

ENV 710

discrete probability

roadmap

- any questions?
- pod work – Bernoulli random variable
- pod work – binomial distribution



Pods

Name	Pod	Name	Pod	Name	Pod
Benaka, Isaac	1	Go, Li Jia	5	Palia, Sophia	9
Bernaus, Katrina	1	Gulino, Justin	5	Pang, MiaoJun	9
Bi, Yuntian	1	Haber, Jordan	5	Pike, Rachel	9
Bliska, Hanna	2	Harvey, Marla	6	Price, Noah	10
Brentjens, Emma	2	Hays, Brandon	6	Rowley, Caroline	10
Campos, Gabriel	2	Healey, Liam	6	Satagopan, Nanditha Ram	10
Carlson, Maria	3	Hyun, Jiwon	7	Seagle, Jenna	11
Davidson, Kelly	3	Jackson, Rachel	7	Sepe, Stevie	11
Diaz, Danae	3	Kuhlmann, Emily	7	Sun, Suri	11
Dye, Logan	4	Martinez, Laura	8	White, Libby	12
Franzetti, Tristan	4	Merritt, Melissa	8	Wong, Richard	12
Freedman, Jacob	4	Owens, Katie	8	Yang, Wookeun	12
				Zungailia, Isabel	12

5 Rules of Probability

(So simple even your cat can understand them)

$$0 \leq P(A) \leq 1$$



1. The probability Felix will eat pizza for dinner tonight is somewhere between zero and 100%.

$$P(A) + P(B) = 1$$



2. Left alone in the yard, the probability Fido will dig up the flower garden = 88%; The probability he will find other "entertainment" = 12%
(Probability he'll get up to some kind of mischief = 88% + 12% = 100%).

$$P(\text{not } A) = 1 - P(A)$$



3. Felix is Asleep 75% of the time. So the probability he is not Asleep is
 $100\% - 75\% = 25\%$.

$$P(A \text{ or } B) = P(A) + P(B)$$



4. Felix is in a box, or he's not in a box (i.e. he can't be both). If he's in the box 40% of the time, then
the probability Felix is in a box OR he's not in a box = $40\% + 60\% = 100\%$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



5. Fido brings you one of his favorite toys.



The probability it's a ball OR it's red = $50\% + 50\% (-25\%) = 75\%$

Probability Rules Cheat Sheet

complement rule

$$P(A) = 1 - P(A')$$

multiplication rules (joint probability)

dependent $P(A \cap B) = P(A) * P(B|A)$

independent $P(A \cap B) = P(A) * P(B)$

mutually exclusive $P(A \cap B) = 0$

addition rules (union of events)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

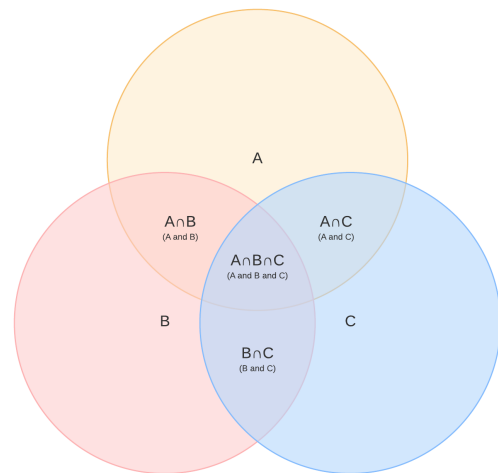
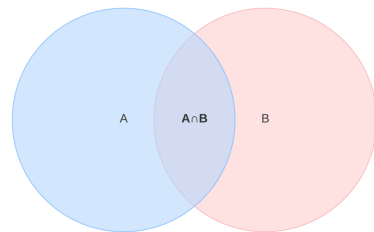
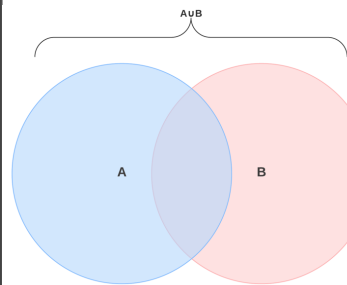
mutually exclusive $P(A \cup B) = P(A) + P(B)$

conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Theorem

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

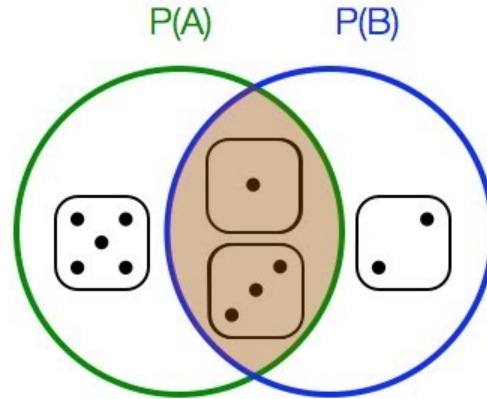


Conditional Probability

What is the Probability of
rolling a dice and it's
value is less than 4

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

knowing that the value is
an odd number



I – Bernoulli random variable

Discuss the following and solve the problems together (10 min).

- what is a probability distribution? why are we learning about them?
- what are the possible outcomes of a Bernoulli random variable?
- what is the probability of rolling a die 3 times and getting a 4 all 3 times?
- what is the probability of rolling a die 3 times and only getting 4 one time?

I – Bernoulli random variable

Discuss the following and solve the problems together (10 min).

