LABORATORY OF HUMAN AND ROBOT INTERACTION - UNIVERSITY OF TEHRAN

Delta Parallel Robot - Kinematic Study

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November 12, 2023

1 BASIC GEOMETRY

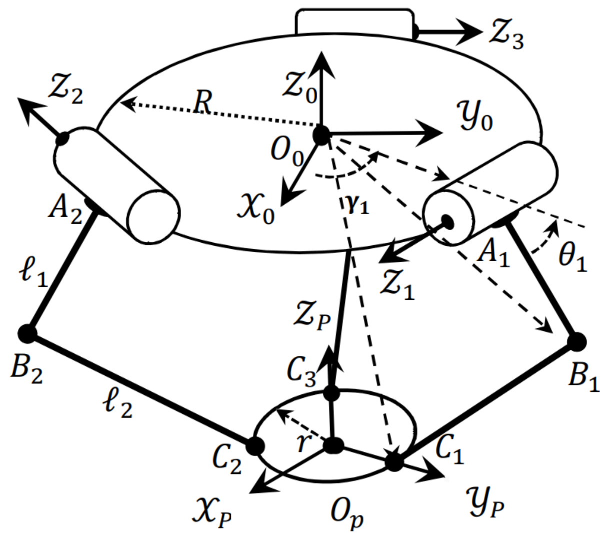

Figure 1.1: Delta Parallel Robot schematic

The first value definition is regarding the distance of the actuator joint and base frame center:

$$R_i = \overline{O_0 A_i}$$

And then the center of the End-Effector's distance with the nearest universal joint:

$$r_i = \overline{O_P C_i}$$

The length of each drive link will be:

$$L_i = \overline{A_i B_i}$$

And the length of the driven link will be:

$$l_i = \overline{B_i C_i}$$

The angles between x_0 and $\overline{O_0 A_i}$ are equal to γ_i. The angles between x_P and $\overline{O_P C_i}$ are equal to β_i. The angles between the horizon and the drive link are θ_i, Where $i = 1,2,3$

2 Forward Kinematics

We'll first write down the kinematic chain. The position of A_i in relation to the {0} frame:

$$\overline{O_0 A_i} = \begin{bmatrix} R_i \cos\gamma_i & R_i \sin\gamma_i & 0 \end{bmatrix}^T \tag{2.1}$$

The position of B_i in relation to the {0} frame:

$$\overline{O_0 B_i} = \begin{bmatrix} (L_i \cos\theta_i + R_i)\cos\gamma_i \\ (L_i \cos\theta_i + R_i)\sin\gamma_i \\ -L_i \sin\theta_i \end{bmatrix} \tag{2.2}$$

The position of {P} frame origin point in relation to the {0} frame:

$$\overline{O_0 O_P} = \begin{bmatrix} X_P & Y_P & Z_P \end{bmatrix}^T \tag{2.3}$$

The position of C_i in relation to the {P} frame

$$\overline{O_P C_i} = \begin{bmatrix} r_i \cos\beta_i & r_i \sin\beta_i & 0 \end{bmatrix}^T \tag{2.4}$$

The position of C_i in relation to the {0} frame:

$$\overline{O_0 C_i} = \begin{bmatrix} X_P + r_i \cos\beta_i \\ Y_P + r_i \sin\beta_i \\ Z_P \end{bmatrix} \tag{2.5}$$

With that the kinematic chain has ended. The next step is to use a "good" constraint as the solution of the kinematic chain which in this case is the lengths of the driven links:

$$l_i = \left| \overline{O_0 B_i} - \overline{O_0 C_i} \right| \tag{2.6}$$

With expanding the mentioned equation we will have the following:

$$\overline{O_0B_i} - \overline{O_0C_i} = \begin{bmatrix} (L_i\cos\theta_i + R_i)\cos\gamma_i - r_i\cos\beta_i - X_P \\ (L_i\cos\theta_i + R_i)\sin\gamma_i - r_i\sin\beta_i - Y_P \\ -L_i\sin\theta_i - Z_P \end{bmatrix} \tag{2.7}$$

