LABORATORY OF HUMAN AND ROBOT INTERACTION - UNIVERSITY OF TEHRAN

Delta Parallel Robot - Kinematic Study

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1 BASIC GEOMETRY AND ROBOT DESCRIPTION

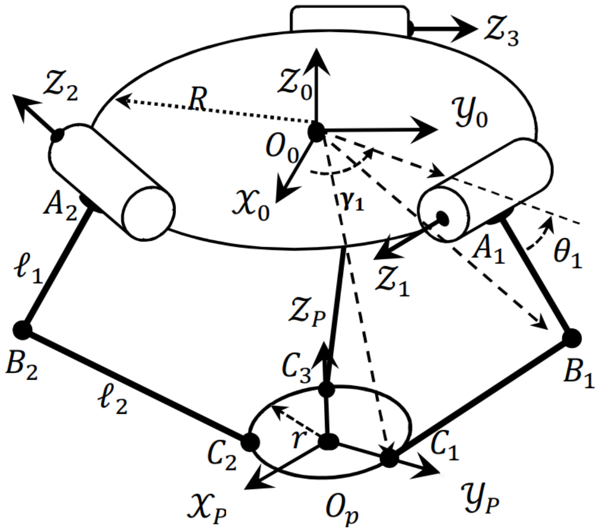

Figure 1.1: Delta Parallel Robot schematic

The Delta Parallel Robot (DPR) consists of three main chains. Each chain start from the base platform (O_0), then there is the upper arm connected to the base via a pin joint (A), then there is the lower arm which is connected to the upper arm via a universal joint (B), and finally the End-Effector (EE) of the robot which is connected to the lower arm via an universal joint (C). This results in the EE moving along three axes. The EE will be always parallel to the base platform.

$$
\begin{cases}
R_i \equiv \overline{O_0 A_i} = \text{The distance of the actuator joint and base platform center} \\
r_i \equiv \overline{O_P C_i} = \text{The distance of the EE's center with the joint connection to the lower arm} \\
L_i \equiv \overline{A_i B_i} = \text{The length of each drive link} \\
l_i \equiv \overline{B_i C_i} = \text{The length of the driven links}
\end{cases}
$$

The angles between x_0 and $\overline{O_0 A_i}$ are equal to γ_i. The angles between x_P and $\overline{O_P C_i}$ are equal to β_i. The angles between the horizon and the drive link are θ_i, Where $i = 1, 2, 3$

2 Simplified Kinematics

The problem of Forward Kinematics (FK) is to find the position of EE in relation to the base platform given the actuator joint angles, and Inverse Kinematics (IK) is the opposite of that. Here we'll have some **assumptions to simplify** the problem:

$$\begin{cases} L = L_1 = L_2 = L_3 \\ l = l_1 = l_2 = l_3 \\ R = R_1 = R_2 = R_3 \\ r = r_1 = r_2 = r_3 \\ \gamma_1 = \frac{1}{2}\gamma_2 = \frac{1}{3}\gamma_3 = \beta_1 = \frac{1}{2}\beta_2 = \frac{1}{3}\beta_3 = 120° \end{cases}$$

2.1 Calculating Positions

About the **position of A_i in relation to the $\{O_0\}$-frame** we can derive the following equation:

$$\overline{O_0 A_i} = [R\cos\gamma_i \quad R\sin\gamma_i \quad 0]^T \tag{2.1}$$

Now we can derive the **position of B_i in relation to the $\{O_0\}$-frame**:

$$\overline{O_0 B_i} = \begin{bmatrix} (R + L\cos\theta_i)\cos\gamma_i \\ (R + L\cos\theta_i)\sin\gamma_i \\ -L\sin\theta_i \end{bmatrix} \tag{2.2}$$

