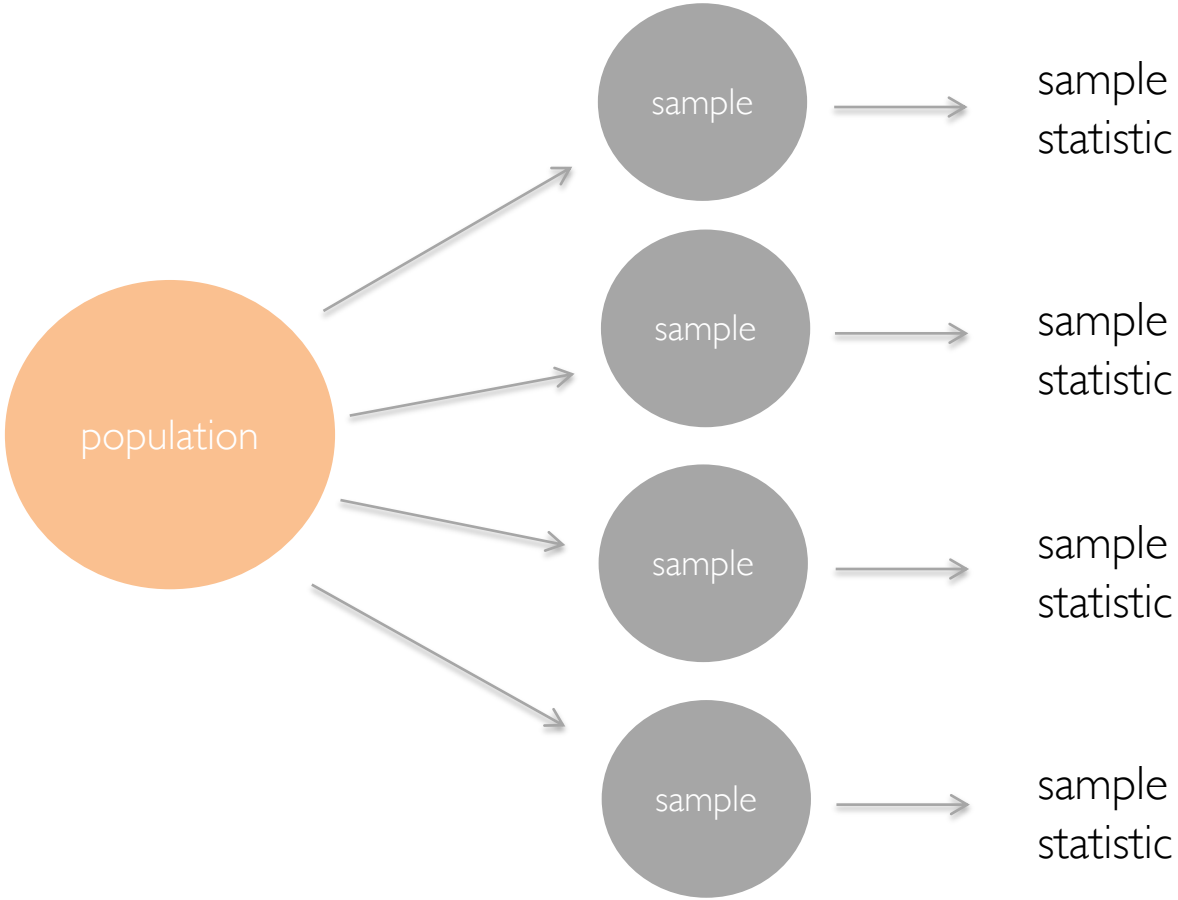
- what is sampling and why do we do it?
- understand concepts such as IID and sampling distributions
- know when to use standard error versus standard deviation



