ENV 710: Lecture 1

descriptive statistics

Duke | NICHOLAS SCHOOL OF THE ENVIRONMENT

learning goals

- what are different types of data and examples of each?
- key terms: population, sample, parameter, etc.
- what are measures of location and spread, and how are they calculated? pros and cons of each?
- how is the shape of a data distribution described?
- how are outliers defined, and how to deal with them?

stuff
you
should
know

research steps

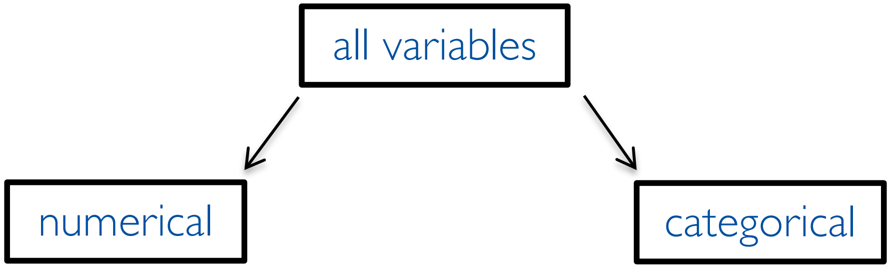
- determine your question
- design the study
- collect the data
- describe the data
- infer from the sample to the population

summary statistics & figures

Attention! we are Starting in the middle of the research process

types of data

a **variable** is a characteristic or measurement that differs from individual to individual

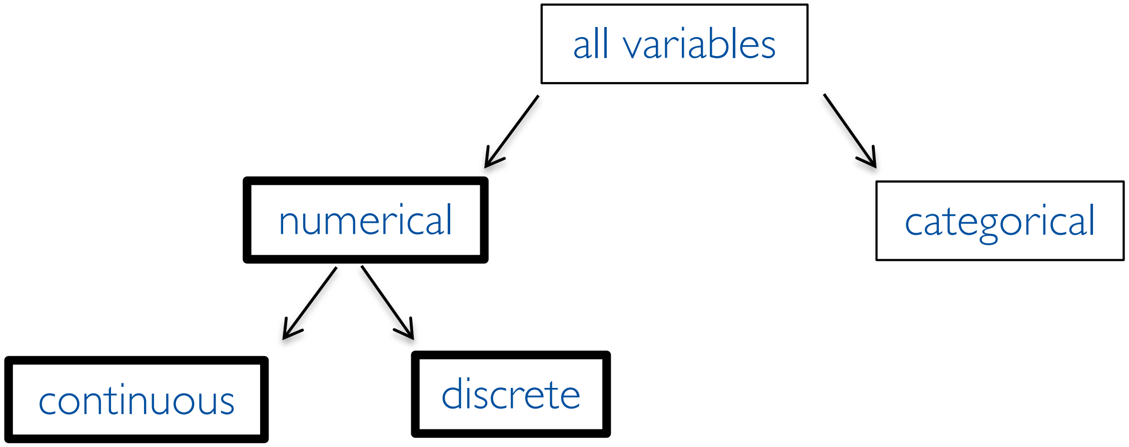


numerical values that can be added, subtracted, etc.

take on limited number of distinct categories: can be numbers, but not sensible to do arithmetic operations