Similarly for **position of C_i in relation to the $\{O_0\}$-frame** we have:

$$\overline{O_P C_i} = [r\cos\gamma_i \quad r\sin\gamma_i \quad 0]^T \tag{2.3}$$

We found the position of A_i and B_i in relation to the initial frame, so now it's time to find the **position of C_i in relation to the $\{O_0\}$-frame** assuming that the position of center of EE is equal to $[X_P \quad Y_P \quad Z_P]^T$:

$$\overline{O_0 C_i} = \begin{bmatrix} X_P + r\cos\gamma_i \\ Y_P + r\sin\gamma_i \\ Z_P \end{bmatrix} \tag{2.4}$$

2.2 Constraint Equation

Given the mentioned equations we can say that we have the following **constraint**:

$$l = |\overline{O_0 B_i} - \overline{O_0 C_i}| \tag{2.5}$$

Re-writing the previous equation we have:

$$\overline{O_0 B_i - O_0 C_i} = \begin{bmatrix} (R - r + L\cos\theta_i)\cos\gamma_i - X_P \\ (R - r + L\cos\theta_i)\sin\gamma_i - Y_P \\ -L\sin\theta_i - Z_P \end{bmatrix} \tag{2.6}$$

2.3 Forward Kinematics

We **numerically** solve the Equation 2.5 for $[X_P, \quad Y_P, \quad Z_P]$ assuming θ_i is given.

2.4 Inverse Kinematics

we assume a **variable change** of $t = \tan(\theta_i/2)$, then $\sin(\theta_i) = \frac{2t}{t^2+1}$ and $\cos(\theta_i) = \frac{1-t^2}{t^2+1}$. We then apply this variable change to the Equation 2.5 and solve a system of linear equations.

3 Detailed Kinematics

The assumptions of previous section aren't present here. The reason being that in real life there are no perfect values. For example if we assume that $\gamma = 120°$ the real life angle might be 119, 121 or even 125. That is why if we want to calibrate the geometry of the robot we need the calculations to be more detailed.

3.1 Calculating Positions

We'll first write down the kinematic chain. The position of A_i in relation to the {0} frame:

$$\overline{O_0 A_i} = \begin{bmatrix} R_i \cos\gamma_i & R_i \sin\gamma_i & 0 \end{bmatrix}^T \tag{3.1}$$

The position of B_i in relation to the {0} frame:

$$\overline{O_0 B_i} = \begin{bmatrix} (L_i \cos\theta_i + R_i)\cos\gamma_i \\ (L_i \cos\theta_i + R_i)\sin\gamma_i \\ -L_i \sin\theta_i \end{bmatrix} \tag{3.2}$$

The position of $\{P\}$ frame origin point in relation to the {0} frame:

$$\overline{O_0 O_P} = \begin{bmatrix} X_P & Y_P & Z_P \end{bmatrix}^T \tag{3.3}$$

The position of C_i in relation to the $\{P\}$ frame

$$\overline{O_P C_i} = \begin{bmatrix} r_i \cos\beta_i & r_i \sin\beta_i & 0 \end{bmatrix}^T \tag{3.4}$$

The position of C_i in relation to the {0} frame:

$$\overline{O_0 C_i} = \begin{bmatrix} X_P + r_i \cos\beta_i \\ Y_P + r_i \sin\beta_i \\ Z_P \end{bmatrix} \tag{3.5}$$

3.2 Constraint Equation

With that the kinematic chain has ended. The next step is to use a "good" constraint as the solution of the kinematic chain which in this case is the lengths of the driven links:

$$l_i = \left| \overline{O_0 B_i} - \overline{O_0 C_i} \right| \tag{3.6}$$

With expanding the mentioned equation we will have the following:

$$\overline{O_0 B_i} - \overline{O_0 C_i} = \begin{bmatrix} (L_i \cos\theta_i + R_i)\cos\gamma_i - r_i \cos\beta_i - X_P \\ (L_i \cos\theta_i + R_i)\sin\gamma_i - r_i \sin\beta_i - Y_P \\ -L_i \sin\theta_i - Z_P \end{bmatrix} \tag{3.7}$$

