A Guidance on the Projects

In this short note, we provide some guidance on the projects.

1. Choice of parameters

The following parameter choices are advised in the experiments:

1. Dimension of the problem $n = 256$.
2. Number of measure measurement $m = 64$. A simple approach to test your code is that you can set $A = \mathrm{Id}$, i.e. A is the identity operator.
3. The variance of the noise ϵ should be small, for instance 10^{-1} or 10^{-2}. In the report, 0.5 variance is too large. (You can still use 0.5 if you like)
4. The choice of the regularization parameter μ can be set as 10 times the variance of the noise.

2. Lipschitz constant of the gradient of model (2.1)

To apply gradient descent to solve the problem, the iterate reads

$$x_{k+1} = x_k - \gamma \nabla \Phi(x_k)$$

where we use the constant step-size γ. For this case, we should set

$$0 < \gamma < \frac{1}{L}$$

where L is the Lipschitz constant of the gradient $\nabla \Phi(x)$. Recall the objective function of (2.1) in our project file

$$\min_x \ \Phi(x) = h(\nabla x) + \frac{1}{2}\|Ax - b\|^2$$

Since we have two choices of $h(x)$, below we discuss their Lipschitz constant

1. For $h(x)$ defined via the $\phi_\eta(x)$, the Lipschitz constant $\nabla \Phi(x)$ reads

$$\frac{2\mu}{\eta} + \|A\|^2$$

   As we can see here, the smaller the η we choose, the larger the Lipschitz constant (hence smaller step-size). In practice, the smaller the value of η, the better reconstruction result we will get. Therefore, we need to make trade-off about the choice of η.

2. For $h(x) = \|\nabla x\|^2$, the Lipschitz constant of $\nabla \Phi(x)$

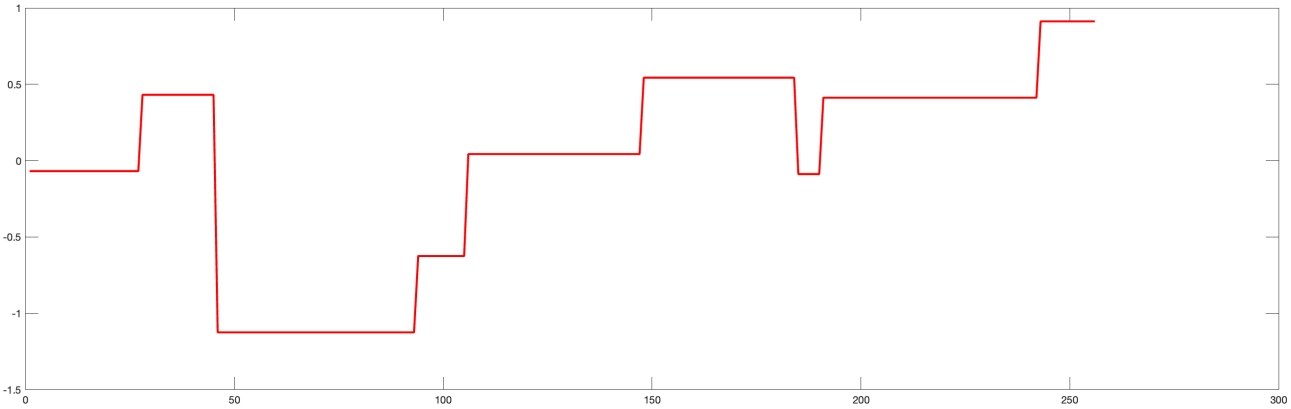
$$8\mu + \|A\|^2$$

3. Observations you are expected to obtain

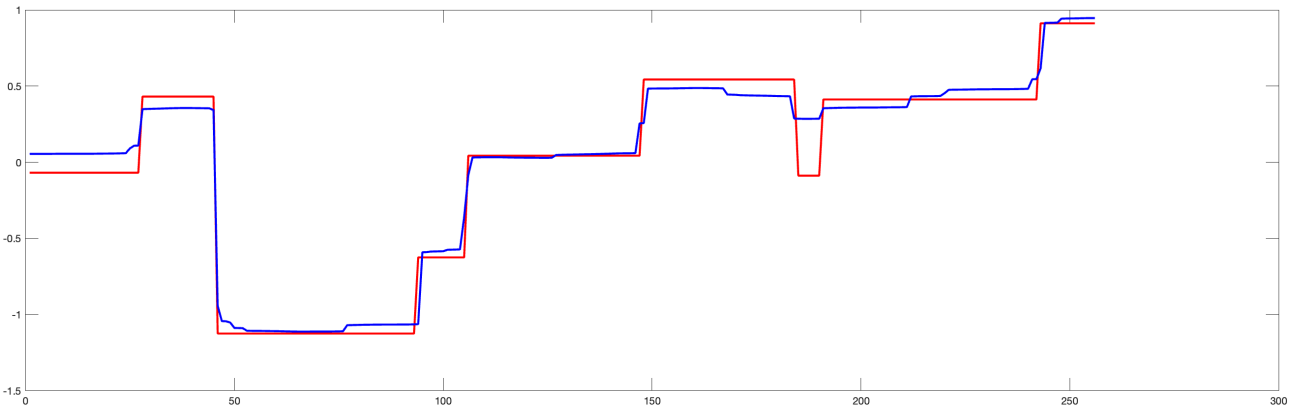
In the numerical experiments, the following quantities should be recorded and shown in the report: reconstructed signal x; the convergence of $\|x_k - x_{k-1}\|$ and $\|\nabla \Phi(x_k)\|$ (you can ignore the function value criterion $\Phi(x_k) - \Phi(x_{k-1})$ mentioned in the project file). But you

can provide other observations if you want. But, in general, the observations are expected to be similar to what I provide below.