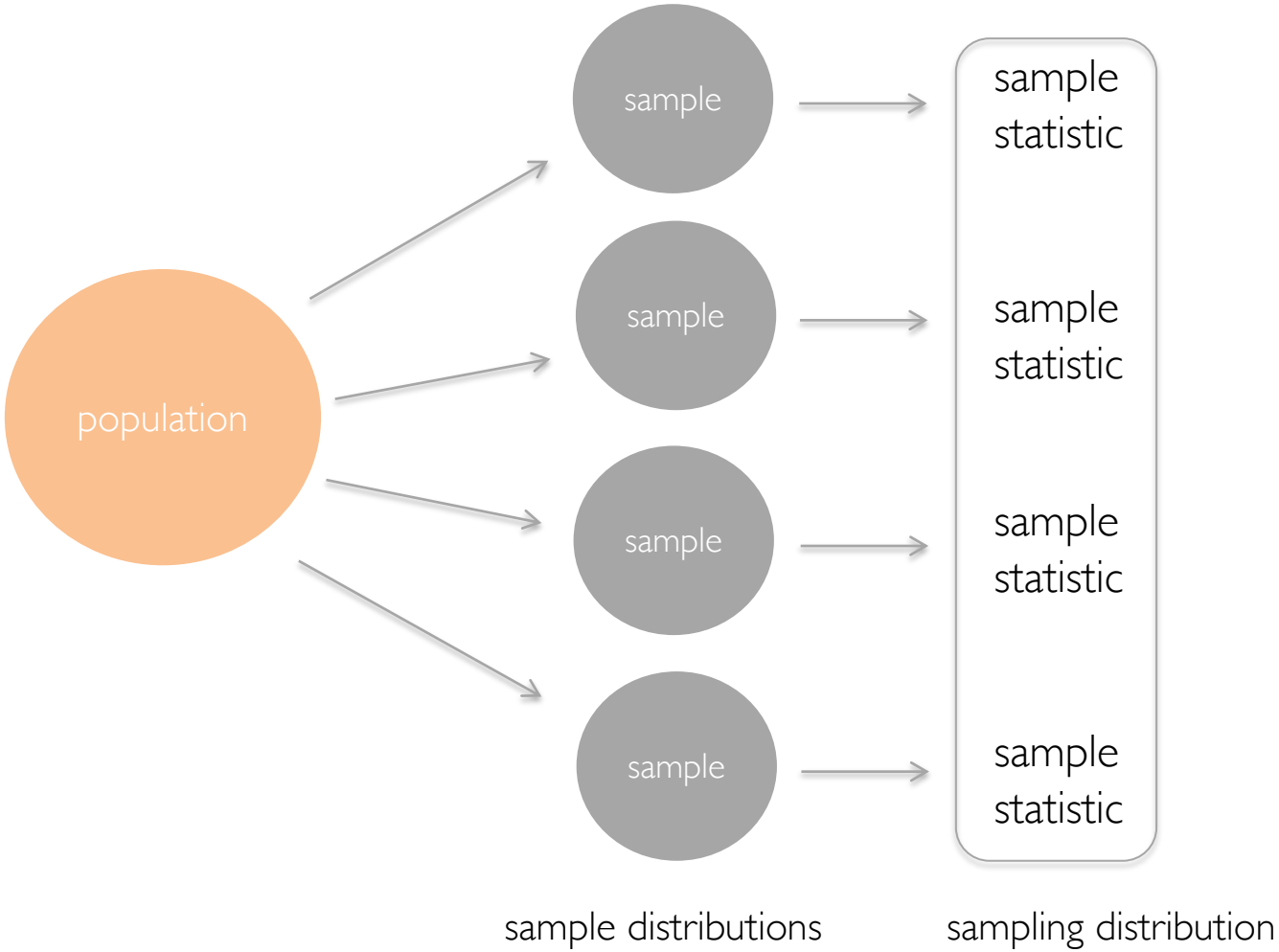


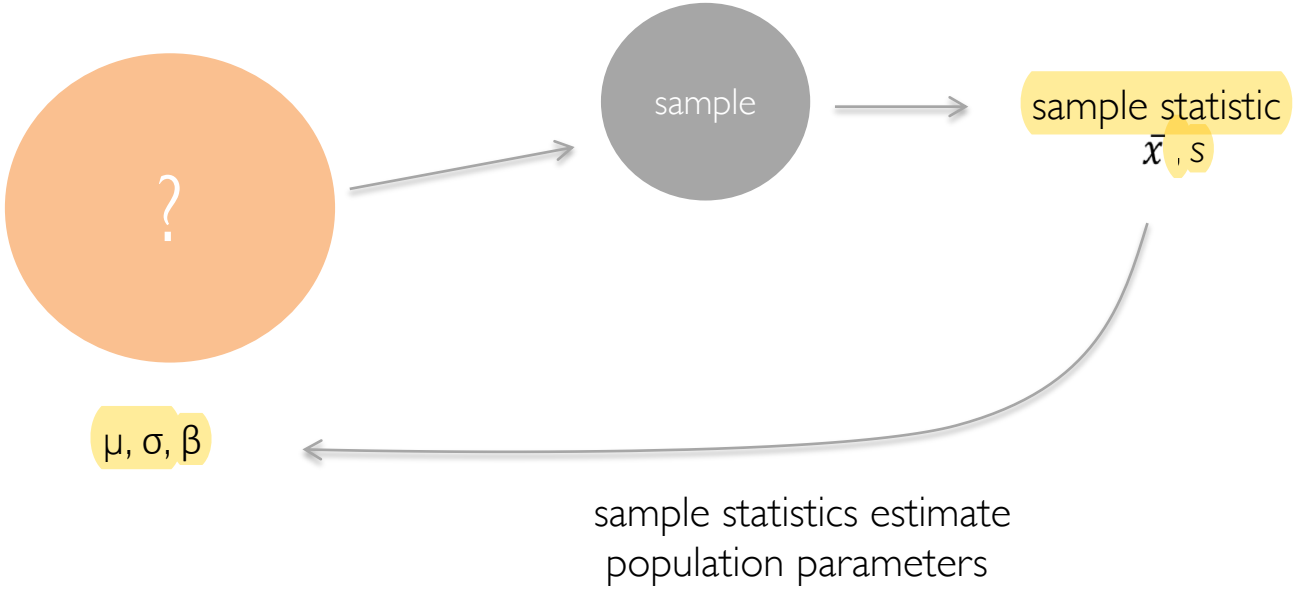
IID
independent and
identically distributed


sample
sample
statistic
sample
sample
statistic
population

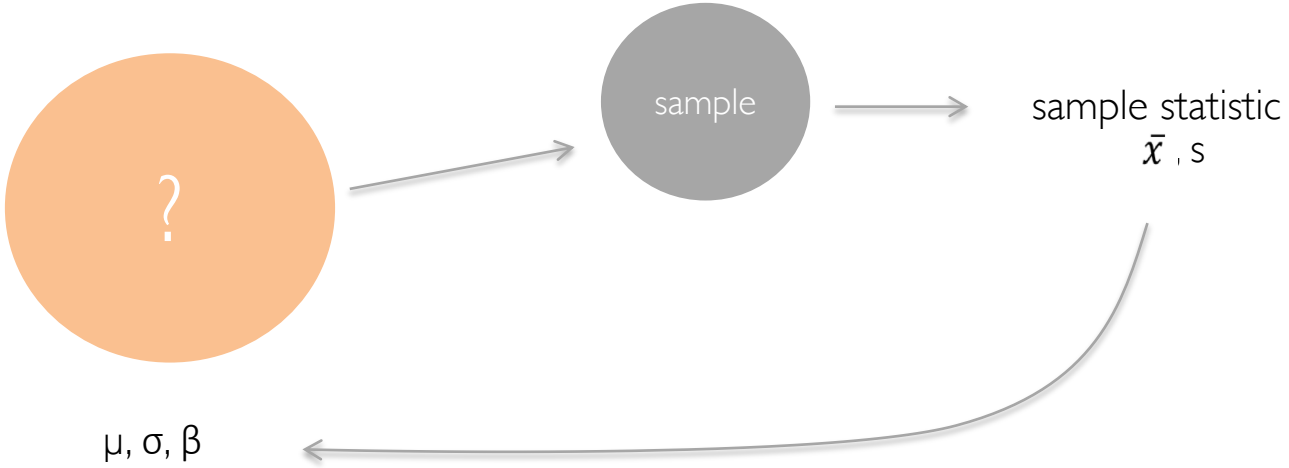


sample distributions


population
sample
sample
sample
sample
sample statistic
sample statistic
sample statistic
sample statistic
sample distributions
sampling distribution

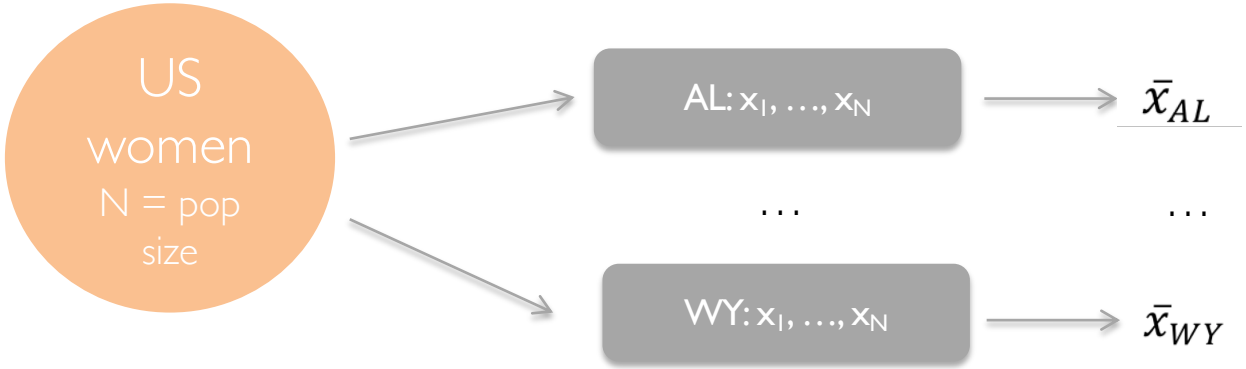


- sample statistics are random variables that vary from sample to sample
- $\bar{x}$ is an estimator of μ



