

NSUPS Bootcamp Week 1

Getting Started with CPPS

Number Theory

Number Theory

the branch of mathematics that deals with the properties and relationships of numbers, especially the positive integers. - Wikipedia

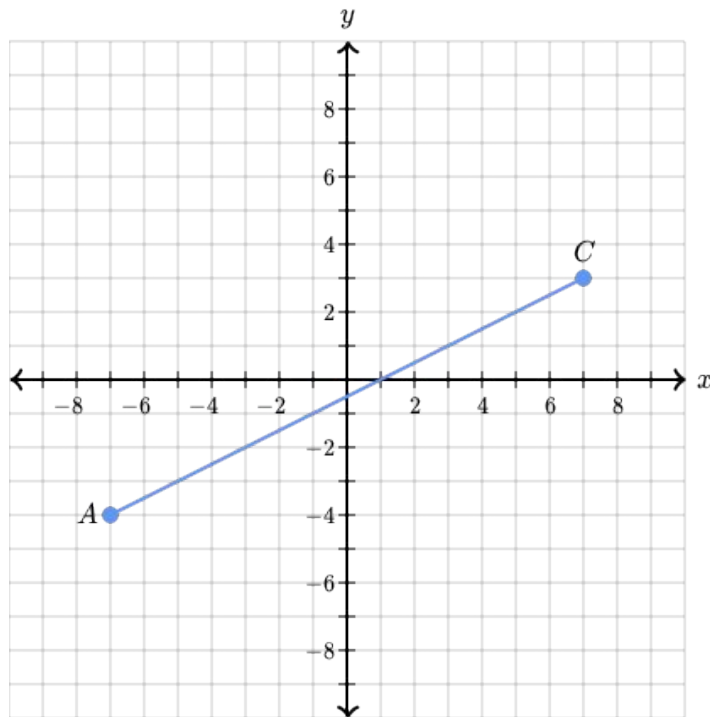
GCD

- **Problem:** Given two positive numbers **A** and **B**, find the greatest number **X** that divides both **A** and **B**.
- Basically, we want to find **Greatest Common Divisor (GCD)** of **A** and **B**, i.e, $\text{GCD}(A,B)$.
- For example, $\text{GCD}(3,4) = 1$ and $\text{GCD}(12,15) = 3$.

Euclidean Algorithm

- $\text{GCD}(A, B) = \text{GCD}(B, A \% B)$
- Proof? It's not necessary, but if you are curious, you can view it from <http://forthright48.blogspot.com/2015/07/euclidean-algorithm.html>
- Watch me code on whiteboard now. Both recursive and iterative version.
- There is also a builtin function in C++ for finding gcd. You can simply write `__gcd(a, b)` to find $\text{GCD}(a, b)$.
- **Property:** $\text{GCD}(A, B, C) = \text{GCD}(\text{GCD}(A, B), C)$
- **Warning:** Careful about $\text{GCD}(0, 0)$ and negative GCD result.

Number of Lattice Points on a Line Segment



Problem: Given a line segment with two lattice points A (x_1, y_1) and C (x_2, y_2), find number of lattice points that fall on that segment.

LCM

- $\text{GCD}(A,B) * \text{LCM}(A,B) = A * B$
- $\text{LCM}(A,B) = (A * B) / \text{GCD}(A,B)$
- Watch me code on whiteboard
- **Warning:** It might overflow!
- **Better:** $\text{LCM}(A,B) = (A / \text{GCD}(A,B)) * B$

Stadium Lights

Two of the lights at the local stadium are flickering. They both just flickered at the same time.

One of the lights flickers every 12 seconds and the other light flickers every 20 seconds.

How many seconds until both lights will flicker at the same time again?

Answer: $\text{LCM}(12,20) = (12 / \text{GCD}(12,20)) * 20 = (12/4) * 20 = 3 * 20 = 60 \text{ sec}$

Prime Numbers

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself. - Wiki

Primality Test

1. $O(N)$ Method
2. $O(\sqrt{N})$ Method
3. Miller Rabin - Advanced Algorithm (we will cover it later)

Sieve Technique

- **Problem:** Number of Divisor of all numbers from 1 to 10^6 .
- Watch me code it on whiteboard.
- What is the complexity? $O(N^2)$.
- No. It's $O(N \cdot \log(N))$! How?
- Watch me work out the complexity on whiteboard.

Harmonic Sequence

In mathematics, the harmonic series is the divergent infinite series:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

There is no closed form formula for partial sum of the sequence, but an approximate formula exists

$$H_n \approx \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2}$$

where $\gamma \approx 0.5772156649$ is the Euler-Mascheroni constant.

More on Sieve

1. **Problem:** Find sum of divisor of all numbers from 1 to 10^6 .
2. **Problem:** Given a number line with some numbers as unlucky. You start from 0 and want to take equal steps until you cross N. You need to find the smallest step size such that you can cross N without stepping on any unlucky numbers.
3. **Problem:** Find all prime numbers from 1 to 10^6 . This is called Prime Sieve.

Prime Sieve

- Watch me do stuff on whiteboard.
- Curious about the stuff on whiteboard? You can find the same things in LightOJ Tutorial:
http://lightoj.com/article_show.php?article=1001&language=english&type=pdf
- You will find code for Prime Sieve in
<http://forthright48.blogspot.com/2015/07/sieve-of-eratosthenes-generating-primes.html>

More Resources

1. ProgKriya Basic Number Theory:
<http://www.progkriya.org/gyan/basic-number-theory.html>
2. forthright48's Blog: <http://forthright48.blogspot.com/p/cpps-101.html>

Bit Manipulation

- Binary - Decimal Conversion
- Bitwise operators
- Application (Set or Reset the k'th bit of an integer)

Resources

- <https://www.geeksforgeeks.org/bitwise-operators-in-c-cpp/>
- <https://codeburst.io/your-guide-to-bit-manipulation-48e7692f314a>
- <https://www.hackerearth.com/practice/notes/bit-manipulation/>
- https://www.cprogramming.com/tutorial/bitwise_operators.html

Practice Contests

1. There will be practice contest each week to practice the topics taught in camp.
2. Contest will be hosted on [Vjudge.net](https://vjudge.net).
3. Everyone will need to open an account in vjudge. You can set username as anything you want, but you need to set your Nickname as your NSUID.

Tips for Practice Contests

1. Remember, the problems in contests are often not sorted in order of difficulty. The easiest problem can be anywhere, even in middle!
2. Don't get stuck on a particular problem. If you can't solve a problem, simply skip it for now, solve other problems that you are able to solve and come back again later to solve it.
3. Follow the ranklist.
4. There is a fine line between learning to solve and cheating to solve. I will let you be the judge.

What's the point of learning theory if you don't solve?

“For the things we have to learn before we can do them, we learn by doing them.” - Aristotle

“Knowledge is of no value unless you put it into practice.” - Anton Chekhov

Sneak Peak of Stuffs we will cover next class

1. STL of the Day
 - a. <https://community.topcoder.com/tc?module=Static&d1=features&d2=082803>
 - b. <http://www.geeksforgeeks.org/cpp-stl-tutorial/>
2. EOF using scanf() and gets()
3. More Number Theory: Prime Factorization, Application of Prime Factorization
4. Modular Arithmetic

Thank You

Any Questions?