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r"""
Updates to the thin SVD using NumPy.

This function is a SAGE replication of Matthew Brand's article on "Fast low-rank modifications of the
thin singular value decomposition." <http://www.stat.osu.edu/~dmsl/thinSVDtracking.pdf>
This function is an approximation to the true thin SVD, therefore, no tests are provided.

AUTHORS:

- Taylor Steiger, James Pak (2013-06-10): initial version


EXAMPLES::

    Update

    sage: X = np.array([[1.0,2.0,3.0,4.0],[3.0,2.0,5.0,5.0],[5.0,3.0,1.0,1.0],[7.0,7.0,7.0,7.0]])
    sage: U, s, V = np.linalg.svd(X, full_matrices = False)
    sage: a = np.reshape(np.array([4.0,5.0,1.0,7.0]), (-1, 1))
    sage: U, S, V = svd_update(U, np.diag(s), V, X, a, update = True)

    Downdate

    sage: X = np.array([[1.0,2.0,3.0,4.0],[3.0,2.0,5.0,5.0],[5.0,3.0,1.0,1.0],[7.0,7.0,7.0,7.0]])
    sage: U, s, V = np.linalg.svd(X, full_matrices = False)
    sage: U, S, V = svd_update(U, np.diag(s), V, X, downdate = True)

    Revise

    sage: X = np.array([[1.0,2.0,3.0,4.0],[3.0,2.0,5.0,5.0],[5.0,3.0,1.0,1.0],[7.0,7.0,7.0,7.0]])
    sage: U, s, V = np.linalg.svd(X, full_matrices = False)
    sage: a = np.reshape(np.array([4.0,5.0,1.0,7.0]), (-1, 1))
    sage: U, S, V = svd_update(U, np.diag(s), V, X, a)

    Recenter

    sage: X = np.array([[1.0,2.0,3.0,4.0],[3.0,2.0,5.0,5.0],[5.0,3.0,1.0,1.0],[7.0,7.0,7.0,7.0]])
    sage: U, s, V = np.linalg.svd(X, full_matrices = False)
    sage: U, S, V = svd_update(U, np.diag(s), V, X)

"""

#*****************************************************************************
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#       Copyright (C) 2013 James Pak <jimmypak@uw.edu>
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#*****************************************************************************

import numpy as np

def svd_update(U, S, V, X, c = None, update = False, downdate = False):
    """
    INPUT:

    - U -- a (nxn) matrix containing singular vectors of X.

    - S -- a (nxn) diagonal matrix containing singular values. the ith diagonal entry is the singular
value corresponding to the ith column of U.
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- V -- a (nxn) matrix containing singular vectors of X.

- X -- a (mxn or nxm) matrix such that U^T*X*V=S.

- c -- (default: None) a column vector for revision or update of decomposition.

- update -- (default: False) boolean whether to add c to the decomposition. If true, c must also be provided.

- downdate -- (default: False) boolean whether to downdate the decomposition.

OUTPUT:

A 3-tuple consisting of matrices in this order:

1. Transformed U.
2. Transformed S.
3. Transformed V.