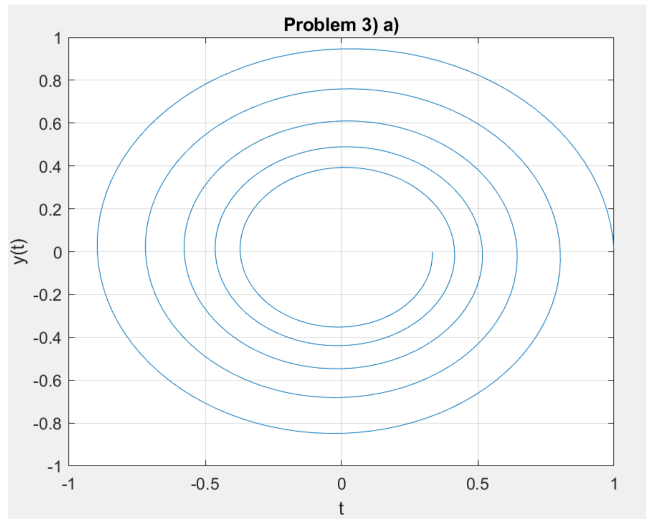
Problem 3

a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

Decay rate: $y(10) = \frac{1}{3}y(0)$, so $e^{10\sigma} = \frac{1}{3}e^{0\sigma} = \frac{1}{3}$. $10\sigma = ln\frac{1}{3} \Rightarrow \sigma \approx - 0.1099$

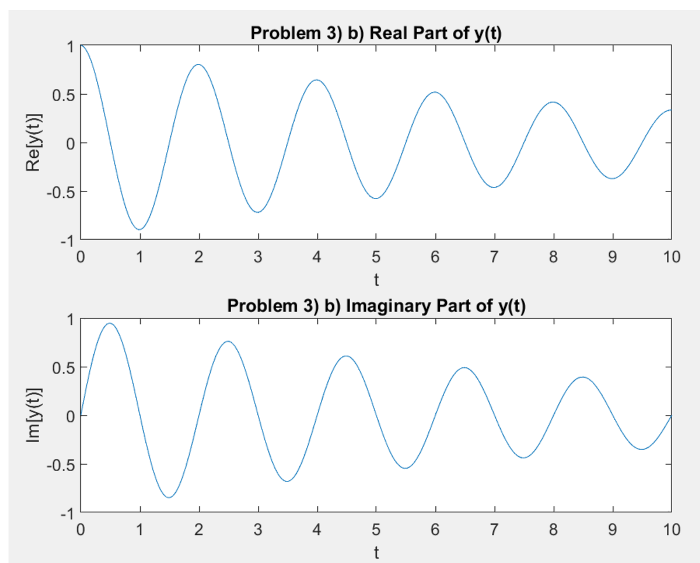
MatLab only graphs the real part of y(t) and ignored the imaginary aspect

```
t = linspace(0, 10, 500);
y = exp((-0.1099 + 1i.*(pi)).*t);
plot(y);
xlabel('t');
ylabel("y(t)");
title("Problem 3) a)");
grid on;
```



b)

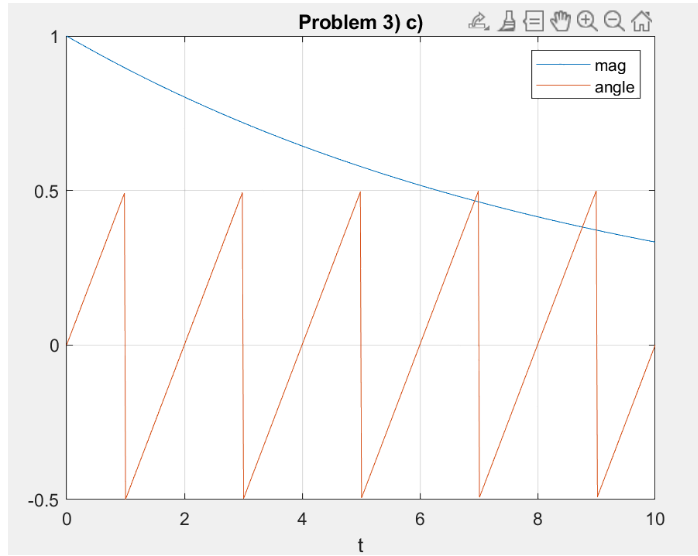
```
t = linspace(0, 10, 500);
y = exp((-0.1099 + 1i.*(pi)).*t);
subplot(211);
plot(t, real(y));
xlabel("t");
ylabel("Re[y(t)]");
title("Problem 3) b) Real Part of y(t)");
subplot(212);
plot(t, imag(y));
xlabel("t");
ylabel("Im[y(t)]");
title("Problem 3) b) Imaginary Part of
y(t)");
```