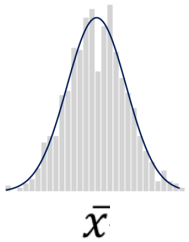
estimators should be unbiased and efficient

- an estimator is unbiased if its expected value equals the population value, e.g. $E(\bar{X}) = \mu$
- an efficient estimator needs relatively few samples to achieve a given performance



$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}}$$

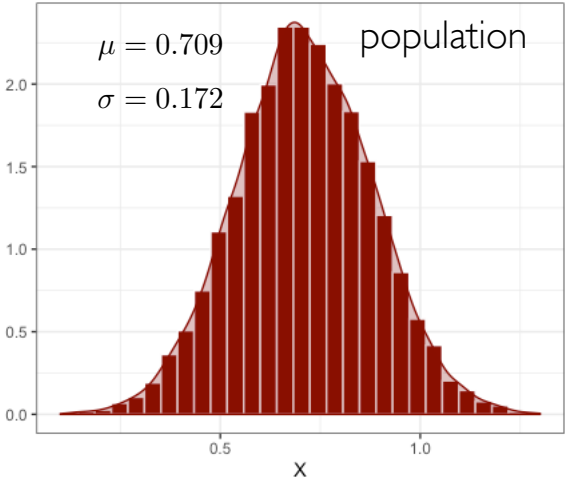


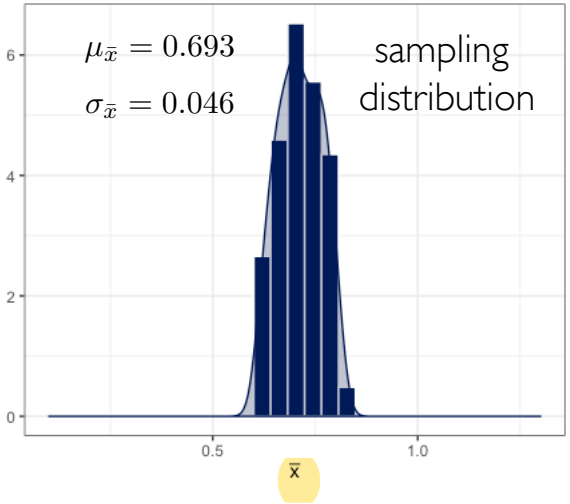
sampling distribution of all the sample means

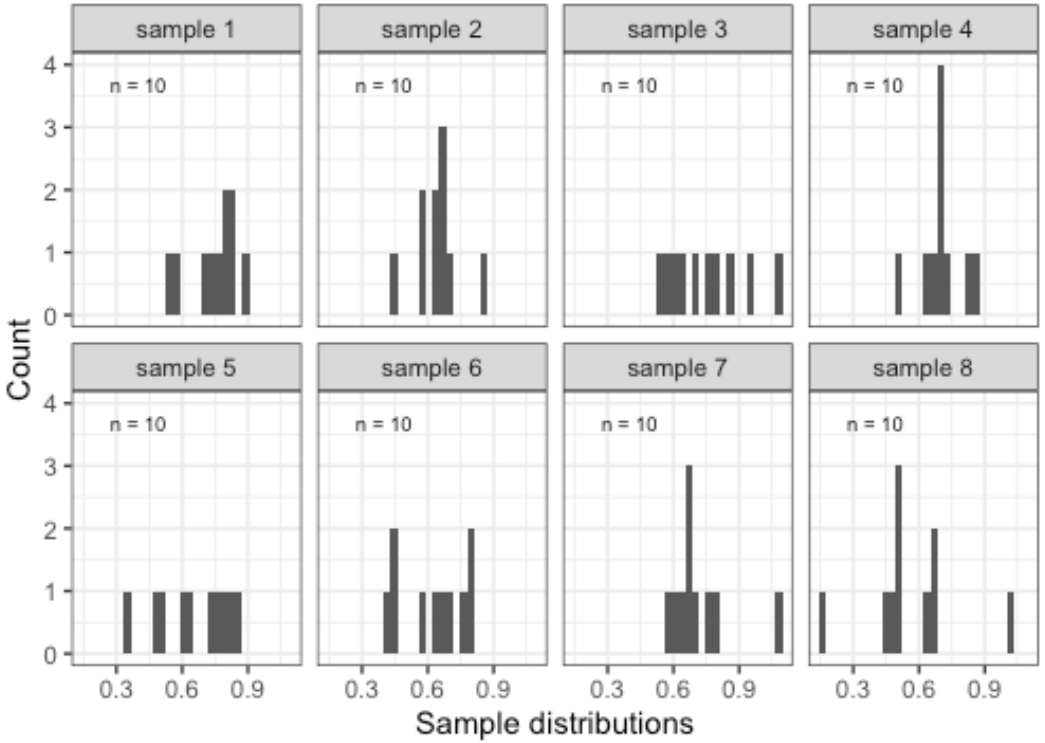
the mean of the sampling distribution, i.e., the average of 51 states' means $\mu(\bar{x}) \sim \mu$

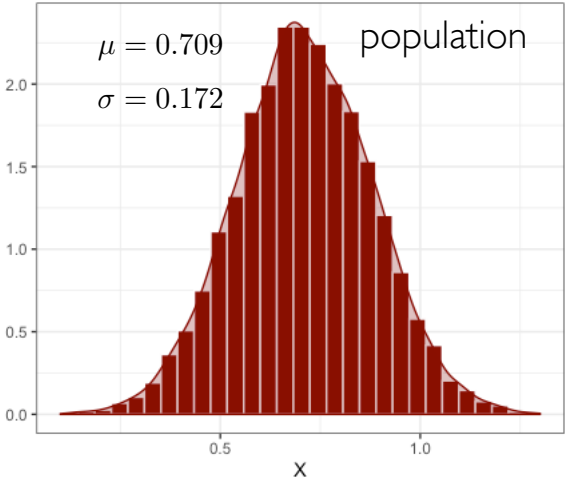
standard deviation of the sampling distribution $\sigma(\bar{x}) < \sigma$ sd of the popultion

