BRIDGING CLASSICAL AND QUANTUM COMPUTATION
FROM TENSOR NETWORK TO QUANTUM CIRCUIT

Pochung Chen
Department of Physics
National Tsing Hua University

Noisy Intermediate-Scale Quantum

- Pre-NISQ
  - Quantum Turing machines and universal quantum computation.
  - Simulation of quantum systems.
  - Quantum algorithms (Peter Shor, 1994).
  - Quantum error correction.

- NISQ
  - Extract the maximum quantum computational power from NISQ devices.
  - Develop teniques toward fault-tolerant quantum computation.

- Reference:
  - “Noisy intermediate-scale quantum algorithms”, Rev. Mod. Phys. **94,** 015004 (2022).

Roger Penrose (1971)

Applications of Negative Dimensional Tensors

ROGER PENROSE

Birkbeck College, University of London, England

I wish to describe a theory of "abstract tensor systems" (abbreviated ATS) and indicate some applications. Unfortunately I shall only be able to give a very brief outline of the general theory here.†

I take as my model, the conventional tensor index notation with Einstein's summation convention, which has become so familiar in physics and in what is now referred to as "old fashioned" differential geometry. The elements of an ATS may be denoted by kernel symbols with indices in a way formally identical with the tensor index notation, but now the meanings of the indices are to be quite different. This will enable more general types of object than ordinary tensors to be considered. Some of these (for example, "negative dimensional" tensors) will not be representable in terms of components in the ordinary way.

Each index is to be simply a *label* and does not stand for, say, 1, 2, ..., *n*. Thus an element ξ^a (a "vector") of an ATS is not a set of components, but a single element of a vector space (or module) $\mathscr{T}^a$ over a field (or ring) $\mathscr{T}$. Since I wish to mirror the ordinary index notation and allow expressions such as $\xi^a\xi^b$ or $\xi^a\eta^b - \eta^a\xi^b$, for example, I shall also need an element ξ^b distinct from ξ^a, and so on. Thus we need another vector space (or module) $\mathscr{T}^b$ which will be canonically isomorphic with $\mathscr{T}^a$, etc. etc. Let me define the labelling set $\mathscr{L}$:

$$\mathscr{L} = (a, b, c, \ldots, z, a_0, b_0, \ldots, a_1, \ldots),$$

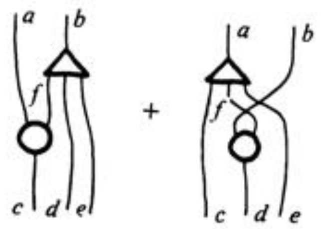
supposed infinite. The elements of $\mathscr{L}$ are to be the allowed "abstract indices". We shall then have an infinite class of canonically isomorphic modules

$$\mathscr{T}^a \cong \mathscr{T}^b \cong \ldots\ldots$$

where corresponding elements are denoted by $\xi^a, \xi^b, \ldots$. Thus $\lambda\xi^a + \mu\eta^a = \zeta^a$ iff $\lambda\xi^b + \mu\eta^b = \zeta^b$, etc. where $\lambda, \mu \in \mathscr{T}$; $\xi^a, \eta^a, \zeta^a \in \mathscr{T}^a$; $\xi^b, \eta^b, \zeta^b \in \mathscr{T}^b$; etc. But

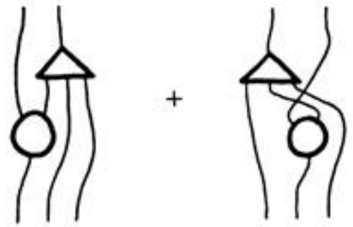
† A more extended account is to be found in [4]. However, the theory is there made unnecessarily complicated, because of an inconvenient (but apparently natural) choice having been made in connection with the notation. The point is avoided here by the use of infinitely many canonically isomorphic copies of each vector space or module. See [5] in connection with the approach used here. Also, compare [3] as regards diagrammatic notation.

Normally, to add two expressions it will be convenient simply to draw the diagram for each term and put a "+" sign between them, e.g.

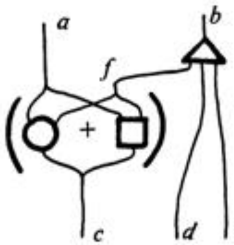


$$= \chi^b_{fde}\theta^{af}_c + \chi^a_{cfe}\theta^{bf}_d$$

We may omit the labels and draw this



making sure that it is clear, from the arrangements of free arms and legs, which are corresponding to which in the different terms. Occasionally it is convenient to employ a notation



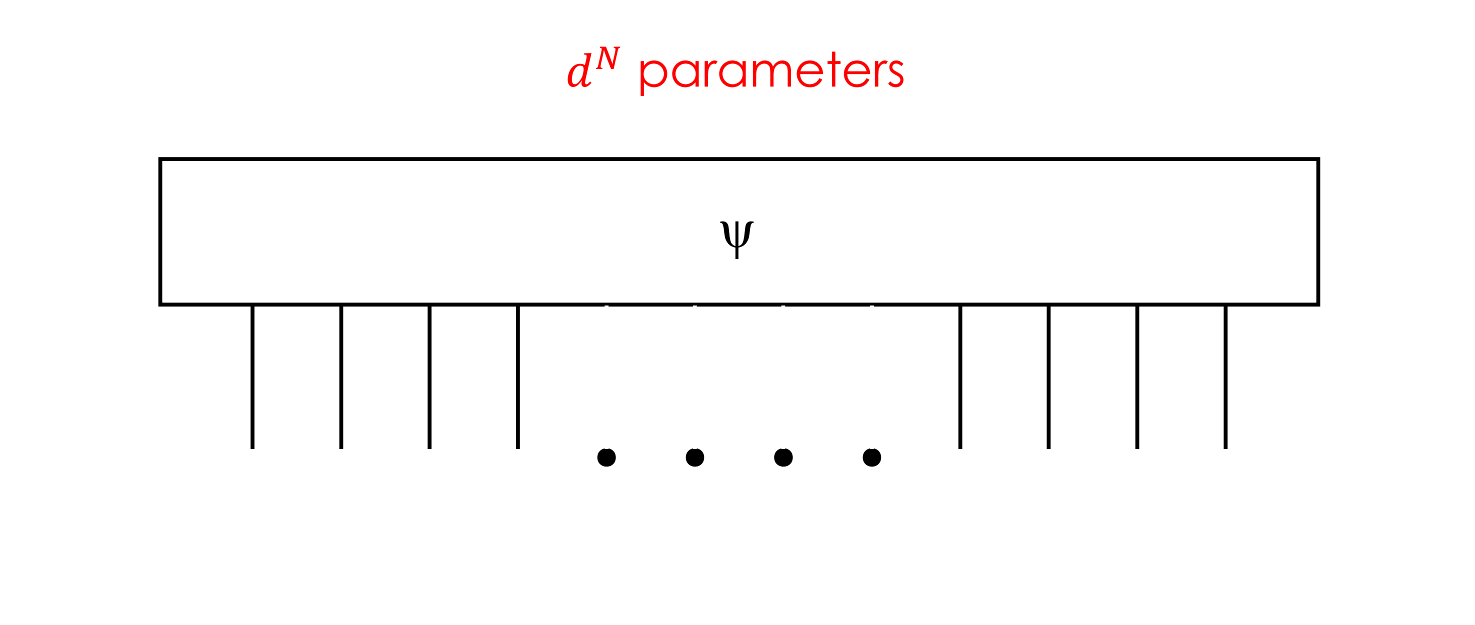
$$= (\theta^{af}_c + \gamma^{af}_c)\chi^b_{fde}$$