3.1 the ground truth signal

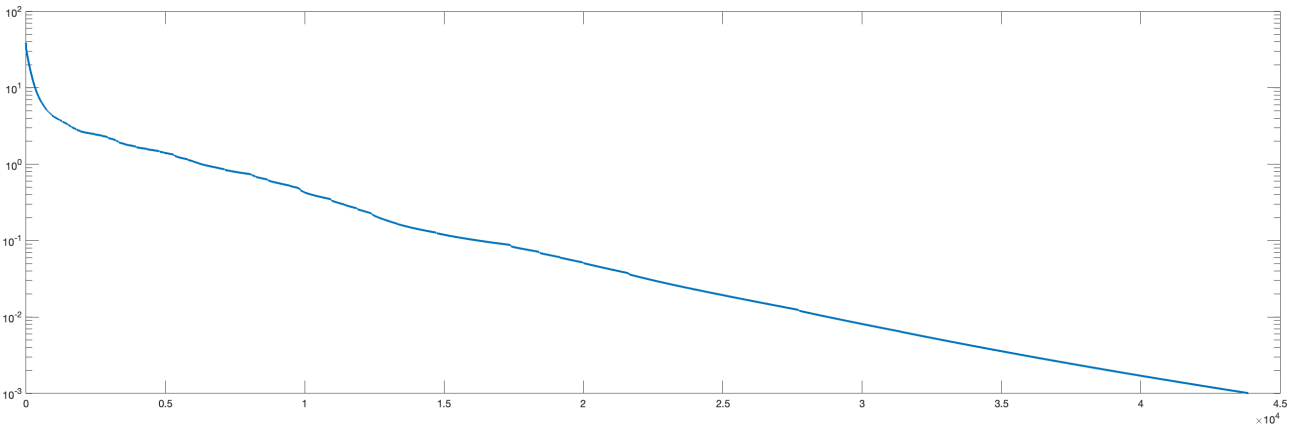


3.2 reconstructed signal via (1.2)



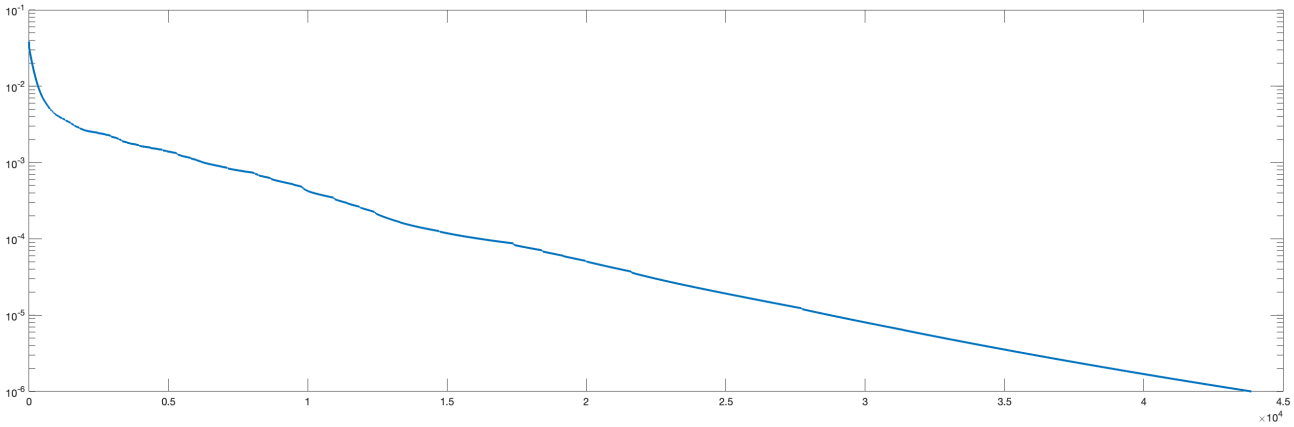
For the above picture, the blue one is the reconstructed signal.

3.3 the convergence of $\|\nabla\Phi(x_k)\|$



Note that the above plot is obtained using “semilogy” in MATLAB.

3.4 the convergence of $\|x_k - x_{k-1}\|$



$\|x_k - x_{k-1}\|$ and $\|\nabla\Phi(x_k)\|$ are actually the same. This is also the reason that I asked you to plot $\Phi(x_k) - \Phi(x_{k-1})$ in the project file.

3.5 Acceleration

In case you want to apply acceleration schemes, then you are expected to obtain the following observation for either $\|x_k - x_{k-1}\|$ or $\|\nabla\Phi(x_k)\|$.