data are measurements of variables

types of data



all variables

numerical

categorical

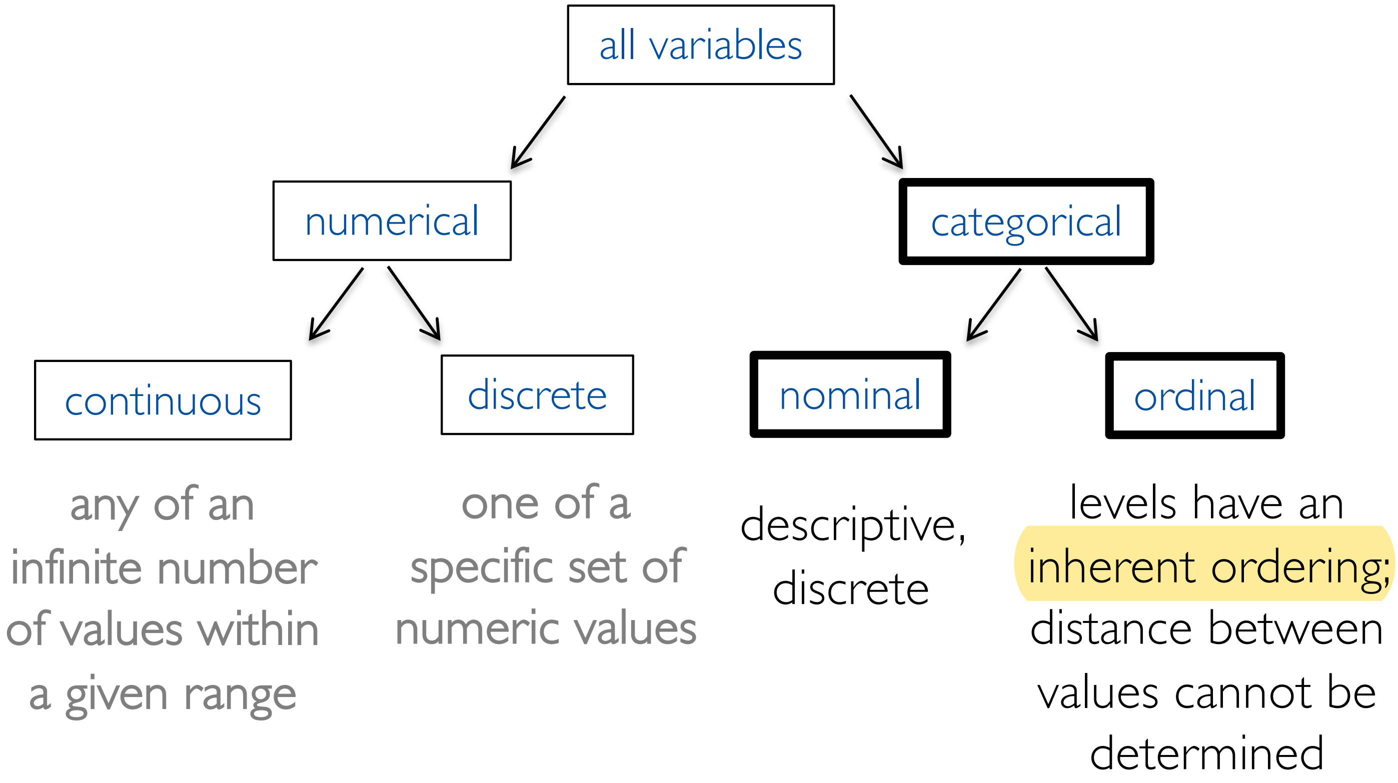
continuous

discrete

any of an infinite number of values within a given range

one of a specific set of numeric values

types of data



all variables

numerical

categorical

continuous

any of an infinite number of values within a given range

discrete

one of a specific set of numeric values

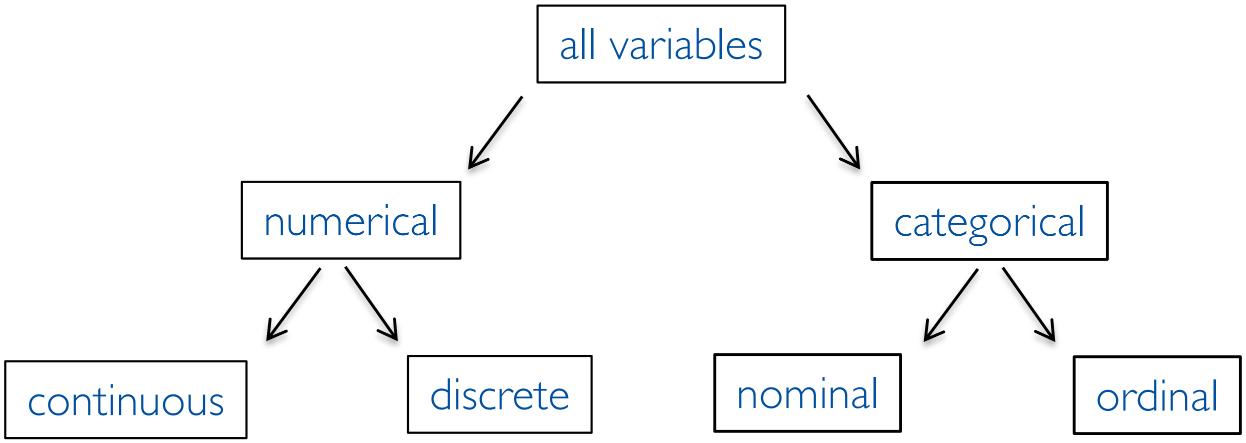
nominal

descriptive, discrete

ordinal

levels have an inherent ordering; distance between values cannot be determined

types of data



all variables

numerical

categorical

continuous

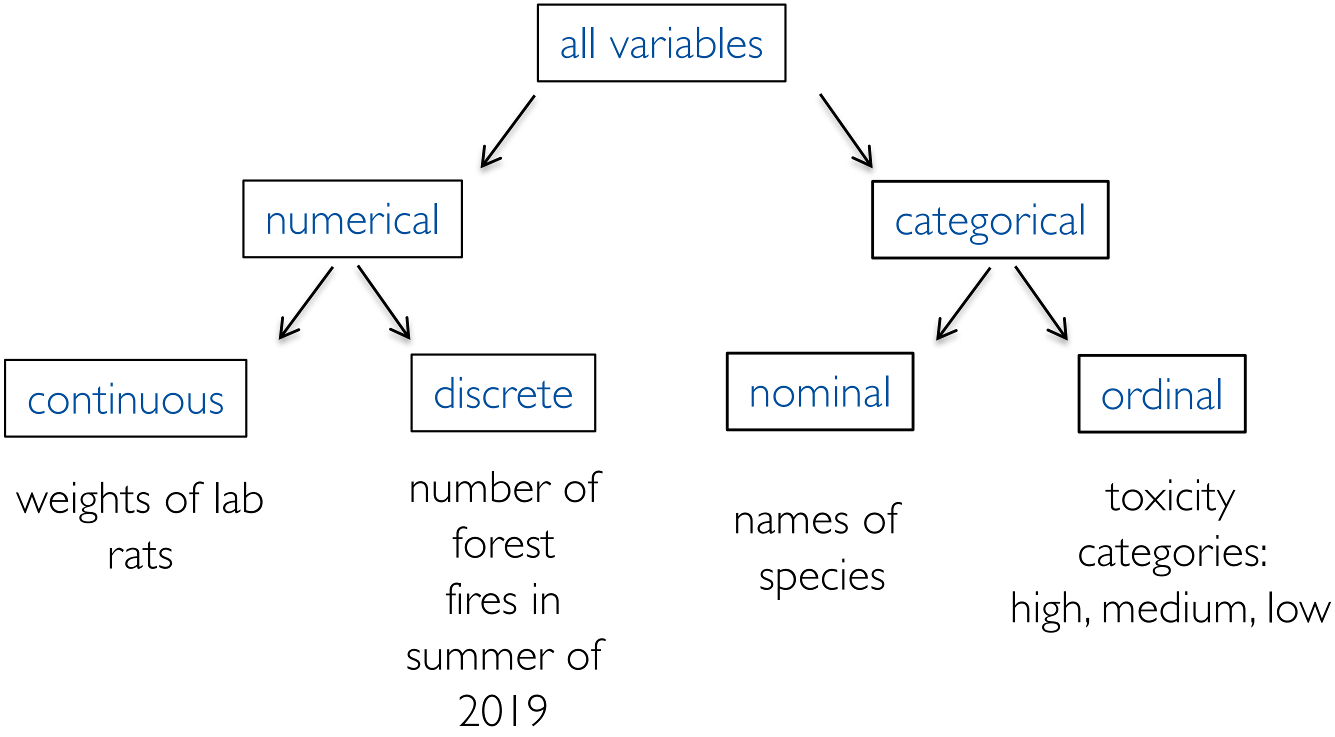
discrete

nominal

ordinal

think of examples of each data type...