when sums in parenthesis are involved.

The notation for the unit δ^a_b is simply a "disembodied" line:

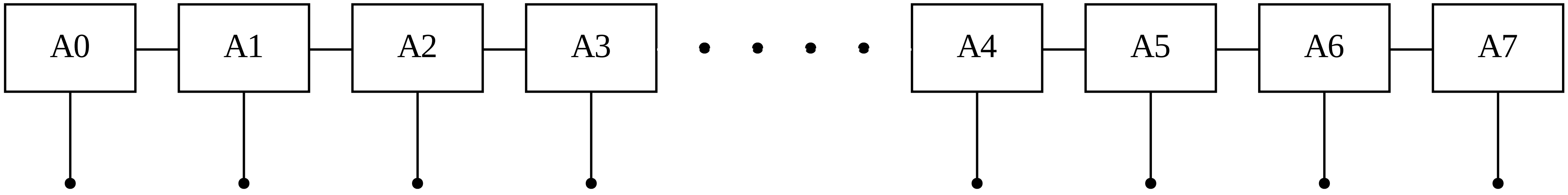
$$= \delta^a_b$$

Quantum State



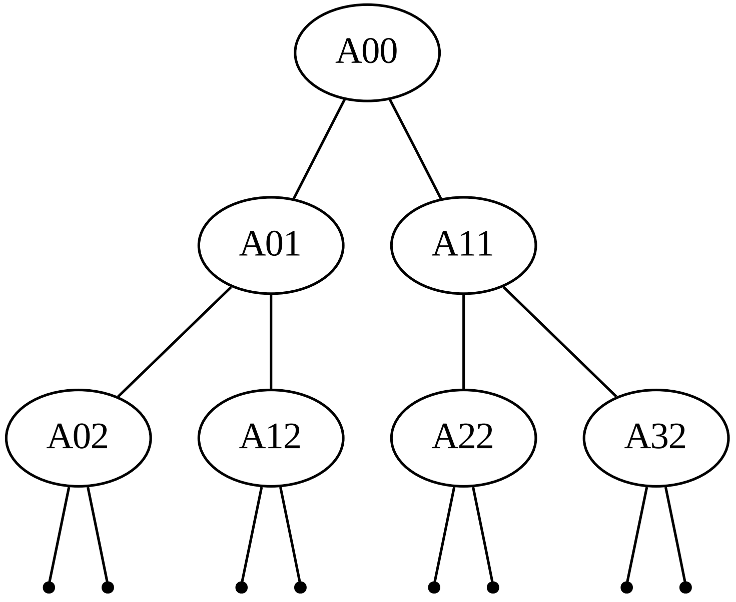
