Hamiltoniano en espacio de momentos.

En las simulaciones vimos que alcanza aproximar el hamiltoniano total a orden cuadrático e incluir el término de interacción:

$$\boxed{\hat{H}_j = \underset{\omega_q^0 = \sqrt{8E_cE_J}}{\hbar\omega_q^0}\, b_j^\dagger b_j - \underset{\hbar\lambda = \frac{E_c}{12}}{\hbar\lambda}\left(12 b_j^\dagger b_j + 6\left(b_j^{\dagger 2} + b_j^2\right) + 6 b_j^{\dagger 2} b_j^2\right)}$$

Calculamos el hamiltoniano total en espacio k:

$$\boxed{\hat{H} = \sum_j \left\{ \hat{H}_j - \hbar g \left(b_j^\dagger - b_j\right)\left(b_{j+1}^\dagger - b_{j+1}\right) \right\}}$$

Transformando $b_j^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-ikj} b_k^\dagger$

Se tiene que $\sum_j b_j^\dagger b_j = \sum_k b_k^\dagger b_k$

$$\sum_j b_j^{\dagger 2} = \sum_k b_k^\dagger b_{-k}^\dagger$$

$$\sum_j b_j^2 = \sum_k b_k b_{-k}$$

$$\sum_j b_j^\dagger b_{j+1} = \sum_k e^{ikj} b_k^\dagger b_k$$

$$\sum_j b_j b_{j+1}^\dagger = \sum_k e^{-ikj} b_k^\dagger b_k$$

$$\sum_j b_j^\dagger b_{j+1}^\dagger = \sum_k e^{-ik} b_k^\dagger b_{-k}^\dagger$$

$$\sum_j b_j b_{j+1} = \sum_k e^{ik} b_k b_{-k}$$

$$\sum_j b_j^{\dagger 2} b_j^2 = \frac{1}{N} \sum_{q,p,r,s} \delta_{q+p}^{r+s}\, b_q^\dagger b_p^\dagger b_r b_s$$

Reemplazando:

$\hbar\omega_q = \sqrt{8E_cE_J} - E_c$

$$\hat{H} = \sum_k \Big\{ (\hbar\omega_q + 2\hbar g\cos(k))\, b_k^\dagger b_k - (6\hbar\lambda + \hbar g e^{-ik})\, b_k^\dagger b_{-k}^\dagger - (6\hbar\lambda + \hbar g e^{ik})\, b_k b_{-k} \Big\} - \frac{6\hbar\lambda}{N}\Big(\sum_{q,p,r,s} \delta_{q+p}^{r+s}\, b_q^\dagger b_p^\dagger b_r b_s\Big)$$

Definiendo:

$$\varepsilon_k = \hbar\omega_q + 2\hbar g\cos(k)$$

$$\Delta_k = \hbar g e^{-ik} + 6\hbar\lambda$$

$$U = 6\hbar\lambda$$

Entonces:

$$\boxed{\hat{H} = \sum_k \Big\{ \varepsilon_k b_k^\dagger b_k - \left[\Delta_k b_k^\dagger b_{-k}^\dagger + \Delta_k^* b_k b_{-k}\right]\Big\} - \frac{U}{N}\sum_{q,p,r,s} \delta_{q+p}^{r+s}\, b_q^\dagger b_p^\dagger b_r b_s}$$

↳ Zero spin, 1D, non-autoconsistent BCS?

Si tuviéramos muchos sitios podríamos usar mean field y sería más parecido al BCS.

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