types of data



all variables

numerical

categorical

continuous

weights of lab rats

discrete

number of forest fires in summer of 2019

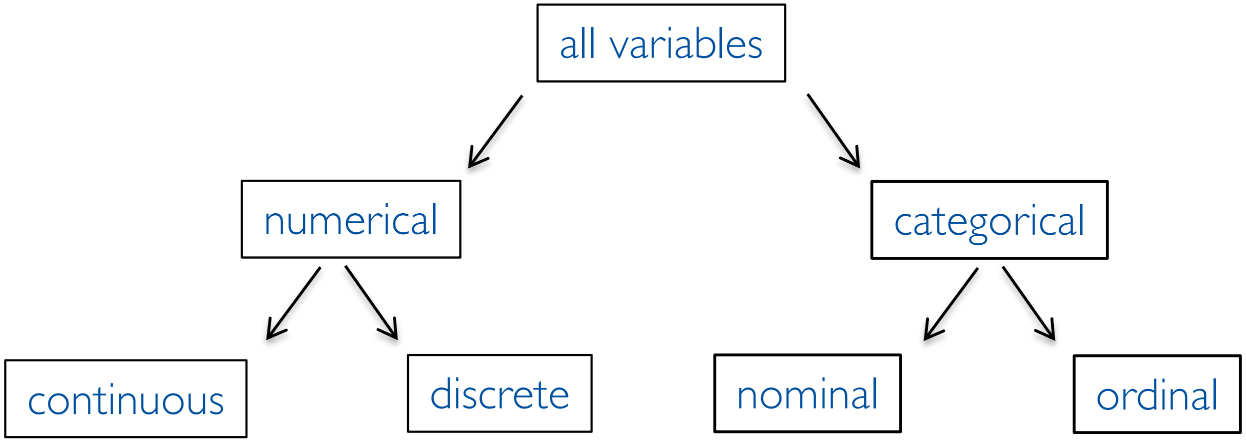
nominal

names of species

ordinal

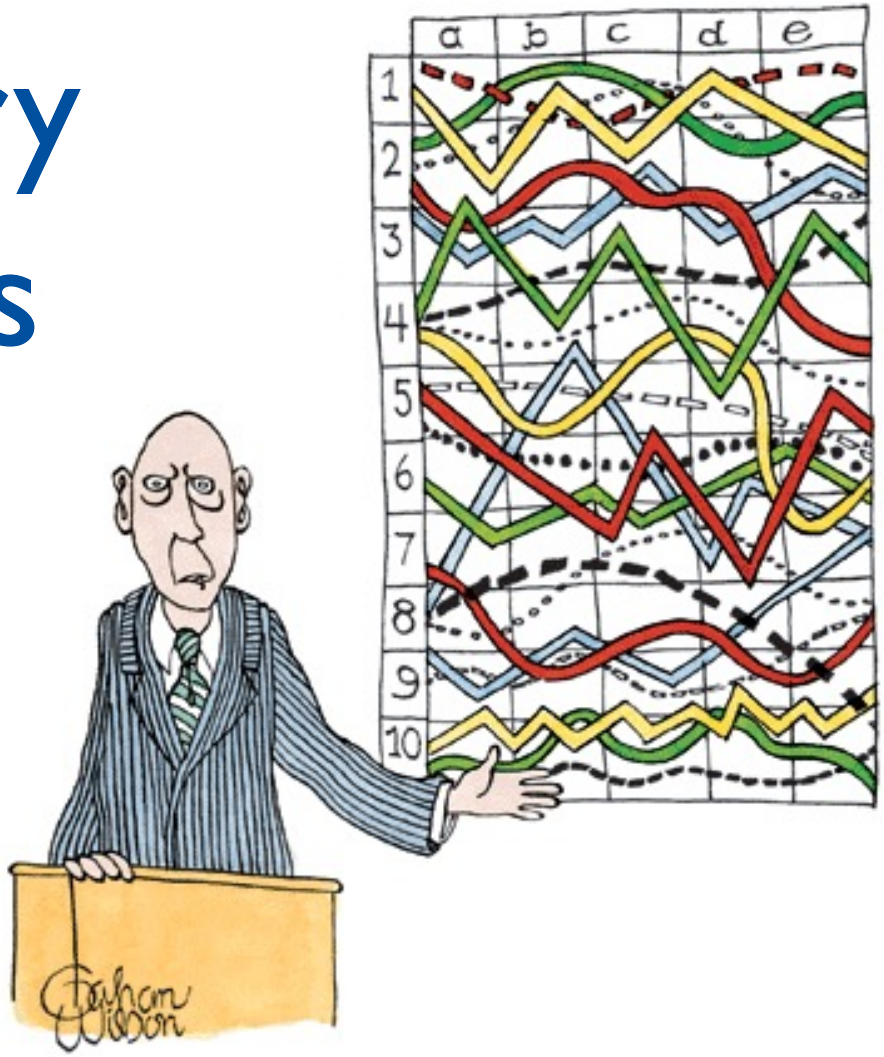
toxicity categories: high, medium, low

types of data



probability distributions of different types of data are different,
therefore we model them in different ways

summary statistics



"I'll pause for a moment so you can let this information sink in."

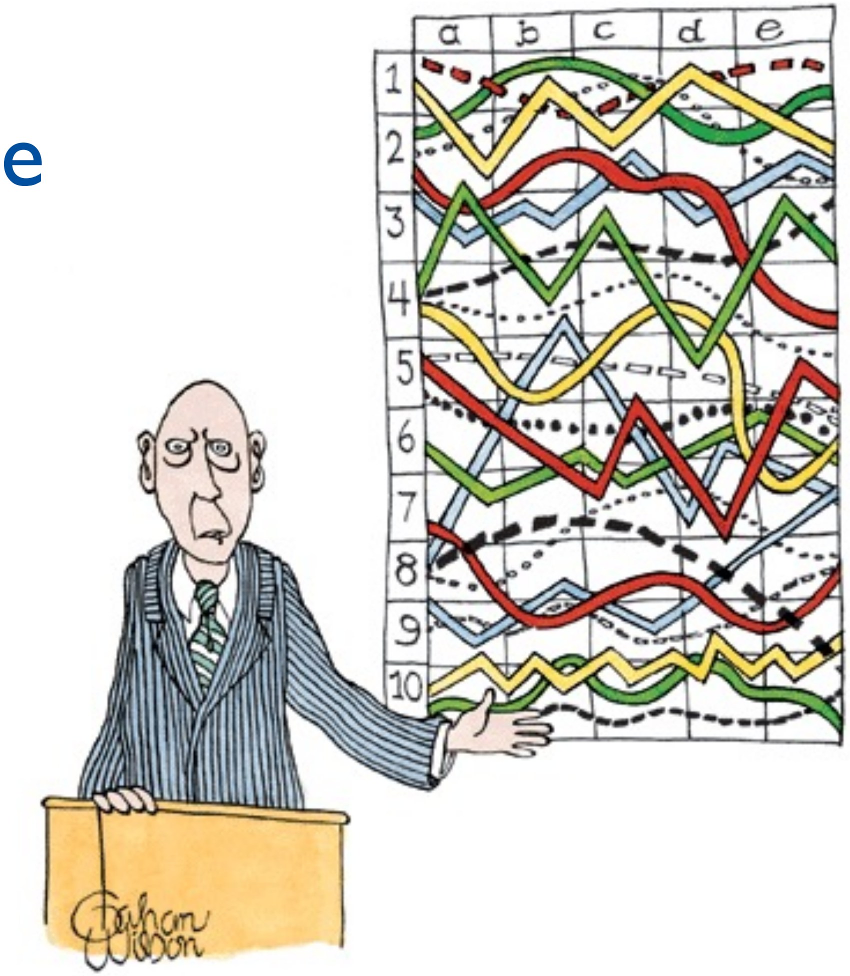
explore and summarize

summarize your data

- summary statistics (e.g., mean, standard deviation, etc.)
- 5-point summary

graph your data

- boxplots, histograms, etc.
- graph, graph, graph



"I'll pause for a moment so you can let this information sink in."