Matrix Product State

dD^2N parameters

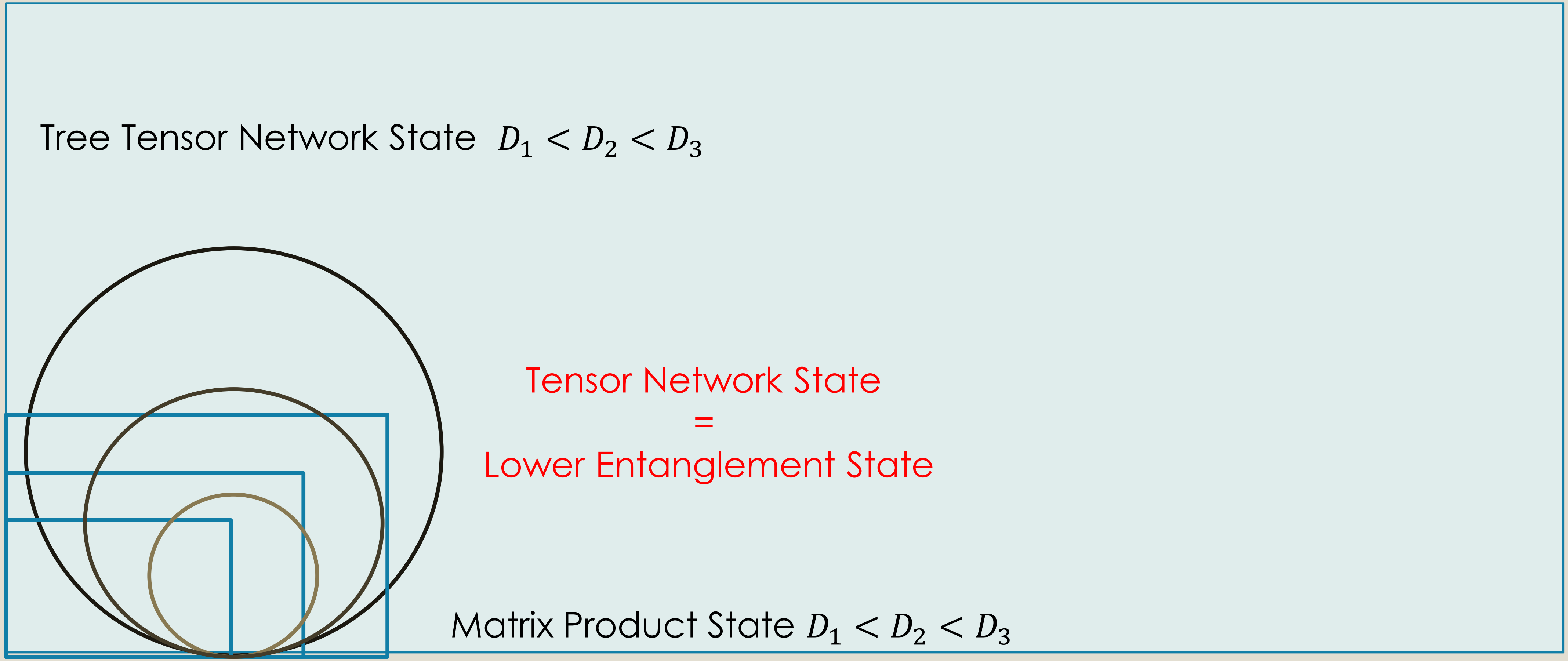


Tree Tensor Network

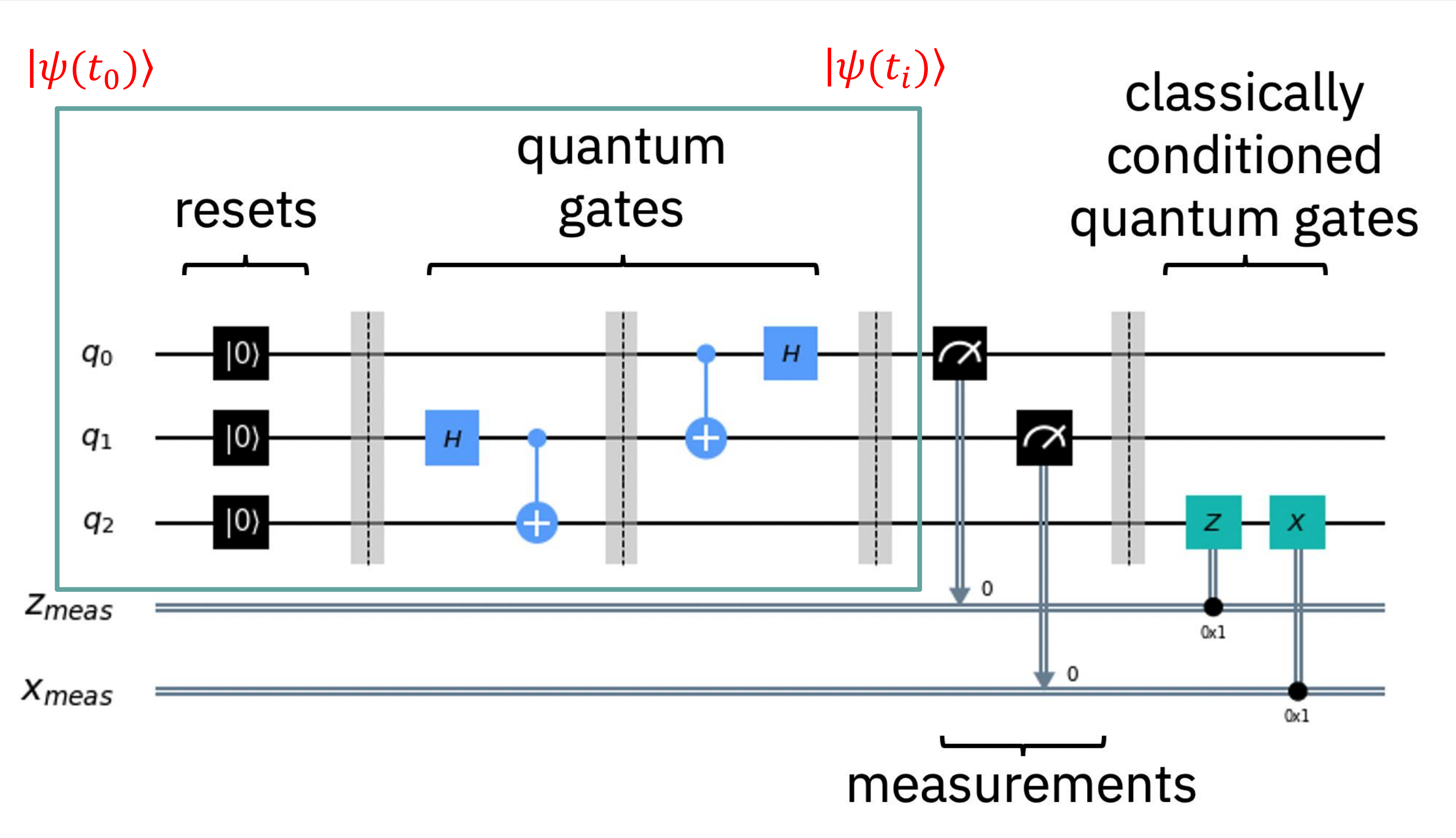
$D^3 N \log(N)$ parameters