c)

```
t = linspace(0, 10, 500);
y = exp((-0.1099 + 1i.*(pi)).*t);
```

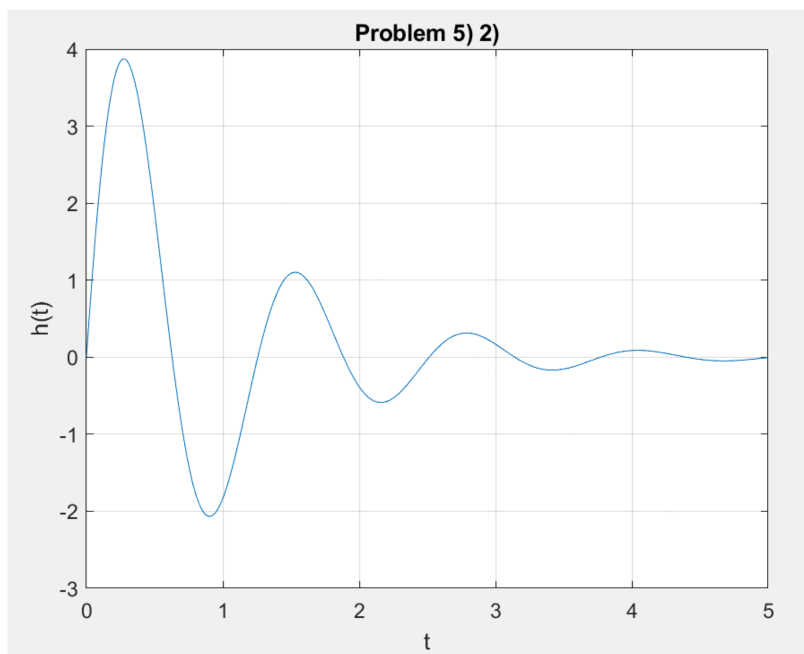
```
plot(t, abs(y), t, angle(y)./(2*pi));
xlabel("t");
title("Problem 3) c)");
legend("mag", "angle");
grid on;
```



Problem 5

2)

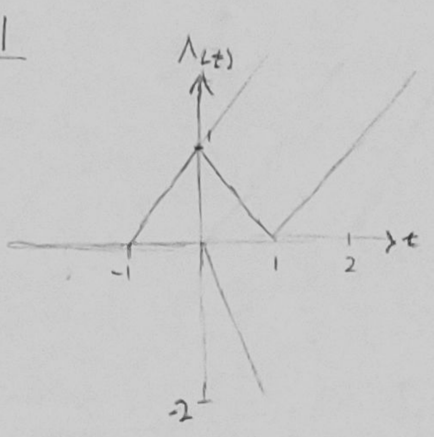
```
t = linspace(0, 5, 500);
ht = 5.2 * exp(-t) .* sin(5 * t);
plot(t, ht);
xlabel("t");
ylabel("h(t)");
title("Problem 5) 2)");
grid on;
```



EE 102 HW2

Problem 1

1)



Difference of 2 ramp functions: $\boxed{r(t+1) - 2r(t) + r(t-1)}$

2) $\Lambda(t) = r(t+1) - 2r(t)$

$$\frac{d\Lambda(t)}{dt} = \frac{d}{dt}(r(t+1)) - 2\frac{d}{dt}r(t) + \frac{d}{dt}r(t-1)$$

$$= \frac{d}{dt}((t+1)(u(t+1))) - 2u(t) + \frac{d}{dt}((t-1)(u(t-1)))$$

$$= u(t+1) + (t+1)\frac{du(t+1)}{dt} - 2u(t) + u(t-1) + (t-1)\frac{du(t-1)}{dt}$$

$$= u(t+1) + \cancel{(t+1)\delta(t+1)} - 2u(t) + u(t-1) + \cancel{(t-1)\delta(t-1)} \; \boxed{= u(t+1) - 2u(t) + u(t-1)}$$

2^{nd} derivative: $\frac{d^2\Lambda(t)}{dt} = \frac{d}{dt}u(t+1) - 2\frac{d}{dt}u(t) - \frac{d}{dt}u(t-1) \; \boxed{= \delta(t+1) - 2\delta(t) + \delta(t-1)}$

Problem 2

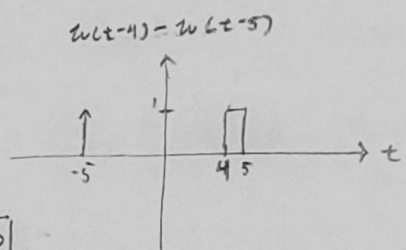
a) $\cos(\frac{\pi}{6}t)\,\delta(t-3) - \int_{-\infty}^{3} \sin(\frac{\pi}{4}\tau)\,\delta(\tau-2)\,d\tau$

$= \cos(\frac{\pi}{2})\,\delta(t-3) - \int_{-\infty}^{3} \sin(\frac{\pi}{2})\,\delta(\tau-2)\,d\tau$