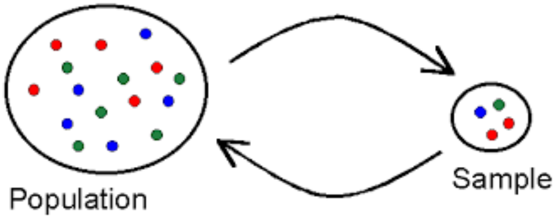
summary stats

- population is the total set of observations
- sample is a portion of a population

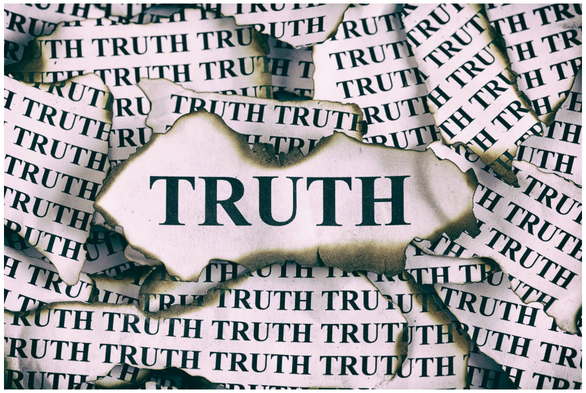
  usually used to calculate stats like the mean

- parameter is any numerical quantity that characterizes a given population or some aspect of it (truth)
- statistics are estimates of population-level parameters (approximation)

  characterize samples, estimate of parameter



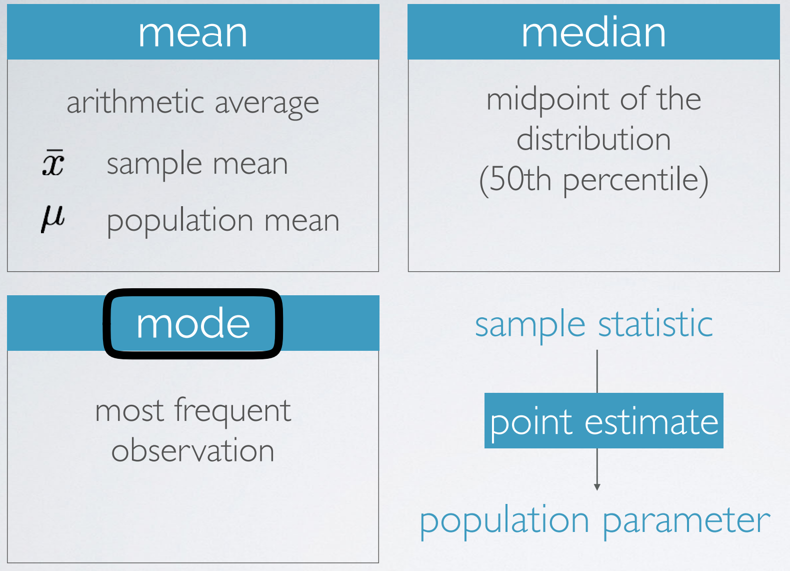
fundamental assumption: there is a true value for each parameter



https://www.jcu.edu.au/__data/assets/pdf_file/0008/115478/Basic-Statistics-6_Sample-vs-Population-Distributions.pdf
https://nednote.com/art-tells-the-truth/

summary stats

- **measures of location**: where most of the data are located



mean

arithmetic average

$\bar{x}$ sample mean

μ population mean

median

midpoint of the distribution (50th percentile)

mode

most frequent observation

sample statistic

point estimate

population parameter

when to use them?

mean: means of large samples of random variables conform to a normal distribution CLT

median/mode: better when distributions of observations cannot be fit by a standard probability distribution, and when there are extreme observations

- arithmetic, geometric, and harmonic means are sensitive to extreme observations

other measures of location

trimmed mean: reduces effects of outliers

- trim a % of the observations and calculate mean

geometric mean: describes multiplicative processes (growth rates)

- normalizes the range being averaged so a given percentage has the same effect
- use when numbers are multiples of each other

harmonic mean: average of rates

```
Yi <- c(10,10,10,10,1000)
```

$$GM_Y = e^{\left[\frac{1}{n}\sum_{i=1}^{n}\ln(Yi)\right]}$$

```
exp(mean(log(Yi)))
```

$$H_Y = \frac{1}{\frac{1}{n}\sum\frac{1}{Y_i}}$$

```
1/mean(1/Yi))
```

```
mean(c(Yi))
```

sample 1		sample 2
10		10
10		10
10		10
10		10
1000		0.1
25.1	geometric mean	4.0
12.5	harmonic mean	0.5
208	mean	8.0

income gap in the US

average compensation in the US climbed from $35,977 (adjusted for inflation) in 1984 to $50,000 in 2018

what's the problem?