Hilbert Space is HUGE

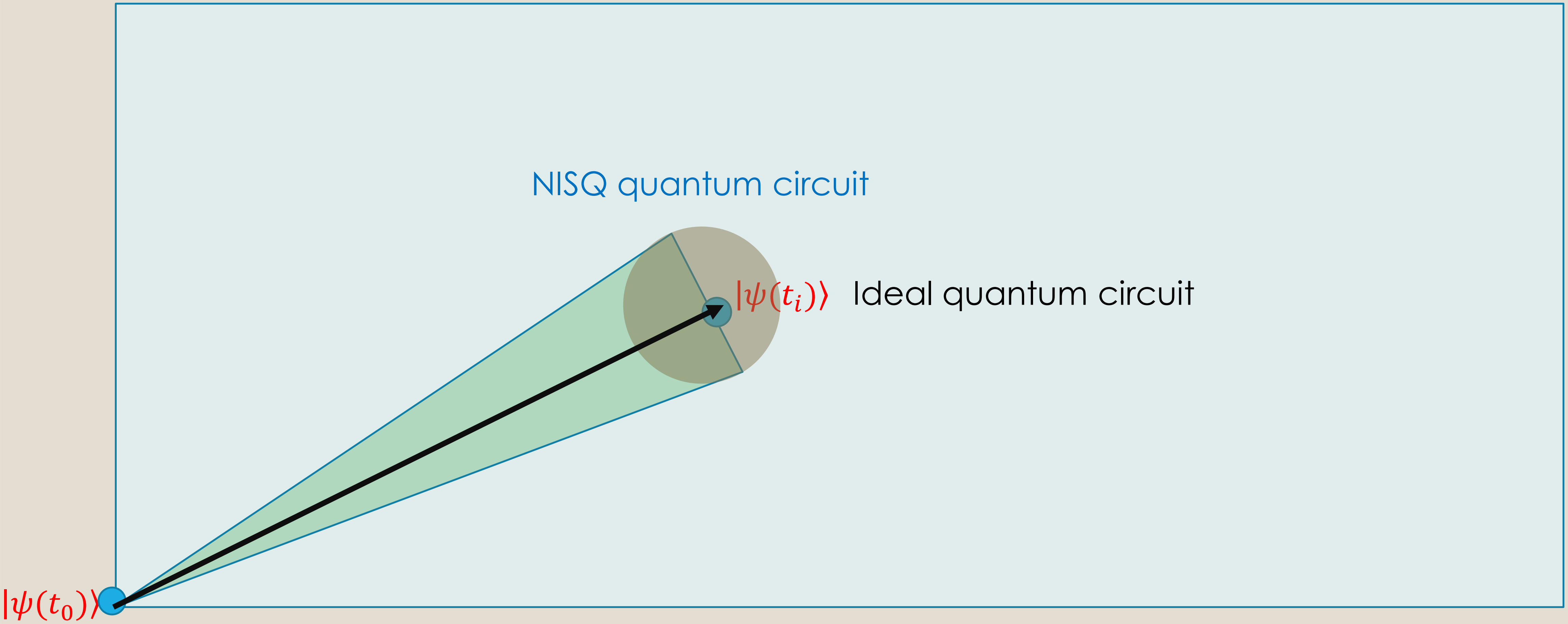


Quantum Circuit → Tensor Network

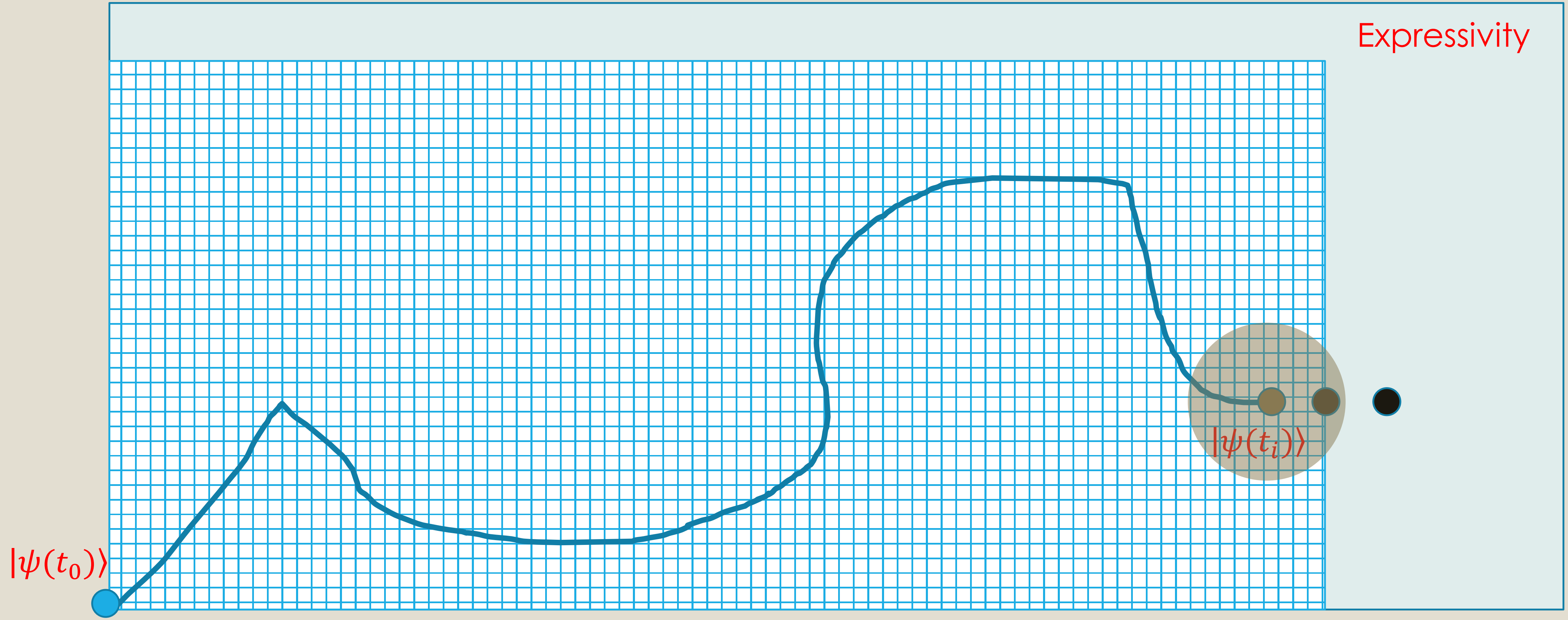


[Quantum computing in a nutshell — Qiskit 0.39.2 documentation]