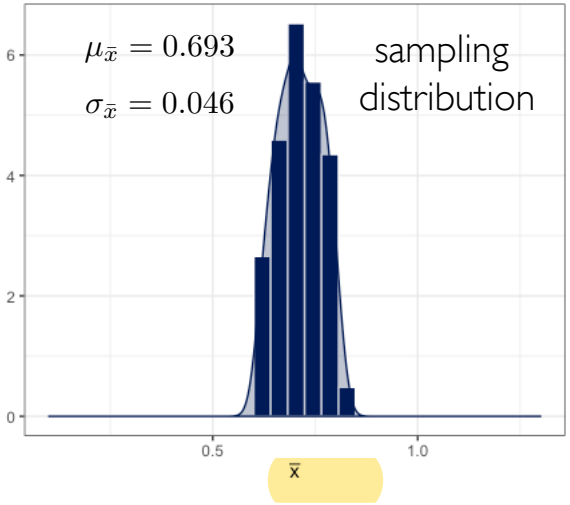
because each of the average sample is less variable than using random sample observations

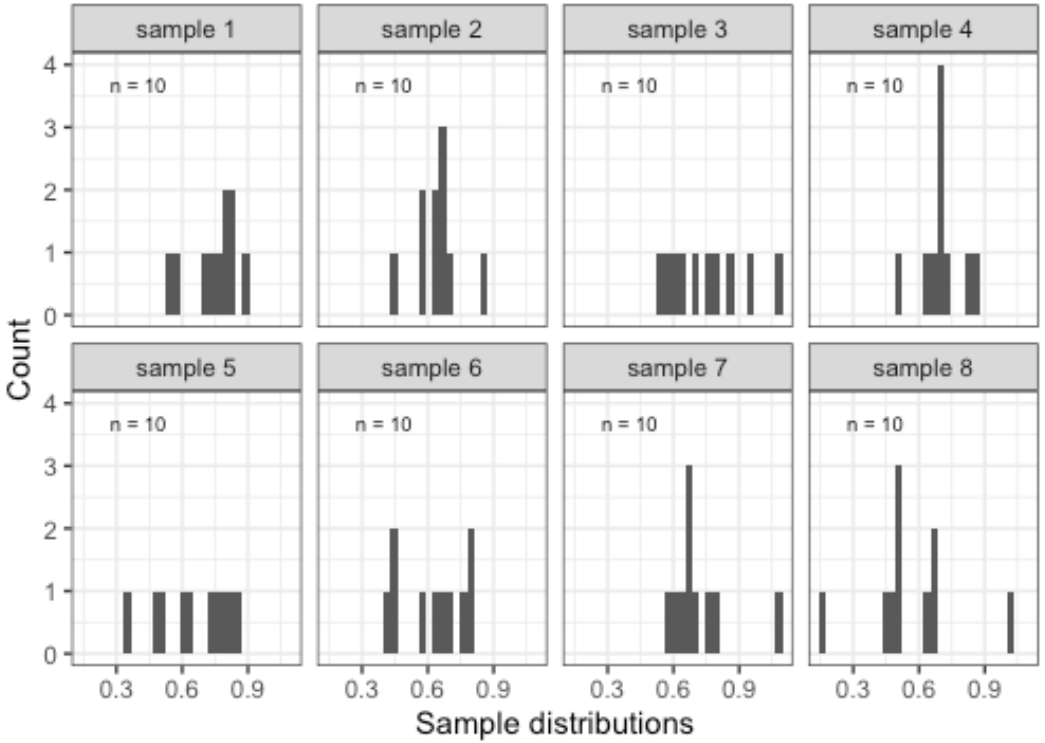

$\mu = 0.709$
$\sigma = 0.172$
population
2.0
1.5
1.0
0.5
0.0
0.5
1.0
x


$\mu_{\bar{x}} = 0.693$
$\sigma_{\bar{x}} = 0.046$
sampling distribution
6
4
2
0
0.5
1.0
$\bar{x}$

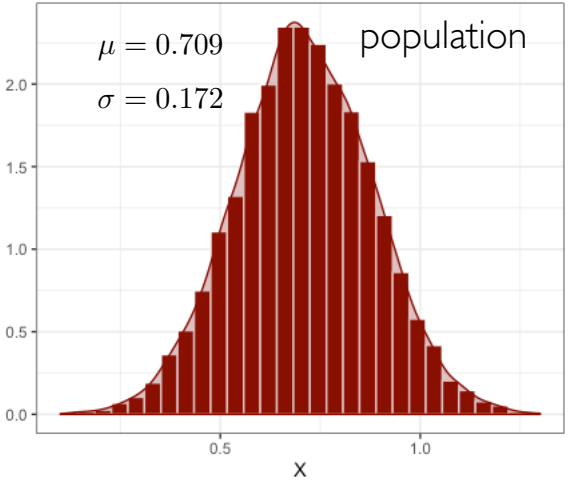

sample 1
sample 2
sample 3
sample 4
sample 5
sample 6
sample 7
sample 8
n = 10
Count
0
1
2
3
4
0.3
0.6
0.9
Sample distributions