Bernie Sanders, Presidential Candidate 2020

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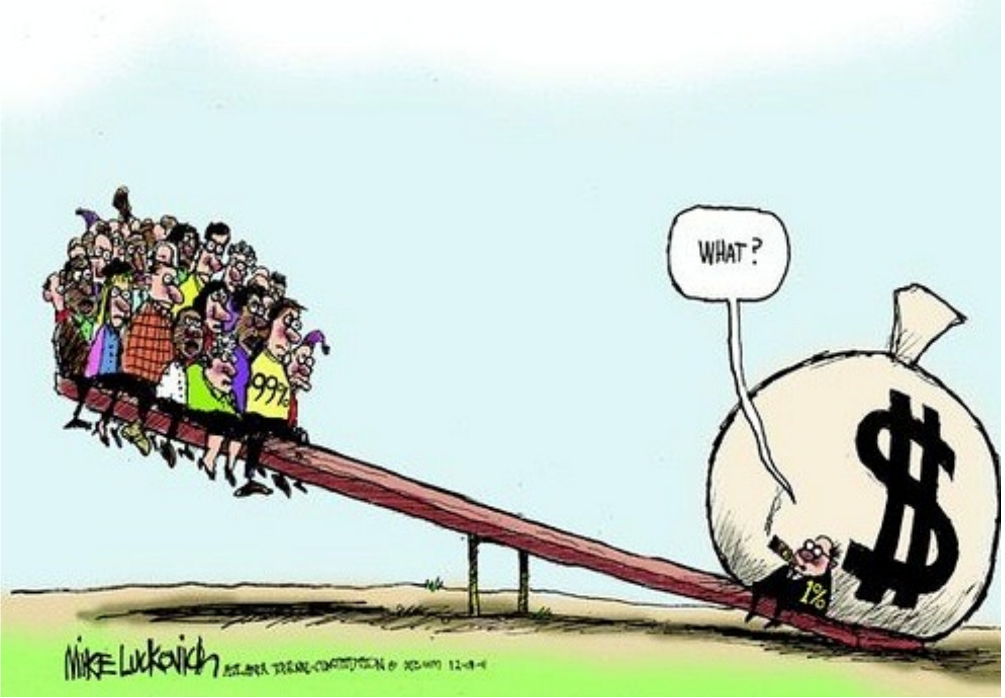
income gap in the US

average compensation in the US climbed from $35,977 (adjusted for inflation) in 1984 to $50,000 in 2018

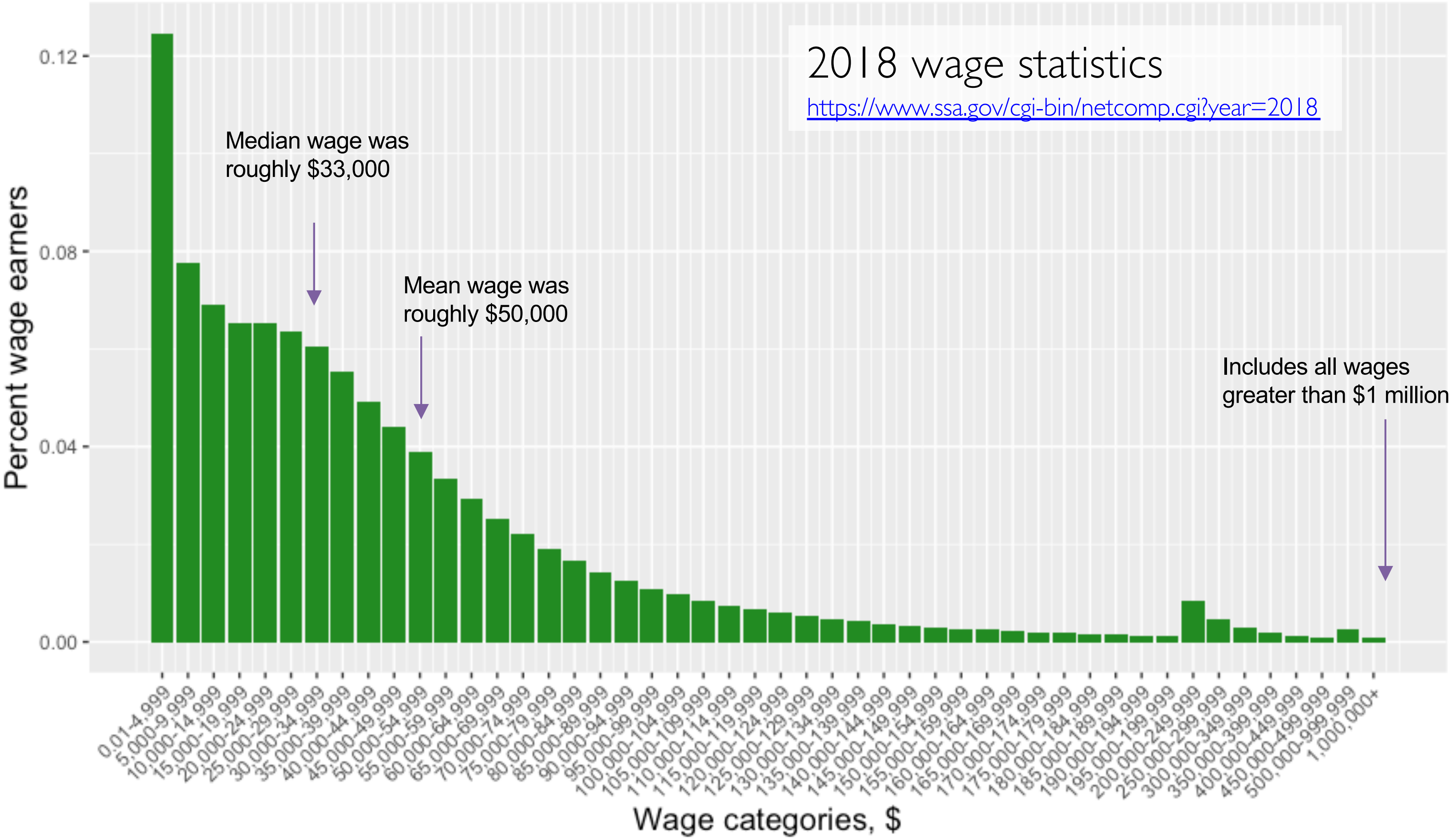
what’s the problem?