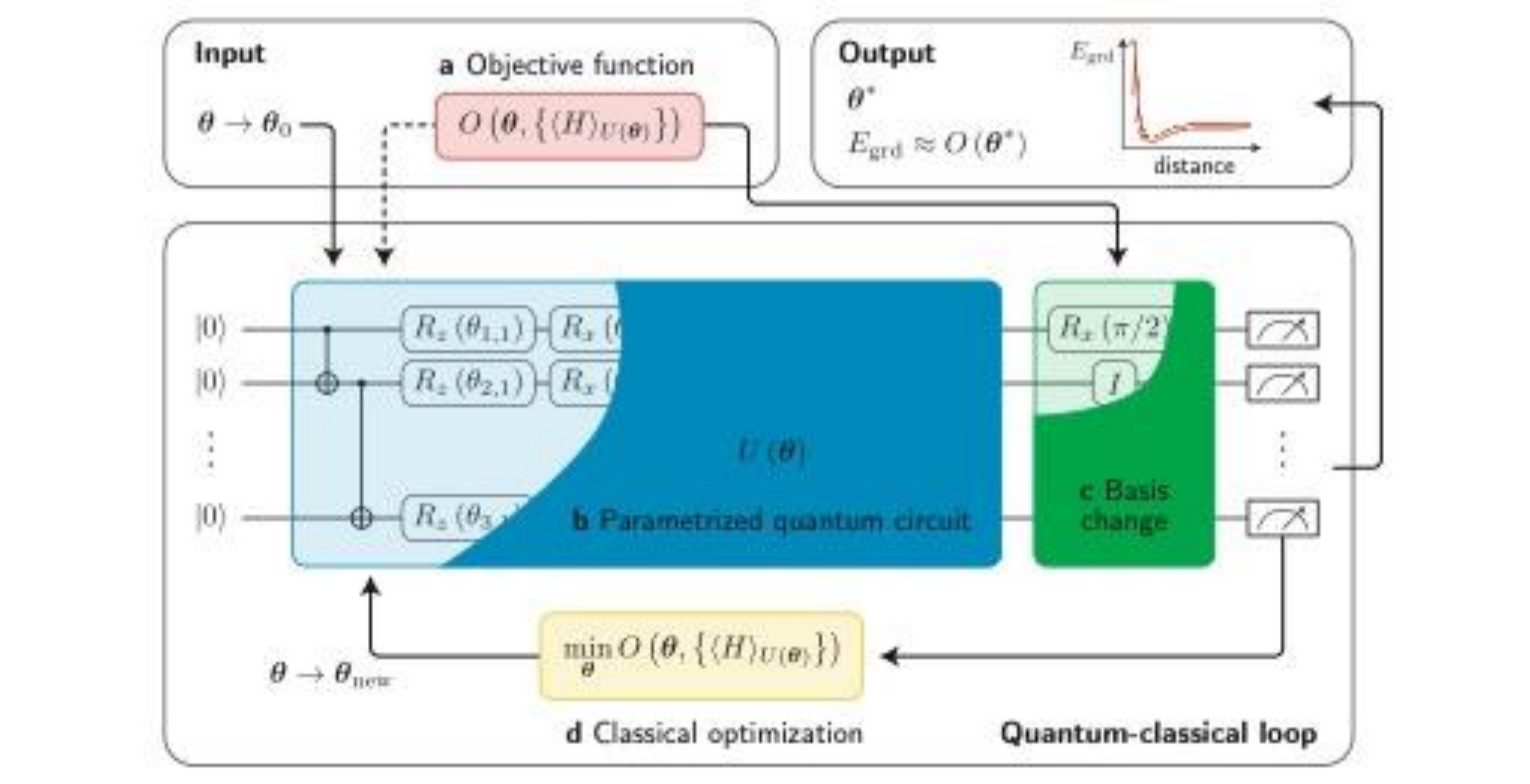
Quantum Circuit on NISQ



Variational Quantum Algorithm & Parameterized Quantum Circuit

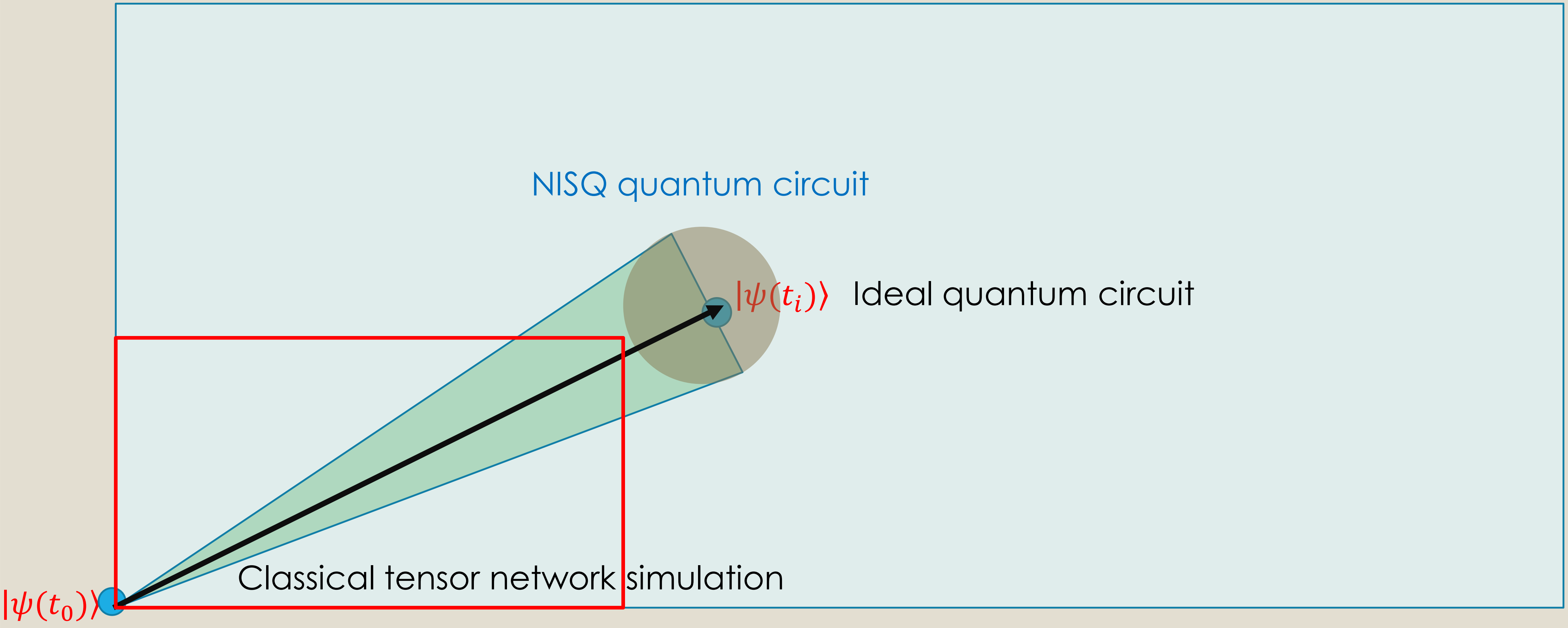


Variational Quantum Algorithm & Parameterized Quantum Circuit

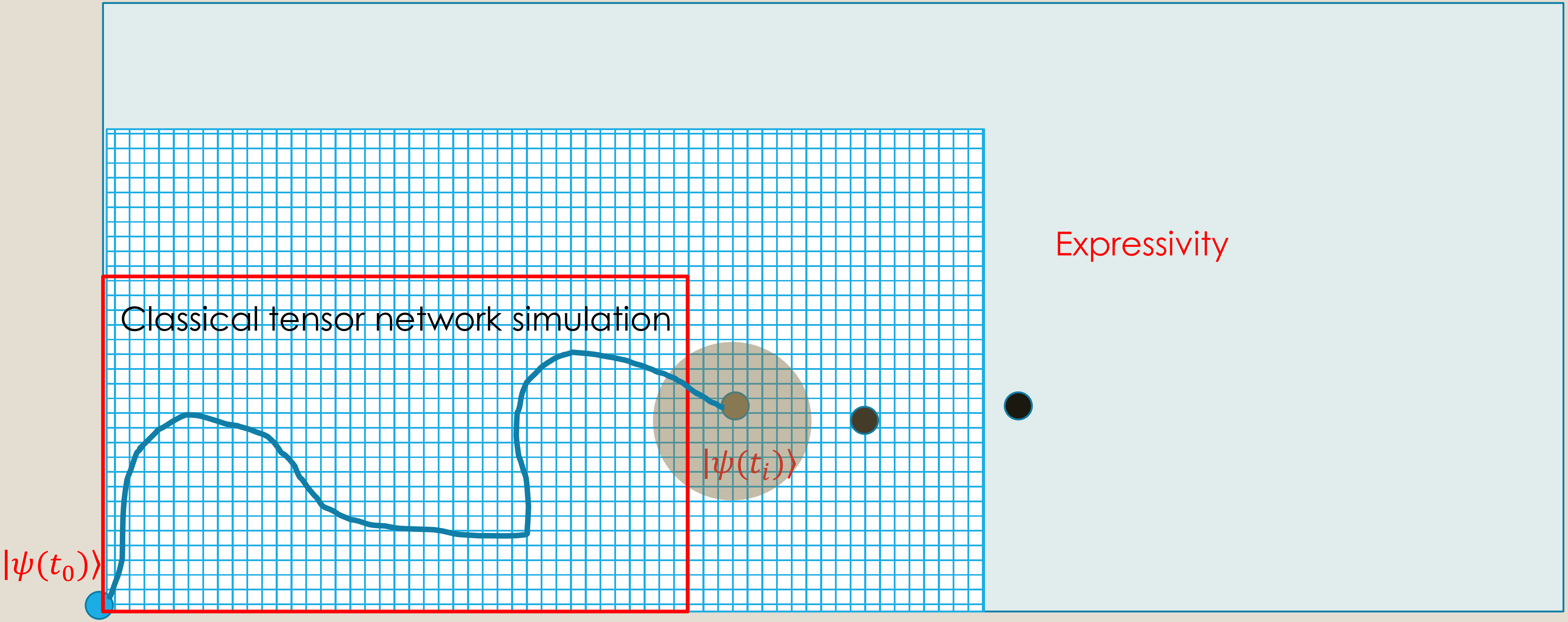


“Noisy intermediate-scale quantum algorithms”, Rev. Mod. Phys. **94,** 015004 (2022).

