Winter 2024
ECE 102
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Systems and Signals

Homework 1
Due 1 PM Friday, Jan. 19, 2024
Submit your solutions on Gradescope.

Note: Answers without justification will not be awarded any marks.

Problem 1 (4 points)

Assume x(t) is even and y(t) is an odd signal. Are the following signals even, odd or neither? provide a justification.

(i) $5x^2(t) + A$, where A is a constant

(ii) $y^n(t)$, where n is an integer that may be even or odd (hint: take cases)

(iii) $\frac{dx(t)}{dt} - \frac{dy(t)}{dt}$

(iv) $x(3t) + 4y(2t) + 4y(-2t)$

Problem 2 (12 points)

For each of the following signals: (a) Find the fundamental period (if the signal is periodic); (b) Decompose the signal into its odd and even parts; (c) Point (and justify) whether it is even,odd or neither.

(i) $x(t) = e^{j4\pi t}\cos(7\pi t) + e^{j3t}$

(ii) $x(t) = (u(t) - \sin(3t))^2$

(iii) $x(t) = \sin(2\pi t + \theta) + \sin(9\pi t)$

(iv) $x(t) = \sin(4t) + e^{-j3t}$

Problem 3 (6 points)

For x(t) indicated in the figure below, sketch by hand or plot in MATLAB the following:

(a) $x(t) + x(2t) + x(4t)$

(b) $x(t-1) - x(t+1)$

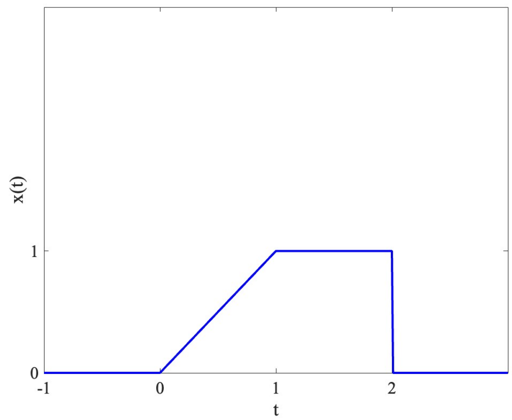

Figure 1: Problem 3

(c) $x^2(-t+5)+2$

Problem 4 (10 points)

Determine if the each of the following signals is a power signal, energy signal or neither.

(i) $\Pi(4t+5)$

(ii) $\sum_{k=-\infty}^{\infty} \Pi(2t+4k)$

(iii) e^{j3t}

(iv) e^{-3t}

(v) $e^{-4|t|+j2t}$

Definition of Unit-pulse function $\Pi(t)$:

$$\Pi(t) = \begin{cases} 1, & \text{if } \ |t| \leq \frac{1}{2} \\ 0, & \text{if } \ |t| > \frac{1}{2} \end{cases}$$

Problem 5 (15 points)

Consider the signal $x(t)$ in Figure 2. Define

$$y(t) = \sum_{n=-\infty}^{\infty} x(t-16n).$$

where n is integer, i.e., $n = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$.

(a) Show that $y(t)$ is periodic and determine its period. We say that $y(t)$ is the periodization of $x(t)$.

(b) Carefully plot the signals $y(t)$ and $z(t) = \frac{d}{dt}y(t)$, the derivative of $y(t)$.

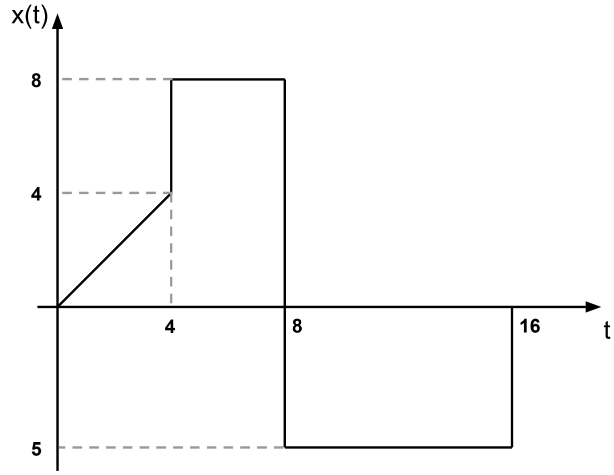

Figure 2: Problem 5

(c) Can you compute

$$q_1 = \int_{-20}^{20} y(t)dt \quad \text{and} \quad q_2 = \int_{-20}^{20} [y(t) + 2]dt$$

For each case, explain your answer.