avg income is misleading and hide the truth. It is highly impacted by the extreme

https://sevenpillarsinstitute.org/consequences-economic-inequality/

Distribution of wages in 2018 in the US



2018 wage statistics

https://www.ssa.gov/cgi-bin/netcomp.cgi?year=2018

Median wage was roughly $33,000

Mean wage was roughly $50,000

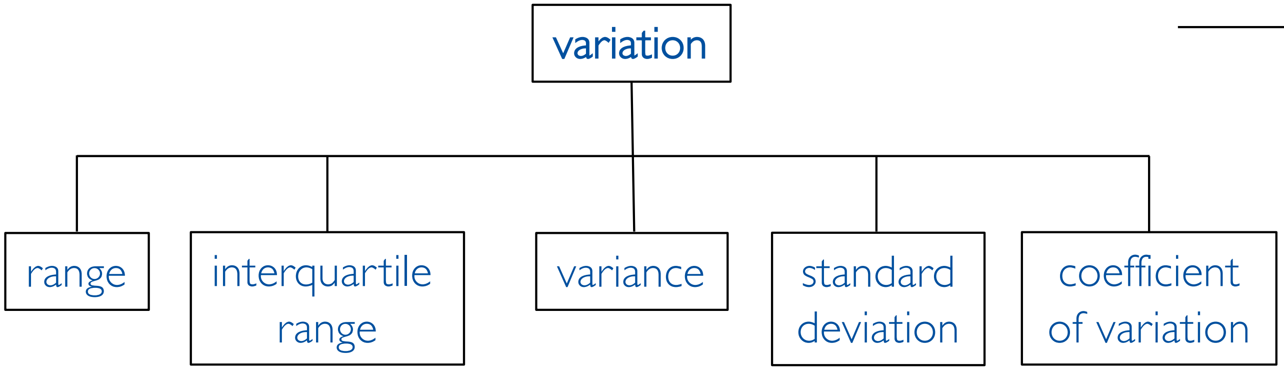
Includes all wages greater than $1 million

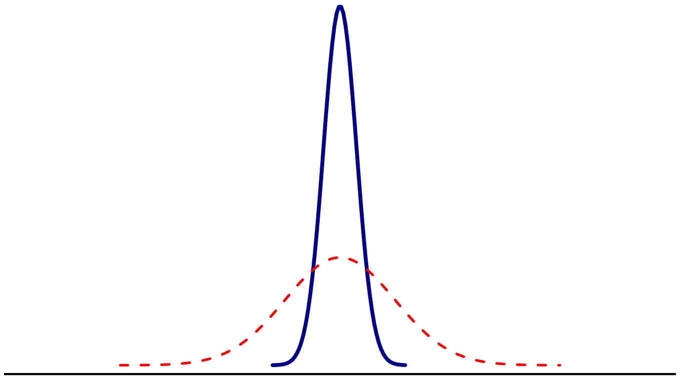
Mean higher than median. not symmetrical

summary stats

- **measures of spread**: spread or variability of the data

variation





same center,
different spread

variance

how far each obs is from the mean

`var()`

gives the sample variance

average squared deviation from the mean

population variance σ^2

sample variance s^2

n	country	life exp.
1	Mozambique	31.3
2	Botswana	32.3
3	Zambia	35.3
...	...	...
180	Andorra	83.5

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

$$s^2 = \frac{(31.3 - 66.0)^2 + (32.3 - 66.0)^2 + ... + (83.5 - 66.0)^2}{180 - 1} = 162.8 \text{ yrs}$$

variance

why do we square the differences?

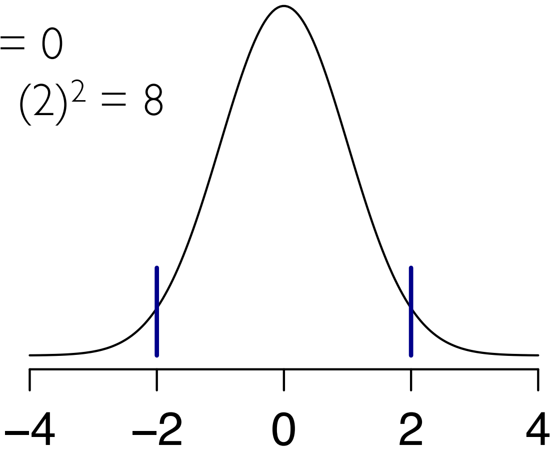
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

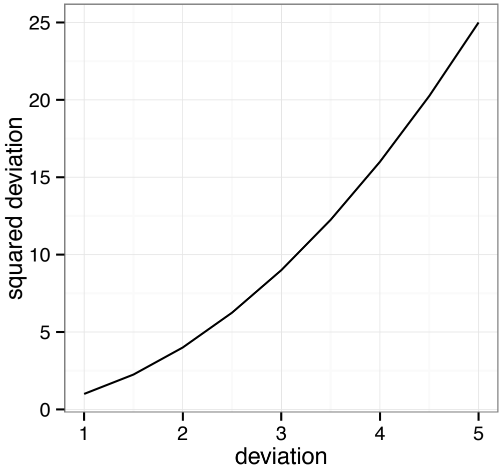
- get rid of negatives so that negatives and positive don’t cancel each other out

- increase larger deviations more than smaller ones so they are weighed more heavily

-2 + 2 = 0

$(-2)^2 + (2)^2 = 8$





standard deviation

average deviation around the mean,
expressed in the same units as the data   benefit

population standard deviation σ

sample standard deviation s

$$s = \sqrt{s^2}$$

$$s = \sqrt{162.8} = 12.8 \text{ yrs}$$

`sd()`
gives the sample
variance

	country	life exp
1	Mozambique	31.3
2	Botswana	32.3
3	Zambia	35.3
...	...	...
180	Andorra	83.5

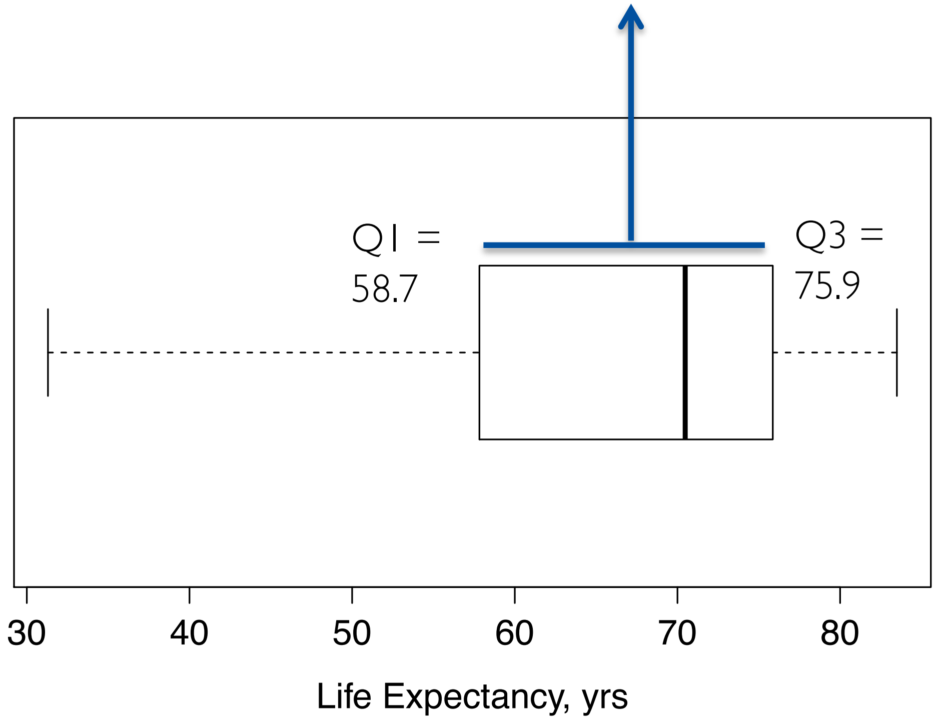
interquartile range

range of middle 50% of the data, distance between the 1st quartile (25th percentile) and 3rd quartile (75% percentile)