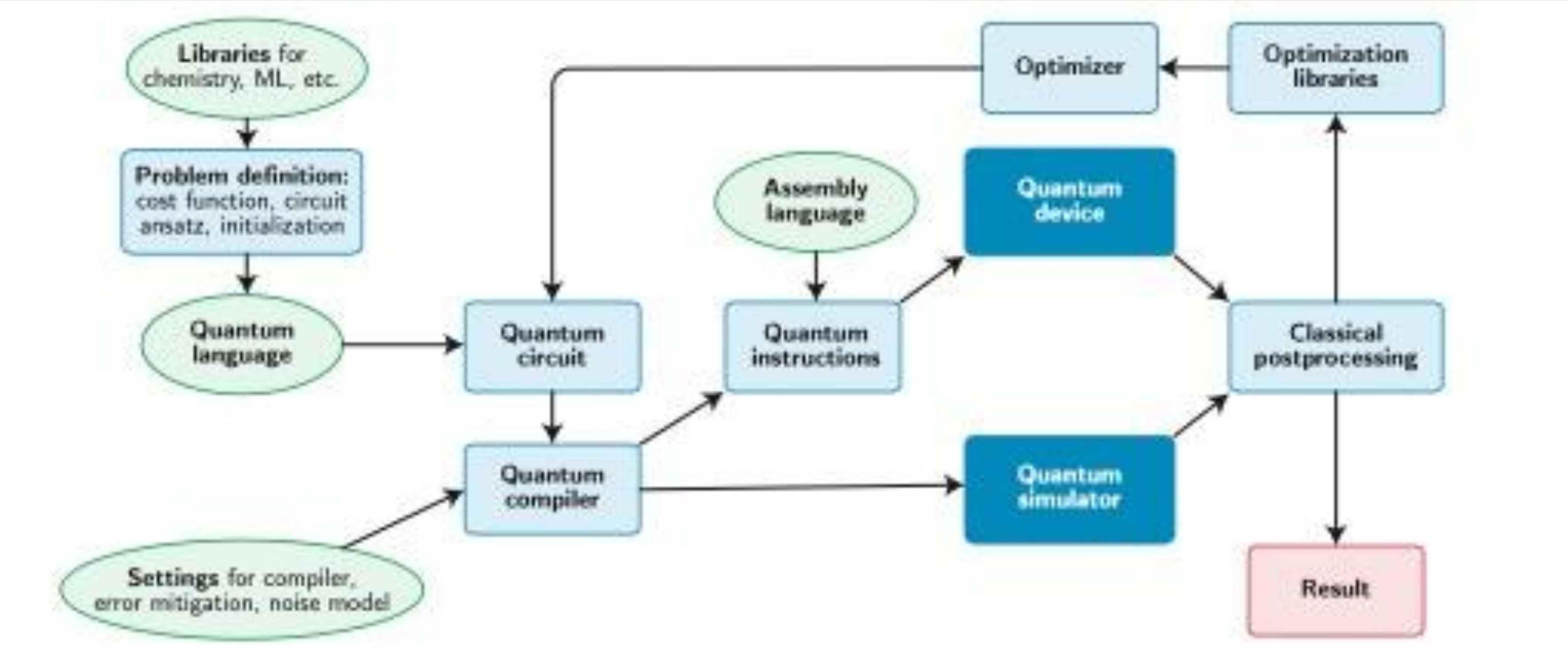
Quantum Circuit on NISQ



Variational Quantum Algorithm & Parameterized Quantum Circuit



Tensor Network v.s. Quantum Circuit Competition or Synergy



"Noisy intermediate-scale quantum algorithms", Rev. Mod. Phys. **94,** 015004 (2022).