- what is a probability distribution? why are we learning about them?
- what are the possible outcomes of a Bernoulli random variable?
- what is the probability of rolling a die 3 times and getting a 4 all 3 times?
- what is the probability of rolling a die 3 times and only getting 4 one time?

mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment

binary outcomes, 0/1

```
> (1/6)^3  
[1] 0.00462963
```

```
> dbinom(x = 1, size = 3, prob = 0.167)  
[1] 0.3476384  
> (1/6*5/6*5/6)+(5/6*1/6*5/6)+(5/6*5/6*1/6)  
[1] 0.3476384
```


2 – Binomial distribution

Discuss the following and solve the problems together (10 min).

- under what situations do you use the binomial distribution?
- describe an example (research scenario) from your field of expertise when you would use the binomial distribution
- what does the binomial function generate? what are the variables and parts of the binomial function?

$$P(X = k) = \frac{n!}{k!(n-k)!} \cdot p^k \cdot (1-p)^{n-k}$$

2 – Binomial distribution

Discuss the following and solve the problems together (10 min).

- under what situations do you use the binomial distribution?
when you have a series of Bernoulli experiments/trials
- describe an example (research scenario) from your field of expertise when you would use the binomial distribution
- what does the binomial function generate? what are the variables and parts of the binomial function?

$$P(X = k) = \frac{n!}{k!(n-k)!} \cdot p^k \cdot (1-p)^{n-k}$$

$X \sim \text{Bin}(n, p)$ where n = number of trials, p = probability of success, k = number of outcomes

3 – Binomial distribution

Discuss the following and solve the problems together (10 min).

- if COVID infects 15% of a local population, what is the probability of finding 7 people in a sample of 20 people with the disease at a testing center? (solve 'by hand' or with R)
- what is the probability of finding more than 4 people infected with the disease in a sample of 20 people? (solve with R)
- what is the mean and standard deviation of this example?

3 – Binomial distribution

Discuss the following and solve the problems together (15 min).

- if COVID infects 15% of a local population, what is the probability of finding 7 people in a sample of 20 people with the disease at a testing center?

```
> dbinom(x = 7, size = 20, prob = 0.15)
[1] 0.01601396
```

- what is the probability of finding more than 4 people infected with the disease in a sample of 20 people?

```
> sum(dbinom(x = 5:20, size = 20, prob = 0.15))
[1] 0.1701532
> 1 - pbinom(q = 4, size = 20, prob = 0.15)
[1] 0.1701532
```

- what is the mean and standard deviation of this example?

```
> mu <- 20 * 0.15
> sigma <- sqrt((20*0.15)*(1-0.15))
```

what is cumulative
probability?

what is the cumulative
density function?

3 – Binomial distribution

Discuss the following and solve the problems together (15 min).

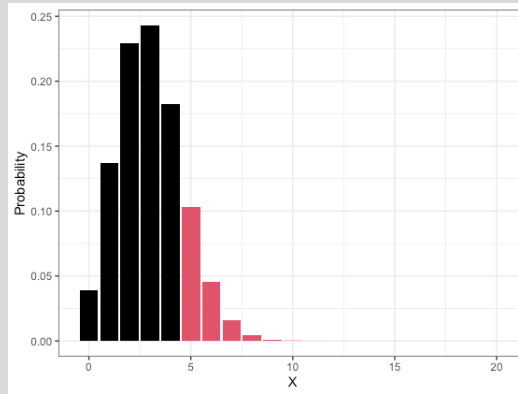
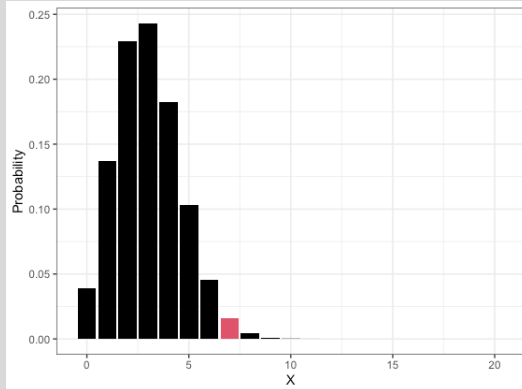
- if COVID infects 15% of a local population, what is the probability of finding 7 people in a sample of 20 people with the disease at a testing center? (solve 'by hand' or with R)
- what is the probability of finding more than 4 people infected with the disease in a sample of 20 people? (solve with R)
- create a figure in R of either of the above scenarios

what should the graphs look like?

3 – Binomial distribution

Discuss the following and solve the problems together (15 min).

- if COVID infects 15% of a local population, what is the probability of finding 7 people in a sample of 20 people with the disease at a testing center? (solve 'by hand' or with R)
- what is the probability of finding more than 4 people infected with the disease in a sample of 20 people? (solve with R)
- create a figure in R of either of the above scenarios



3 – Binomial distribution

Discuss the following and solve the problems together (10 min).

- if COVID infects 15% of a local population, what is the probability of finding 7 people in a sample of 20 people with the disease at a testing center? (solve 'by hand' or with R)
- what is the probability of finding more than 4 people infected with the disease in a sample of 20 people? (solve with R)
- create a figure in R of either of the above scenarios

```
df <- data.frame(p = dbinom(0:20, size = 20, prob = 0.15),  
  n = c(0:20))  
  
df$col <- c(rep(1,5), rep(2,16)) # scenario 2  
df$col <- c(rep(1,7), 2, rep(1,13)) #scenario 1  
my_gg <- ggplot(df, aes(y=p, x=n, fill = col)) +  
  geom_bar(stat="identity") +  
  xlab("X") + ylab("Probability") +  
  scale_fill_identity() +  
  theme_bw()
```



Questions?