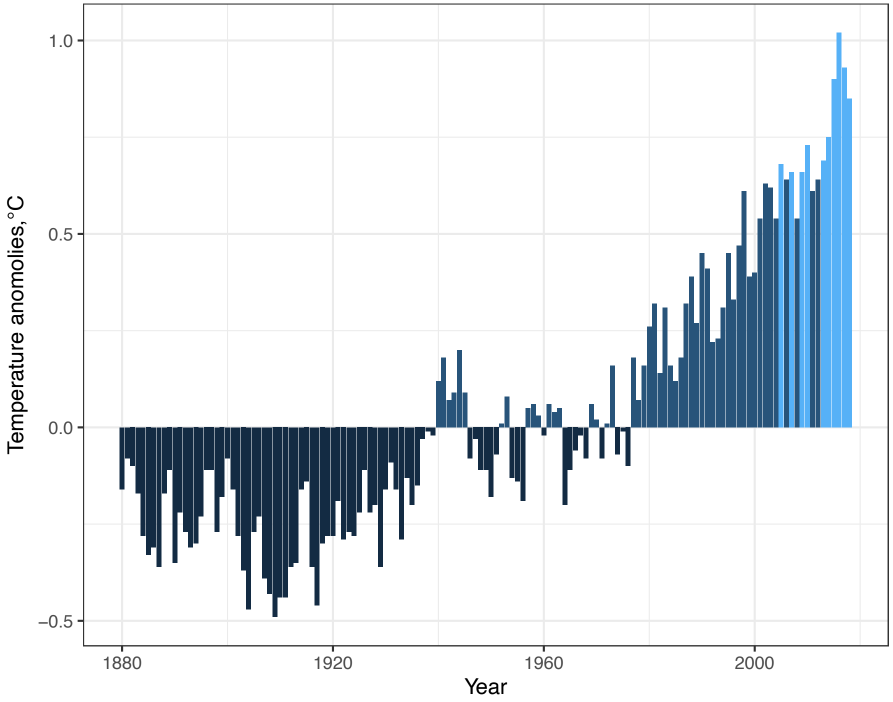
IQR = Q3 – Q1

where the bulk of the data is located

IQR = 75.9 - 58.7.3 = 17.2



summary



deviations from the 1951-1980 mean surface temperatures

deviations of the means are growing higher in recent years

5-point summary

minimum = -0.49

1st quartile = -0.21

median = -0.07

3rd quartile = 0.21

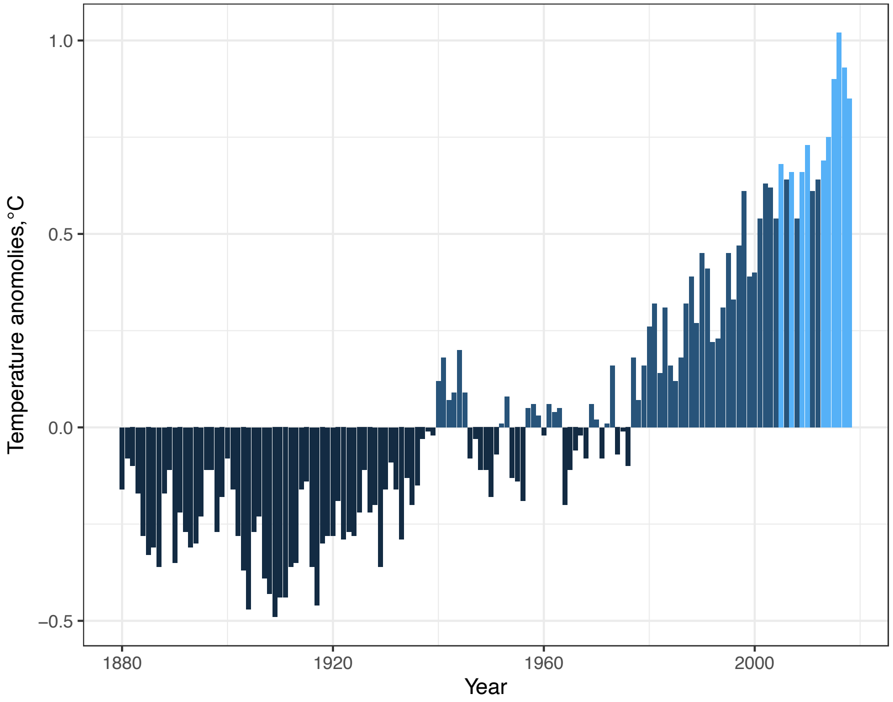
maximum = 1.02

IQR = 0.21—0.21 = 0.42

`summary()`

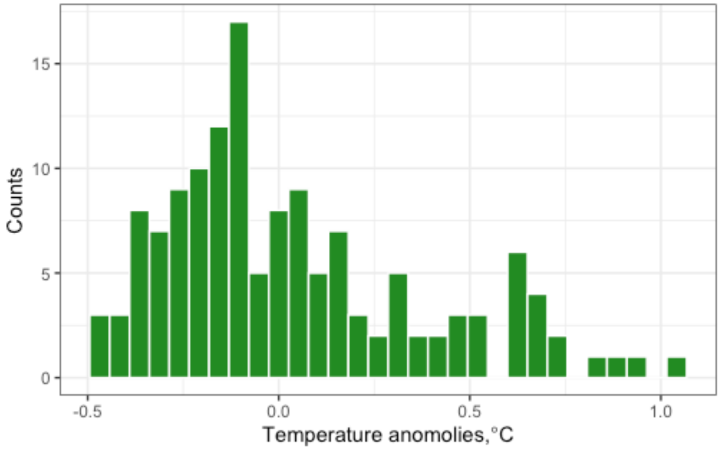
provides the 5-point summary (plus the mean)