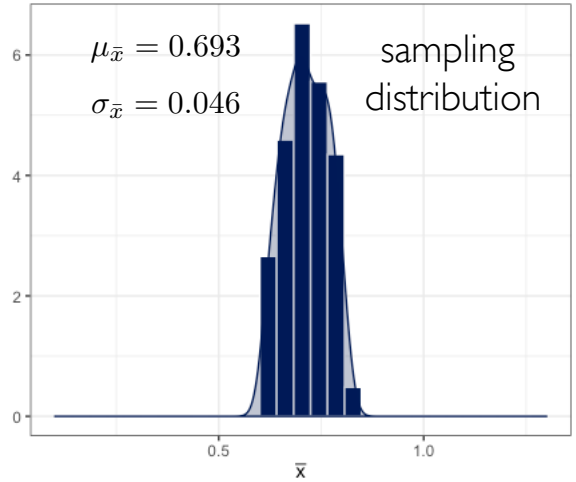


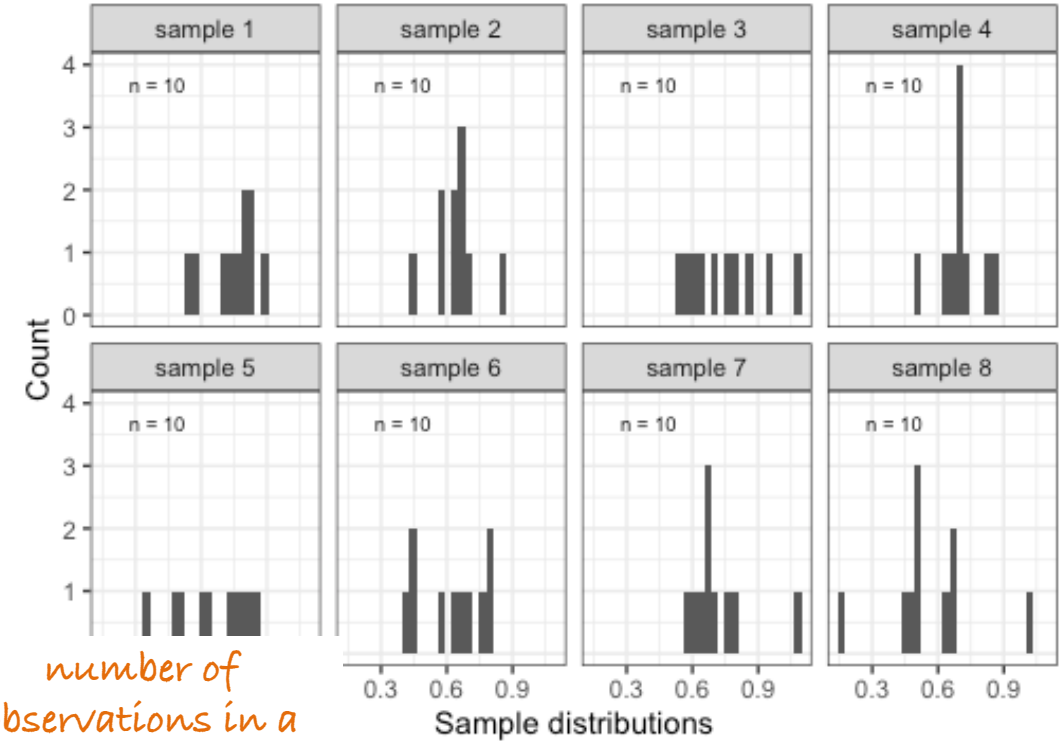




$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.172}{\sqrt{10}} = 0.054$$

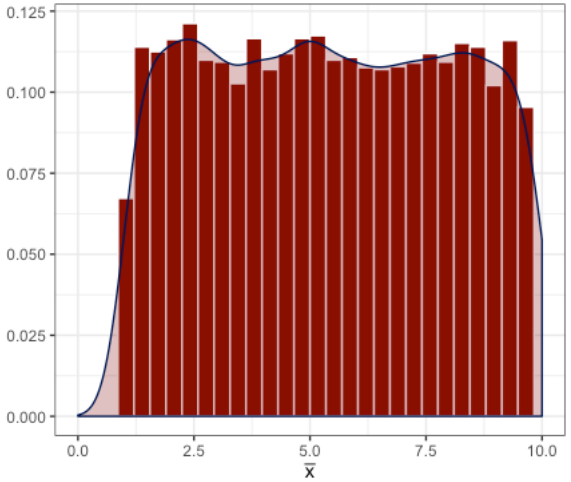






number of observations in a single sample

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 0.054$$

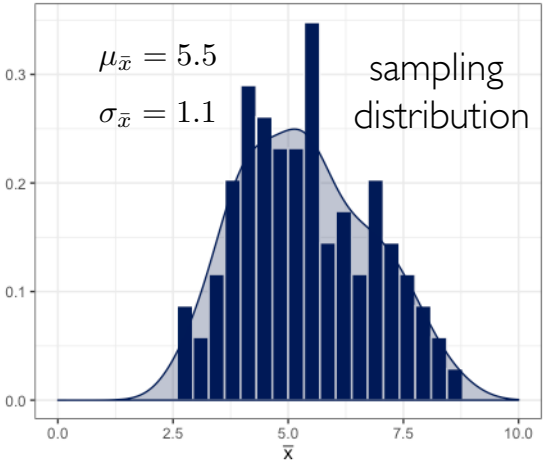


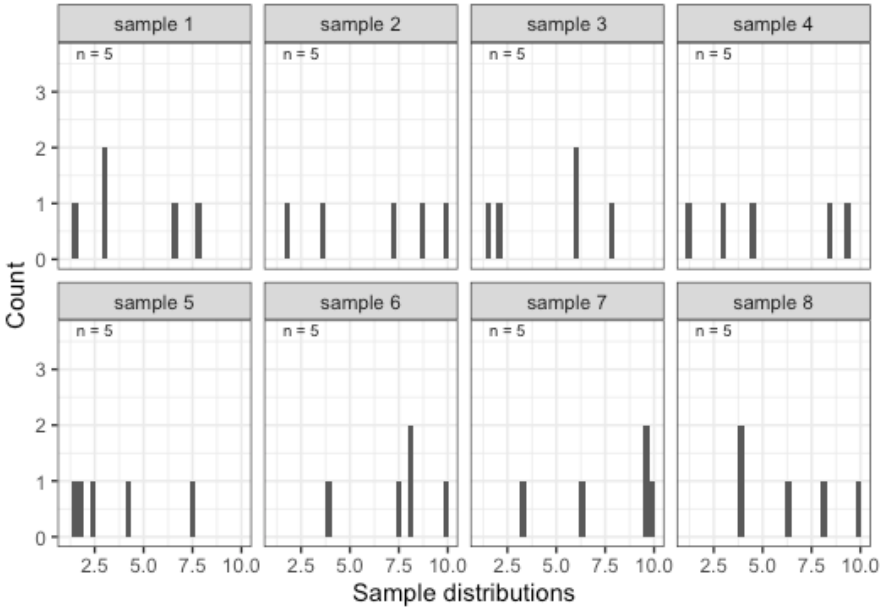
population

$X \sim U(1, 10)$

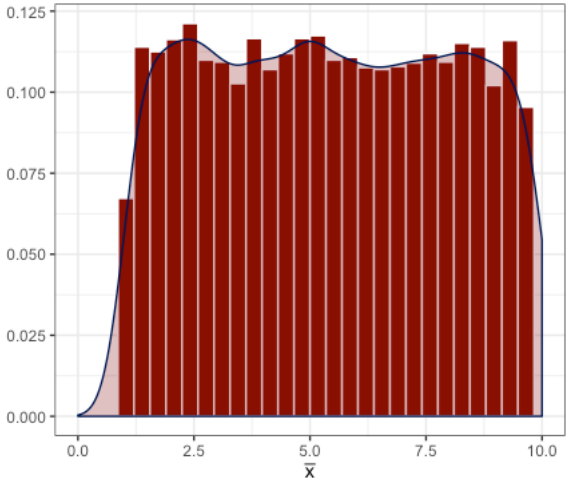
$\mu = 5.4$

$\sigma = 2.6$





$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.6}{\sqrt{5}} = 1.2$$



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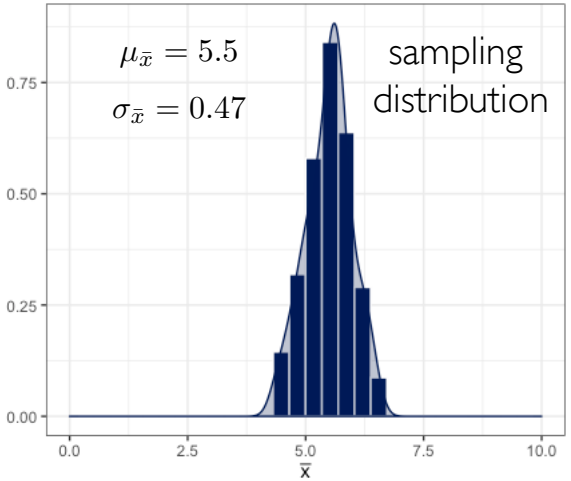
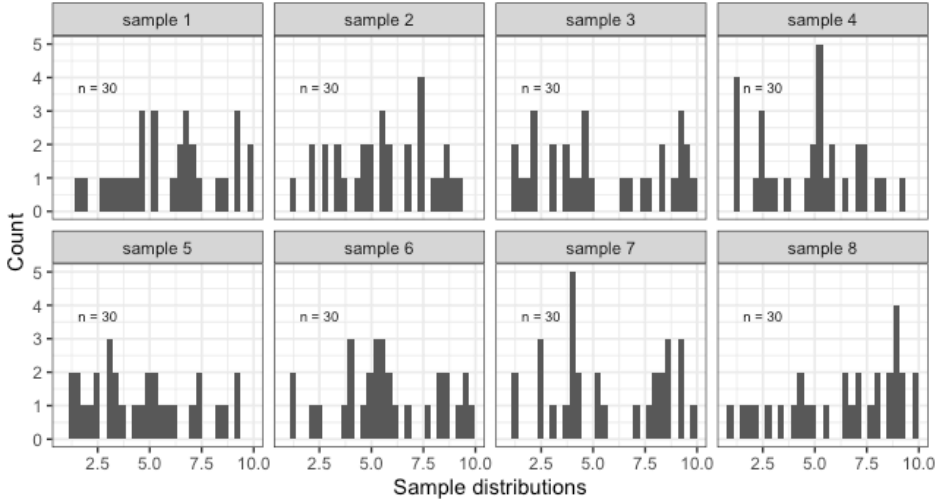




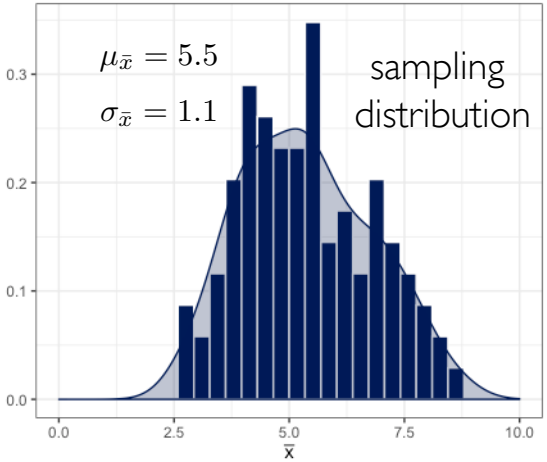
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.6}{\sqrt{30}} = 0.47$$

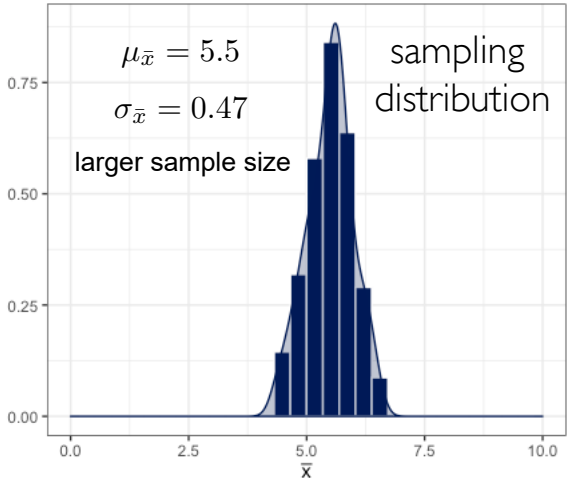
note on standard error

- SE or SEM (standard error of the mean) is the standard deviation of the sampling distribution of the sample means, where s is the sample standard deviation and n is the sample size

$$se = \frac{s}{\sqrt{n}}$$