(d) Is $y(t)$ finite-energy signal? How about $z(t)$ and $x(t)$? Is $y(t)$ finite-power signal?

(e) Define

$$g(t) = \sum_{n=-\infty}^{\infty} y(t - 16n).$$

What can you say about $g(t)$?

Problem 6 (16 points)

MATLAB tasks

For this question, please include all relevant code in text format. For plots, please include axis labels and preferably include a grid.

(a) (5 points) **Task 1**

Plot the waveform

$$x(t) = e^{-t}cos(2\pi t) \tag{1}$$

for $-10 \leq t \leq 10$, with a step size of 0.2.

(b) (5 points) **Task 2**

Create a function **relu(t)** that implements $ReLU$ function:

$$x(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \tag{2}$$

You will need to create a file called "relu.m" containing:

```
function out = relu(t)
out = 0; %replace this line with appropriate implementation
    of the relu function.
end
```

Then plot the function for $-5 \leq t \leq 5$, with a step size of 0.1.

(c) (6 points) **Task 3**

Create functions **even(t, f)** and **odd(t, f)** that take inputs time **t** and function (handle) **f** that compute the respective even and odd parts of **f(t)** at points **t**. For example, the square of a function could be implemented in a file **square.m** as:

```
function out = square(t,f)
out = f(t).^2;
end
```

and run as:

```
t = -10:0.5:10;
y = square(t, @relu);
```

where @*relu* is called a function handle of the function **relu**, and is necessary for passing a function as input to another function.

Running

```
plot(t,y); grid;
```

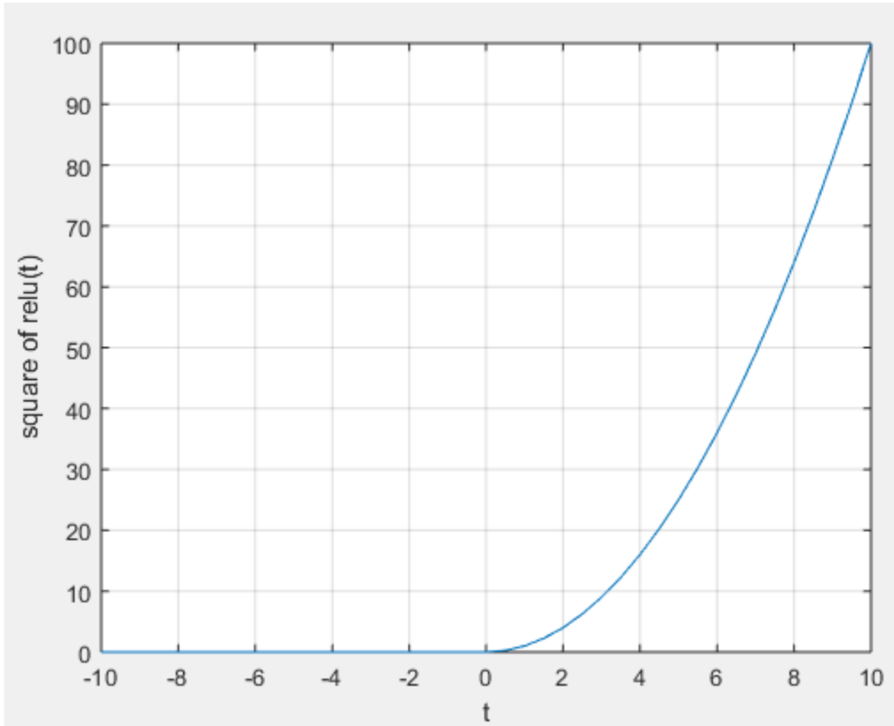

Figure 3: Problem 6

yields the result:

For this question, plot the even and odd components of **relu(t)** for $5 \leq t \leq 5$, with a step size of 0.1 using the functions **even(t, f)** and **odd(t, f)**. Feel free to also define and play around with arbitrary functions to look at their even and odd components.