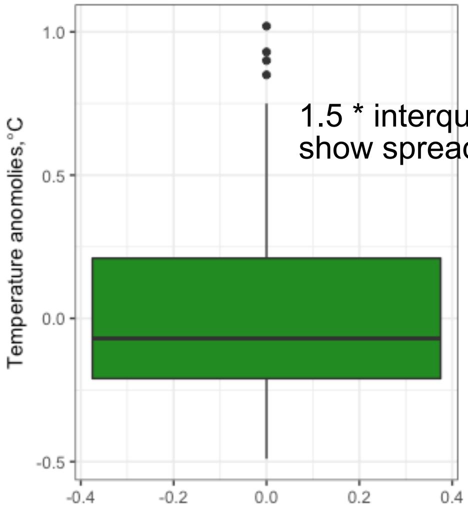
summary



deviations from the 1951-1980 mean surface temperatures

good to see the data distribution: skewed/ multiple modes, etc





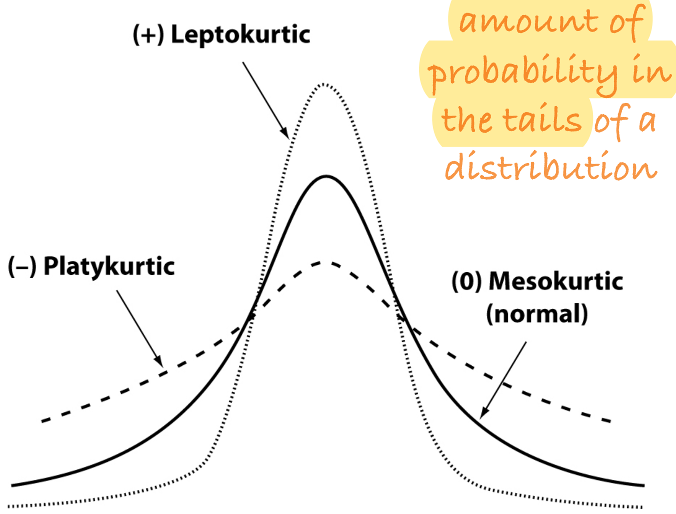
1.5 * interquartile range
show spread and skewness

median

boxplot: more formally

shape of distributions

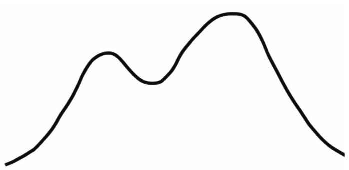
kurtosis – extent to which a distribution is distributed in the tails versus the center



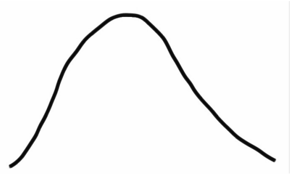
amount of probability in the tails of a distribution

forms of kurtosis

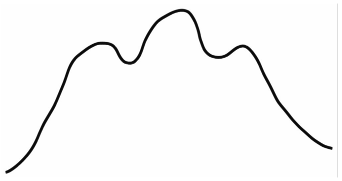
modality – describes the peak of the distribution



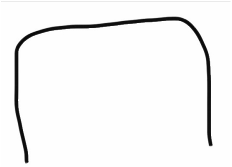
bimodal



unimodal



multimodal

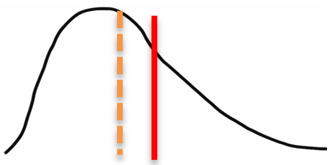


uniform

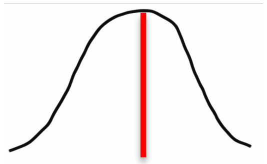
shape of distributions

skewness – how the sample differs in shape from a symmetrical distribution; measure of symmetry

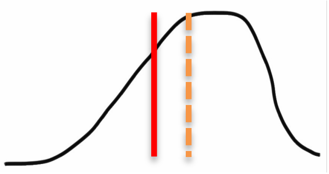
measure of
symmetry



right skewed
positive skew
mean > median



symmetric
mean ≈ median



left skewed
negative skew
mean < median

outliers

"an observation in a data set which appears to be inconsistent with the remainder of the set of data." (Johnson, 1992)

"…an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism." (Hawkins, 1980)

Thus, according to these two def, the outliers do not belong to the rest of the data but others may agree they belong to

outliers

- measurement error
- data entry error
- may occur by chance
- observation generated by a different distribution, mechanism, or process

rule of thumb: data point falls outside the lower and upper fences:

- 3rd quartile + 1.5 x IQR
- 1st quartile – 1.5 x IQR

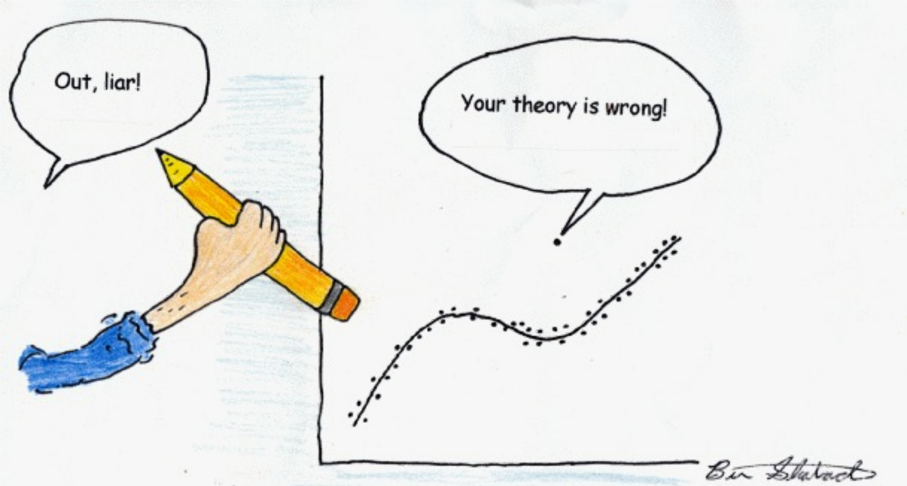
this may not be
the best way!

http://clipart-library.com/thumbs-up-images.html

what do we do?

- think about larger context of data and data collection
- was there a mistake in measurement?
- use alternative outlier criteria, e.g., Chauvenet's criterion
- use estimators that are robust to outliers (median, not the mean)
- consider removal, but only if defensible (in case of measurement error)

need to be transparent



check whether the existence of outliers will affect the inference result
if so, reconsider the theory

http://davidmlane.com/ben/cartoons.html

Post your questions to be answered during lecture