- SE is a measure of the precision of the sampling mean
- SE demonstrates how close the sample mean is to the population mean





standard error vs. standard deviation

standard deviation

- measure of variability in a population
- standard deviation of a sample, s, estimates the variability in a population

standard error

- measure of how precisely a sample statistic estimates a population parameter
- SE is an inferential measure describing uncertainty in your measured effects and depends heavily on sample size

The standard deviation describes variability within a single sample.
The standard error estimates the variability across multiple samples of a population

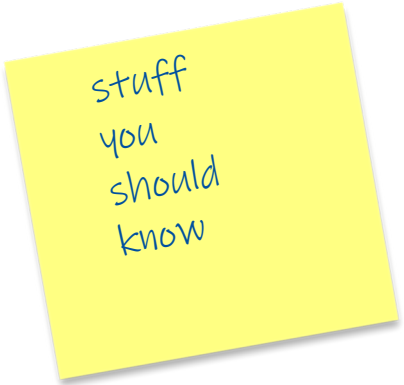
https://www.scribbr.com/statistics/standard-error/#:~:text=What's%20the%20difference%20between%20standard,across%20samples%20of%20a%20population.

inference

central limit theorem

learning goals

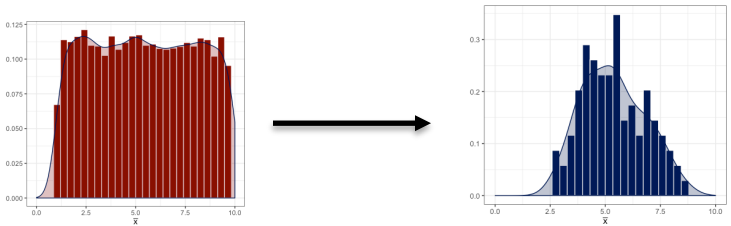
- what is the Central Limit Theorem (CLT)?
- what are the conditions for the CLT?
- are the three tenets of the CLT?
- why is the CLT an earth shattering, big deal?



