Exercise –1
The state space is S = {0, 1, 2, ... N} Class 1: {0}
- state 0 is absorbing
- stays forever

Class 2: {1, 2, 3... N- 1}
- These are transient states

Class N:
- state N is absorbing

Exercise –2
The states in A = {r+1, ... N} are all absorbing state because:
1. The identity block I ensures $P_{\{j,j\}}$= 1 for all j ∈ A
2. The zero block) ensures no return to transient staes T
3. Each absorbing state forms its own closed commuincation class

Exercise –3
T is a stopping time because at each time n, we can determine whether t =n using only the history of the chain up to time n

T is adapted to the natural filtration because the deecision "have we been absorbed yet?" depends only on the past and present states, not on future states.

Exercise 4
For this problem we should model X_n as a **discrete-time Markov Chain** where state represents Theseus position in the maze at time n.

State space: S = {0, 1, 2, E}

State 0: Initial Chamber
State 1: Fork in the path
State 2: Right Path
State E: Exit

Transisition Probabilities:

State 0 -> State 2: 1/2
State 0 -> State 1: 1/2

State 1 -> State E: 1/3
State 1 -> State 0: 2/3

State 2 -> State 0: 1
State E -> State E: 1

Part 2
use first-step analysis approach from optimal stopping theory.
$w_i = g(i) + \sum_j P_{ij} w_j$

w_i = expected time to exit from state i
g(i) = some immediate cost time to transition to this state.
P_{ij} = transition probability form i to j.
w_j = Expected time to exit start from state j.
$\sum_j$ = The sum of all possible next state j.

Exercise 5

Absorption probability dependss only on which states you visit, not on how long you spend in each state.

CTP $X(*)$

Let u_i = probability of eventually being absorbed into state A, starting from state i

Starting from state i:
- The process spends some random time in state i(distributed as G_i)
- Then jums to state j with probability P_{ij}
- From state j, probabilit yof absorption is u_j

Therefore

$u_i = \sum_j P_{ij} * u_j$

DTMC X_n

Starting from state i in the discrete chain we
- jump to state j with probability P_{ij}
- From state j, probability of absorption is u_j

Therefore

$u_i = \sum_j P_{ij} * u_j$

Conculsion

Both continuous time and discrete time satisfy the exact same system of linear equations with the same boundary conditions.

$u_i = u_i$ for all states i.

The distribution doesn't matter
- where absorption occurs
- which absorbing state is reached
- the probability of absorption

does effect
- when absorption occurs
- the rate at which transtions happens.

Exercise 6
First you hang out in state i then you jump to state j with some average probability.

Let W_i denote the expected time to absorption for the continuous-time process X(.) starting from transient state i.

Using first-step analysis, we condition on the first trasition:

W_i = E[time in state i] + E[time after leaving state i | X(0) = i]

The time spent in state i has mean $1/v_i$ by definition.

After spending time T_i in state i, the process jumps to state j with probability Pij, If j is absorbing, no additional time is needed. If j is transisent, the expectred additional time is Wj.

By the law of total expectation:

E[time after leaving state i | X(0) = i]
=$\sum_j P_{ij} * E[\text{time to absorption from } j]$
=$\sum_j \in T P_{ij} * W_j$

The sum is only over trasient states T since absorbing state contribute 0.)

Therefore:

$W_i = \frac{1}{v_i} + \sum_j \in T\ W_j P_{ij}$

Exercise 7
we need to prove
- The future doesn't depends on the past only present
- it doesn't matter

Exercise 8
You can reach any state i, you can reach any state j in exactly n steps. Therefore i -> j for all pairs(i, j) This means all states communicates

All are single communicating class no separate groups.

Exercise 9
It oscilates back and force with o -> 1.