From the Eq. 2.7 we can write:

$$X_P^2 + Y_P^2 + Z_P^2 + a_iX_P + b_iY_P + c_iZ_P + d_i = 0 \tag{2.8}$$

Where:

$$\begin{aligned} a_i &= -2(L_i\cos\theta_i + R_i)\cos\gamma_i + 2r_i\cos\beta_i \\ b_i &= -2(L_i\cos\theta_i + R_i)\sin\gamma_i + 2r_i\sin\beta_i \\ c_i &= 2L_i\sin\theta_i \\ d_i &= (R_i + L_i\cos\theta_i)^2 + r_i^2 + L_i^2\sin^2\theta_i - 2r_i(R_i + L_i\cos\theta_i)\cos(\gamma_i - \beta_i) - l_i^2 \end{aligned} \tag{2.9}$$

There will be a system of equations as depicted below:

$$\begin{aligned} X_P^2 + Y_P^2 + Z_P^2 + a_1X_P + b_1Y_P + c_1Z_P + d_1 = 0 \\ X_P^2 + Y_P^2 + Z_P^2 + a_2X_P + b_2Y_P + c_2Z_P + d_2 = 0 \\ X_P^2 + Y_P^2 + Z_P^2 + a_3X_P + b_3Y_P + c_3Z_P + d_3 = 0 \end{aligned} \tag{2.10}$$

or:

$$\begin{aligned} A_1X_P + B_1Y_P + C_1Z_P + D_1 = 0 \\ A_2X_P + B_2Y_P + C_2Z_P + D_2 = 0 \end{aligned} \tag{2.11}$$

Where:

$$\begin{matrix} A_1 = a_1 - a_2 & B_1 = b_1 - b_2 & C_1 = c_1 - c_2 & D_1 = d_1 - d_2 \\ A_2 = a_1 - a_3 & B_2 = b_1 - b_3 & C_2 = c_1 - c_3 & D_2 = d_1 - d_3 \end{matrix} \tag{2.12}$$

So it becomes a linear system of equations.

$$\begin{cases} X_P = m_1Z_P + n_1 \\ Y_P = m_2Z_P + n_2 \end{cases} \tag{2.13}$$

Where:

$$\begin{matrix} m_1 = \dfrac{B_2C_3 - B_3C_2}{A_2B_3 - A_3B_2} & n_1 = \dfrac{B_2D_3 - B_3D_2}{A_2B_3 - A_3B_2} \\ m_2 = \dfrac{A_3C_2 - A_2C_3}{A_2B_3 - A_3B_2} & n_2 = \dfrac{A_3D_2 - A_2D_3}{A_2B_3 - A_3B_2} \end{matrix} \tag{2.14}$$

3 Inverse Kinematics

We'll start off with using the Eq. 2.7 which will help us find θ_i:

$$\begin{aligned}
&\left[-2r_i L_i \sin\beta_i \sin\gamma_i - 2r_i L_i \cos\beta_i \cos\gamma_i - 2L_i X_P \cos\gamma_i - 2L_i Y_P \sin\gamma_i + 2R_i L_i\right]\cos\theta_i \\
&+ 2L_i Z_P \sin\theta_i + X_P^2 + Y_P^2 + Z_P^2 + L_i^2 - l_i^2 + R_i^2 + r_i^2 - 2R_i r_i \sin\beta_i \sin\gamma_i - 2R_i r_i \cos\beta_i \cos\gamma_i \\
&+ 2r_i X_P \cos\beta_i + 2r_i Y_P \sin\beta_i - 2R_i X_P \cos\gamma_i - 2R_i Y_P \sin\gamma_i = 0
\end{aligned} \tag{3.1}$$

or:

$$A_i \cos\theta_i + B_i \sin\theta_i + D_i = 0 \tag{3.2}$$

and using the half angle solution:

$$\theta_i = 2\arctan\left(\frac{-B_i \pm \sqrt{B_i^2 + A_i^2 - D_i^2}}{D_i - A_i}\right) \tag{3.3}$$

4 References

Kinematic Analysis of Delta Parallel Robot: Simulation Study - A. Eltayeb