3.3 Forward Kinematics

For solving FK from the Eq. 3.7 we can write:

$$X_P^2 + Y_P^2 + Z_P^2 + a_i X_P + b_i Y_P + c_i Z_P + d_i = 0 \tag{3.8}$$

Where:

$$\begin{aligned}
a_i &= -2(L_i \cos\theta_i + R_i)\cos\gamma_i + 2r_i \cos\beta_i \\
b_i &= -2(L_i \cos\theta_i + R_i)\sin\gamma_i + 2r_i \sin\beta_i \\
c_i &= 2L_i \sin\theta_i \\
d_i &= (R_i + L_i \cos\theta_i)^2 + r_i^2 + L_i^2 \sin^2\theta_i - 2r_i(R_i + L_i\cos\theta_i)\cos(\gamma_i - \beta_i) - l_i^2
\end{aligned} \tag{3.9}$$

There will be a system of equations as depicted below:

$$\begin{aligned}
X_P^2 + Y_P^2 + Z_P^2 + a_1 X_P + b_1 Y_P + c_1 Z_P + d_1 = 0 \\
X_P^2 + Y_P^2 + Z_P^2 + a_2 X_P + b_2 Y_P + c_2 Z_P + d_2 = 0 \\
X_P^2 + Y_P^2 + Z_P^2 + a_3 X_P + b_3 Y_P + c_3 Z_P + d_3 = 0
\end{aligned} \tag{3.10}$$

or:

$$\begin{aligned}
A_1 X_P + B_1 Y_P + C_1 Z_P + D_1 = 0 \\
A_2 X_P + B_2 Y_P + C_2 Z_P + D_2 = 0
\end{aligned} \tag{3.11}$$

Where:

$$\begin{array}{llll}
A_1 = a_1 - a_2 & B_1 = b_1 - b_2 & C_1 = c_1 - c_2 & D_1 = d_1 - d_2 \\
A_2 = a_1 - a_3 & B_2 = b_1 - b_3 & C_2 = c_1 - c_3 & D_2 = d_1 - d_3
\end{array} \tag{3.12}$$

So it becomes a linear system of equations.

$$\begin{cases} X_P = m_1 Z_P + n_1 \\ Y_P = m_2 Z_P + n_2 \end{cases} \tag{3.13}$$

Where:

$$\begin{array}{ll}
m_1 = \dfrac{B_2C_3 - B_3C_2}{A_2B_3 - A_3B_2} & n_1 = \dfrac{B_2D_3 - B_3D_2}{A_2B_3 - A_3B_2} \\
m_2 = \dfrac{A_3C_2 - A_2C_3}{A_2B_3 - A_3B_2} & n_2 = \dfrac{A_3D_2 - A_2D_3}{A_2B_3 - A_3B_2}
\end{array} \tag{3.14}$$

and etc.

3.4 Inverse Kinematics

We'll start off with using the Eq. 3.7 which will help us find θ_i:

$$\begin{aligned}\left[-2r_iL_i\sin\beta_i\sin\gamma_i - 2r_iL_i\cos\beta_i\cos\gamma_i - 2L_iX_P\cos\gamma_i - 2L_iY_P\sin\gamma_i + 2R_iL_i\right]\cos\theta_i \\ + 2L_iZ_P\sin\theta_i + X_P^2 + Y_P^2 + Z_P^2 + L_i^2 - l_i^2 + R_i^2 + r_i^2 - 2R_ir_i\sin\beta_i\sin\gamma_i - 2R_ir_i\cos\beta_i\cos\gamma_i \\ + 2r_iX_P\cos\beta_i + 2r_iY_P\sin\beta_i - 2R_iX_P\cos\gamma_i - 2R_iY_P\sin\gamma_i = 0\end{aligned} \tag{3.15}$$

or:

$$A_i\cos\theta_i + B_i\sin\theta_i + D_i = 0 \tag{3.16}$$

and using the half angle solution:

$$\theta_i = 2\arctan\left(\frac{-B_i \pm \sqrt{B_i^2 + A_i^2 - D_i^2}}{D_i - A_i}\right) \tag{3.17}$$

4 REFERENCES

Kinematic Analysis of Delta Parallel Robot: Simulation Study - A. Eltayeb