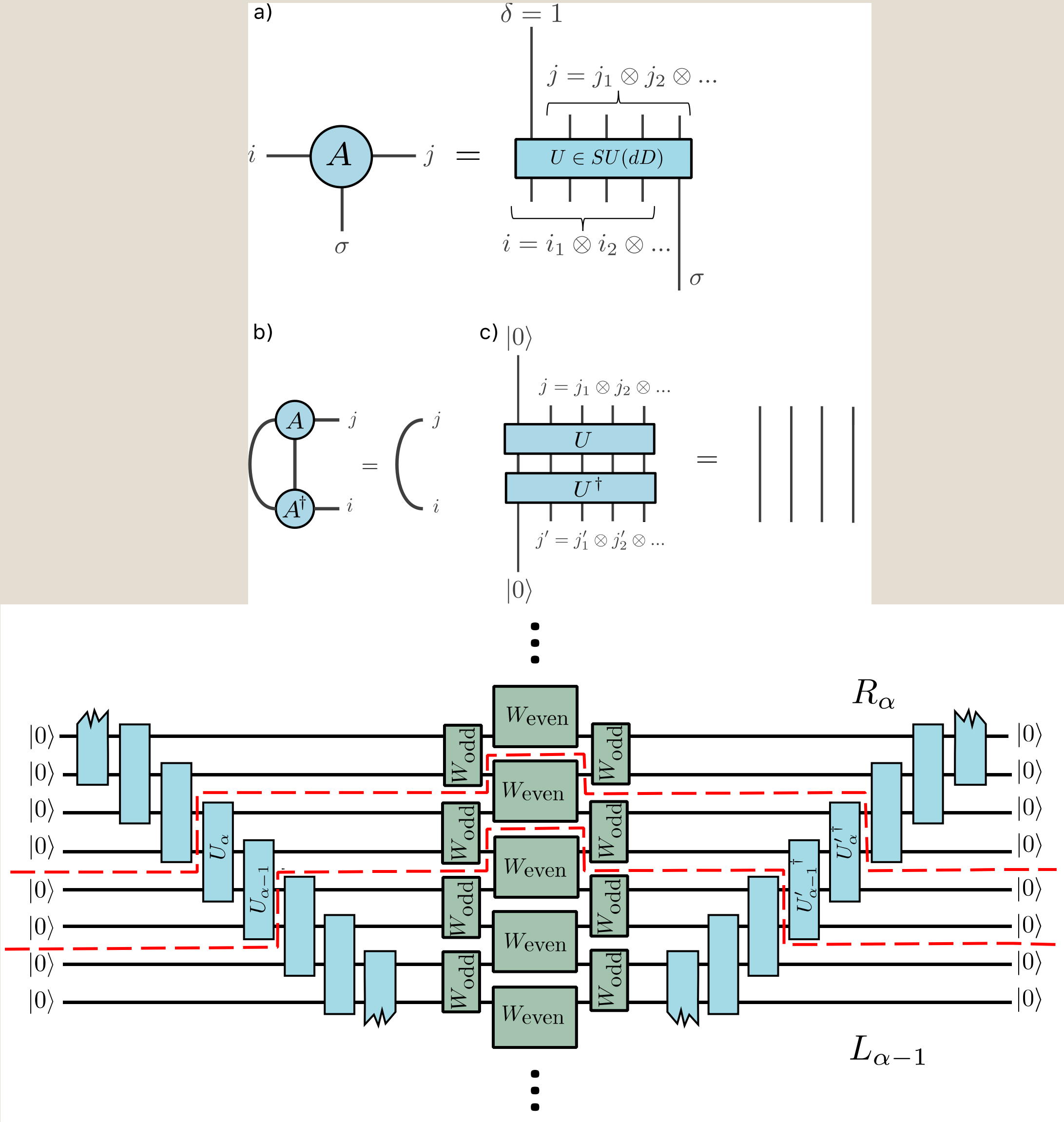
Tensor Network v.s. Quantum Circuit Competition or Synergy

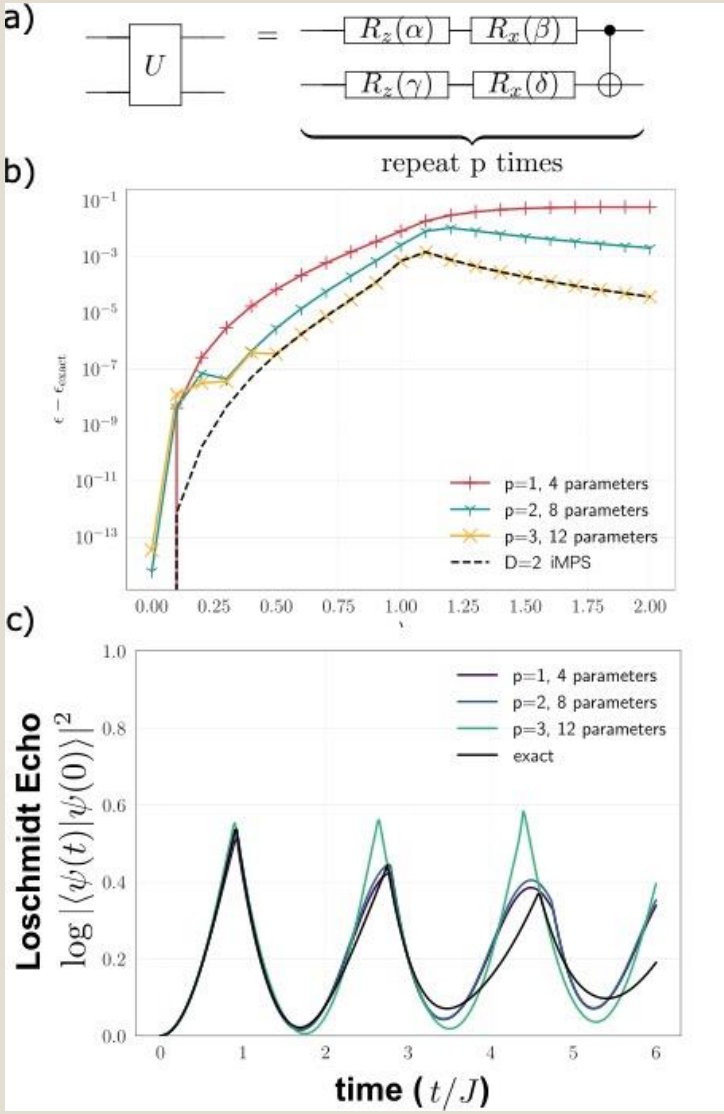
- Classical tensor network algorithms → Quantum circuit algorithms
  - Quantum simulation of many-body systems.

- Classical tensor network state → Quantum tensor network state
  - Increase the expressivity!