Central Limit Theorem (CLT)

the distribution of sample statistics is nearly normal, centered at the population mean, and with a standard deviation equal to the population standard deviation divided by the square root of the sample size

$$\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$



Conditions

CLT describes the "population of the means" created from sets of independent, identically distributed random variables

iid

1. **independence**: sampled observations must be independent
   - random sample/assignment
   - if sampling without replacement, n < 10% of the population

2. **sample size/skew**: either the population is normal, or if the distribution is skewed, the sample size is large (n > 30)

regardless of the parent population…

1. the mean of the population of means is always equal to the mean of the parent population

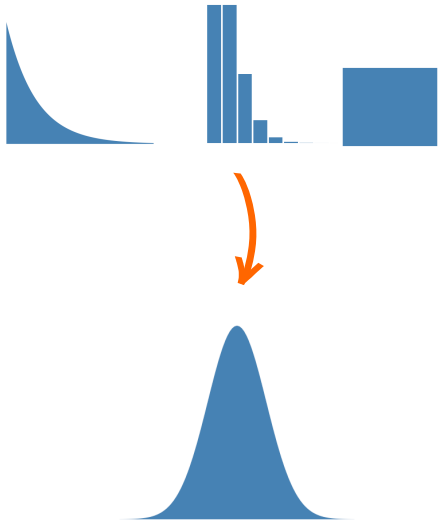
2. the standard deviation of the population of means is always equal to the standard deviation of the parent population, σ, divided by the square root of the sample size

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

3. the distribution of means will approximate a normal distribution as the sample size, *n*, of samples increases

this is a big deal…

as sample size gets large, even if you start with a non-normal distribution, the sampling distribution approaches a normal distribution



simulation
https://jrp-projects.shinyapps.io/cltapp/

why do we care…?

with data that are approximately normally distributed, it is relatively easy to calculate

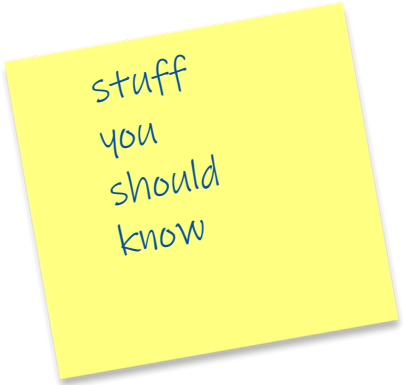
- probabilities
- confidence intervals
- conduct statistical inference and hypothesis testing

inference

checking for normality

learning goals

- why do you need to check the distribution of your data?
- how do you check the fit of a distribution to the data?
- what is a quantile-quantile plot?
- how do you test for the normality of your data?



choice of distribution

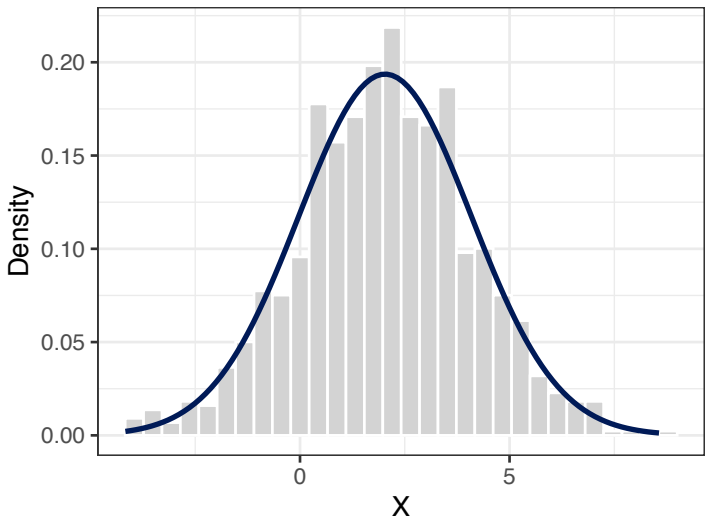
do the data meet the assumptions of the underlying distribution or statistical model?

- fitting a statistical distribution to data is *part art and part science*
- maintain a balance between getting a good distributional fit and preserving ease of estimation
- may settle for a distribution that less completely fits the data because estimating the parameters is easier

choice of distribution

1. are the data continuous or discrete?
2. how symmetric are the data?
   - symmetric: normal, logistic, Cauchy
   - positively skewed: log-normal, gamma, Weibull
   - negatively skewed: beta
3. are there upper or lower limits to the data?
   - lognormal – values never less than 0
4. how likely are extreme values?
   - logistic, cauchy

choice of distribution

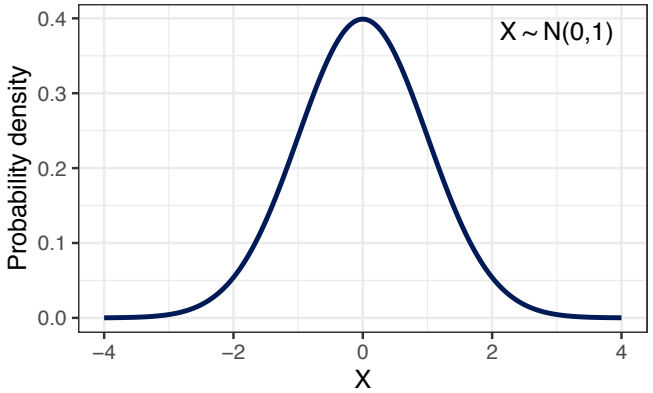


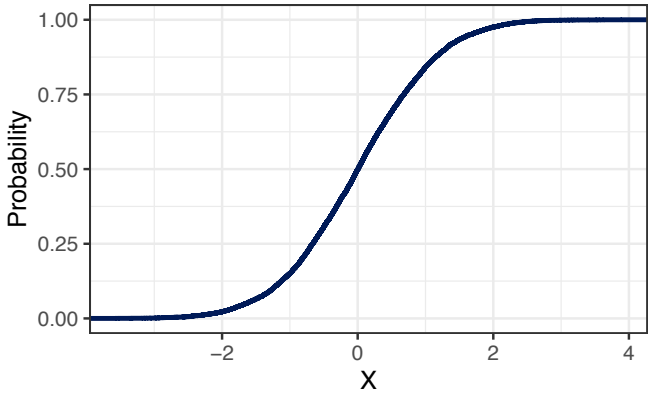
1. compare a histogram of observed data to the fitted data
2. compute the moments of the data distribution (μ, σ, skewness, kurtosis)
3. compare the data to the cumulative distribution function (CDF) to test whether the fitted distribution fits the data

cumulative distribution function

"area in so far function"

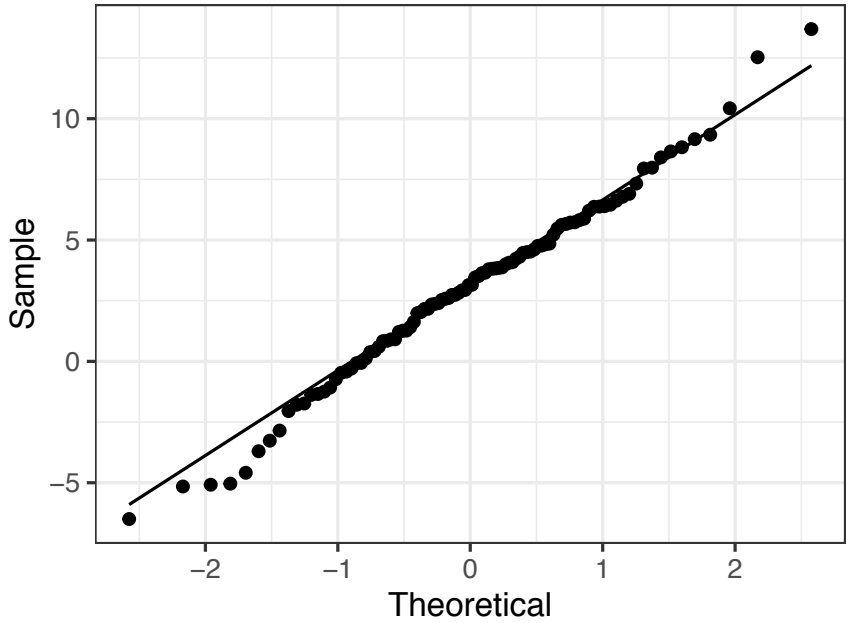
cdf is the probability that a random variable *X* will be found at a value less than or equal to *X*





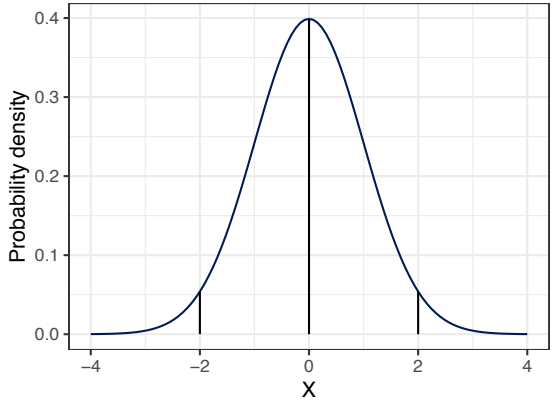
quantile-quantile (Q-Q) plot

- graphical method for comparing 2 probability distributions by plotting their quantiles against each other
- graphical assessment of 'goodness of fit'
- what is a quantile?

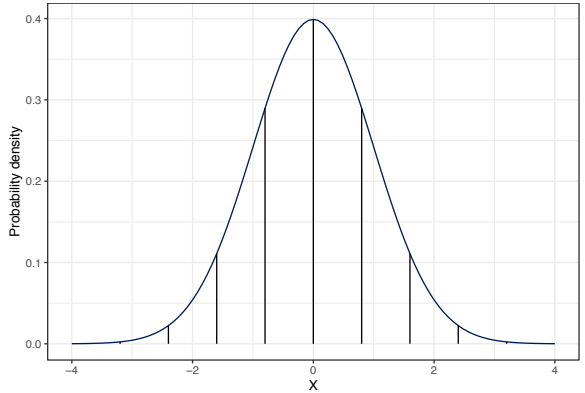


quantiles

- quantiles are cut points dividing the range of a probability distribution into intervals with equal probabilities
- there are one fewer quantiles than the number of groups created
- quartiles are the three cut points that divide a dataset into four equal-size intervals
- 10 quantiles are deciles



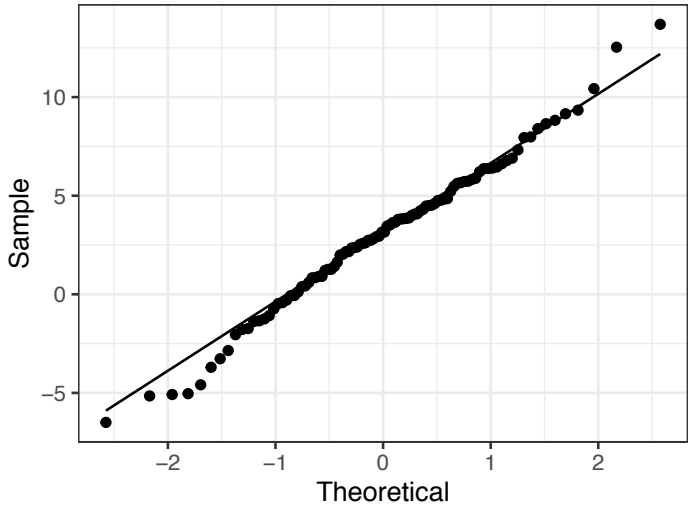

```
x.vals <- seq(-4, 4, length = 1000)
quantile(x.vals)
```




```
quantile(x.vals, c(seq(0, 1, by = 0.1)))
```

quantile-quantile (Q-Q) plot

- does the dataset match a theoretical distribution (e.g. normal)?
- do two datasets come from populations with a common distribution?
- one-to-one relationship between the data and theoretical quantiles, suggests the data follow a nearly normal distribution
- the closer the points are to a perfect straight line, the more confidence we have that they follow a normal distribution



testing for normality

Shapiro-Wilk test

- H_0 that a sample comes from a normal distribution
- H_a that a sample does not come from a normal distribution

```
shapiro.test()
```

```
set.seed(1001)
x <- rnorm(20, mean = 0, sd = 1)
shapiro.test(x)

Shapiro-Wilk normality test
W = 0.9818, p-value = 0.9554
```

testing for normality

but is this a good idea…?

- p-value of 0.05 rejects the H_0 that the data come from a normal distribution
- therefore, $p \leq 0.05$ will rarely happen, leading to the acceptance of H_0 most of the time

```
set.seed(450)
x <- runif(50, min=2, max=4)
shapiro.test(x)

Shapiro-Wilk normality test
W = 0.9601, p-value = 0.08995
```

SW tests that the data are not normally distributed, rather than testing that they are normally distributed

testing for normality

Kolmogorov-Smirnov Test

- compare a sample to a reference probability distribution
- H_0: *x* comes from the probability distribution

```
ks.test(x, "pnorm", mean, sd)
```

```
set.seed(999)
s1 <- rnorm(30, mean=0, sd=1)
ks.test(x=s1, "pnorm", mean=0, sd=1)

    One-sample Kolmogorov-Smirnov test

data:  s1
D = 0.21188, p-value = 0.1164
alternative hypothesis: two-sided

set.seed(999)
s2 <- rnorm(30, mean=2, sd=3)
ks.test(x=s2, "pnorm", 0, 1)

    One-sample Kolmogorov-Smirnov test

data:  s2
D = 0.41961, p-value = 2.656e-05
alternative hypothesis: two-sided
```

testing for normality

Kolmogorov-Smirnov Test

- compare whether two samples are from the same distribution
- H_0: *x1* and *x2* come from the same distribution

```
ks.test(x = sample1, y = sample2)
```

```
set.seed(1001)
s.norm <- rnorm(n=20, mean=0, sd=1)

set.seed(1001)
s.pois <- rpois(n=20, lambda=0.7)

ks.test(x=s.norm, y=s.pois)

Two-sample Kolmogorov-Smirnov test
D = 0.55, p-value = 0.004716
alternative hypothesis: two-sided
```

Post your questions to be answered during lecture