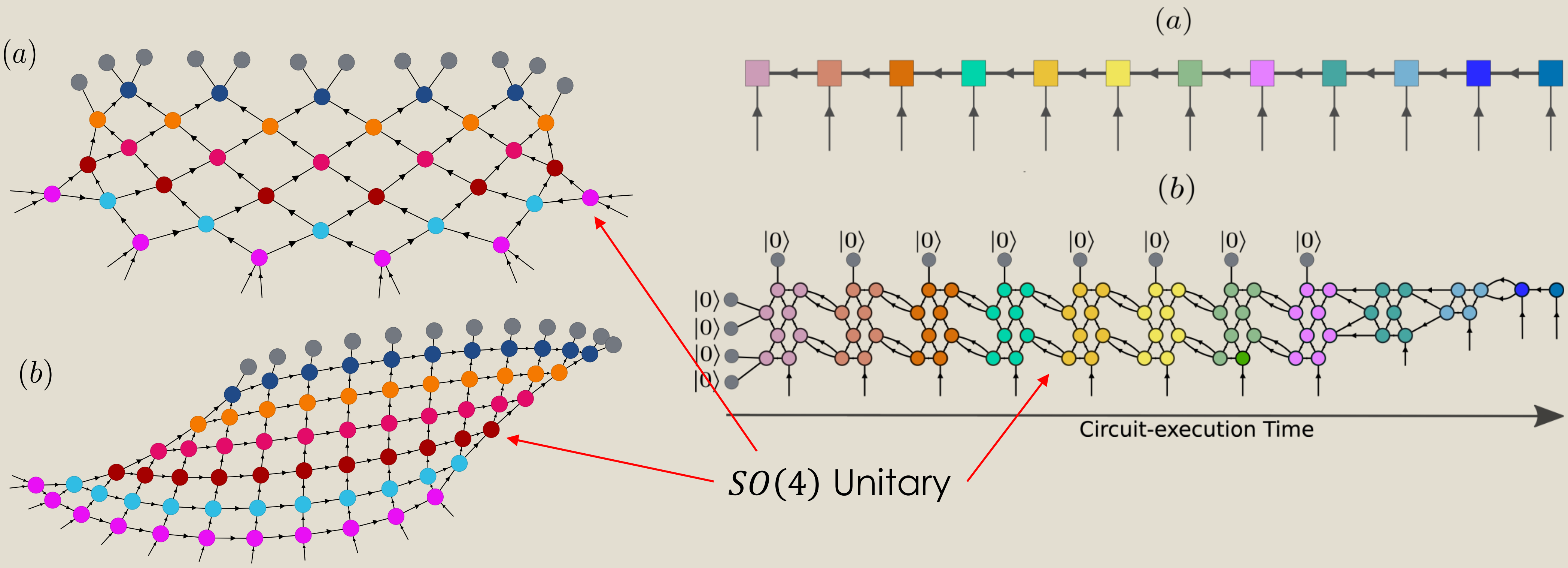
- Classical tensor network simulation → Parameterized quantum circuit (PQC).
  - Identify promosing quantum state from classical tensor network simulation.
  - Convert the tensor network state to a PQC.
  - Apply classically infeasible gates to further optimize the PQC

iMPS → Quantum Circuit Machine



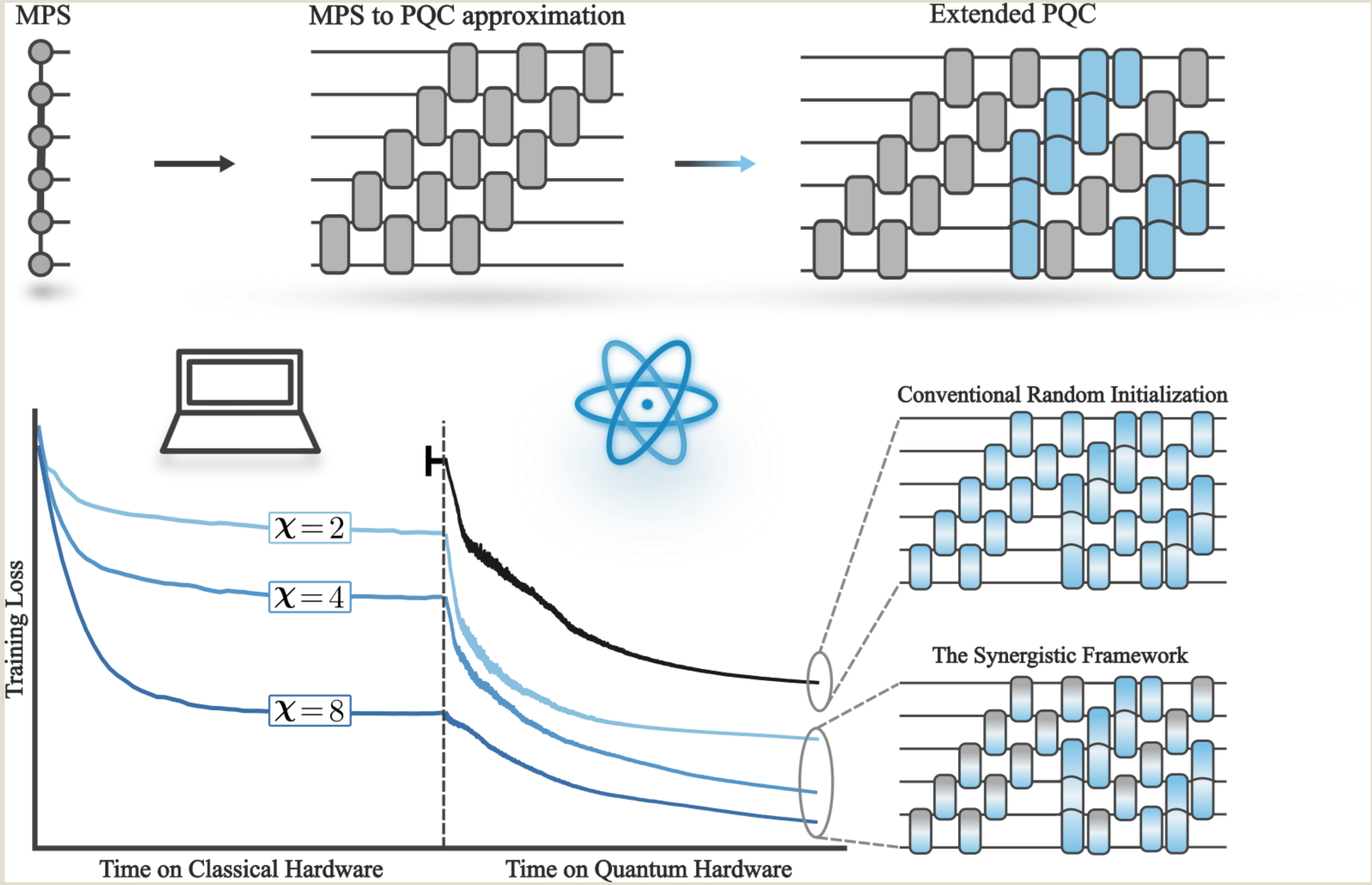


Quantum Circuit Tensor Network



Reza Haghshenas, Johnnie Gray, Andrew C. Potter, and Garnet Kin-Lic Chan, “Variational Power of Quantum Circuit Tensor Networks”, PRX **12**, 011047 (2022).

Classical TN simulation → PQC



Manuel S. Rudolph, Jacob Miller, Jing Chen, Atithi Acharya, Alejandro Perdomo-Ortiz, arXiv:2208.13673.

Tensor Network

is

the Natual Language

for

Classical and Quantum Computation

High-Performance Tensor Network Library is Needed