$= \cancel{\cos(\frac{\pi}{2})}\,\delta(t-3) - \sin(\frac{\pi}{2}) \int_{-\infty}^{3} \cancel{\delta(\tau-2)}\,d\tau$

$\boxed{= -1}$

b) $e^{-t^2}[u(t-4) - u(t-5)]\,\delta(t+5)$



$= e^{-t^2}\,\delta(t+5)\,\Pi(t-4.5)$

$= e^{-25}\,\delta(t+5)\,\Pi(t-4.5) = e^{-25}\,\delta(t+5)\,\Pi(\cancel{-9.5}) \; \boxed{= 0}$

c) $\int_{t}^{\infty} e^{-3\tau}\,u(\tau-1)\,d\tau$

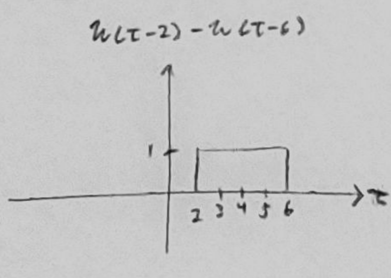
$\frac{dr(t)}{dt} = u(t)$

If t is ≥ 1: $\int_{t}^{\infty} e^{-3\tau}\,d\tau \; \boxed{= \frac{e^{-3t}}{3}}$

$u(\tau-1) = \begin{cases} 1 & \tau \geq 1 \\ 0 & \tau < 1 \end{cases}$

If t is otherwise: $\int_{1}^{\infty} e^{-3\tau}\,d\tau \; \boxed{= \frac{1}{3}e^{-3}}$

d) $\int_{-\infty}^{\infty} \cos(\frac{\pi}{2}\tau)\,(u(\tau-2) - u(\tau-6))\,d\tau$



$= \int_{2}^{6} \cos(\frac{\pi}{2}\tau)\,d\tau = \cos(3\pi) - \cos(\pi)$

$\boxed{= 0}$

Problem 4:

a) $g(t) = \sin(\frac{2\pi m}{r}t) + \cos(\frac{2\pi}{r}t) + \sin(\frac{2\pi}{k}t)$

$T_1 = \frac{r}{m}$ $\quad T_2 = r$ $\quad T_3 = k$

Period: LCM of T_2, and T_3: $\boxed{rk}$

$n_1 T_1 = n_2 T_2 = n_3 T_3$

$\frac{r}{m}$ will always fully divide rk

g(t) is periodic as you can express the ratio of any combination of T_1, T_2, and T_3 as rational #'s.

b) $q(t)$ should be periodic because S is time-invariant and due to the periodicity property $q(t)$ should remain periodic. The period should still be $\boxed{T}$.

$$q(t+T) = S[x(t+T)] = S[x(t)] = q(t)$$

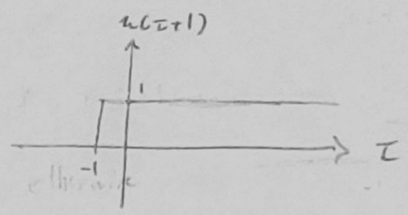
Problem 5:

1)

a) $y(t) = \int_{t-2}^{t+2} x(\tau)\,d\tau \Rightarrow h(t) = \int_{t-2}^{t+2} \delta(\tau)\,d\tau$. If $-2 \le t \le 2$, then $h(t)$ is 1, 0 otherwise.

$$\boxed{h(t) = \Pi\left(\tfrac{t}{4}\right)}$$

b) $y(t) = \int_{t-2}^{t+2} x(\tau)\,d\tau = \int_{t-2}^{t+2} u(\tau+1)\,d\tau$



if $t < -3$

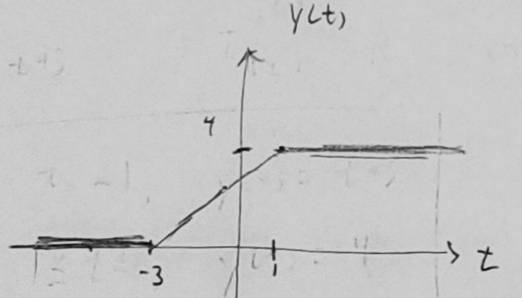
$y(t) = 0$

if $t > 1$

$y(t) = 4$ integrating 1 over an interval of 4

$$r(\tau+1)\Big|_{t-2}^{t+2}$$

$$= r(t+3) - r(t-1)$$



2) $y(t) = -e^{-t}\cos(5t) - 0.2e^{-t}\sin(5t) + 1$

$$h(t) = \frac{dy(t)}{dt} = e^{-t}\cancel{\cos(5t)} + 5e^{-t}\sin(5t) + 0.2e^{-t}\sin(5t) - e^{-t}\cancel{\cos(5t)}$$

$$\boxed{= 5.2e^{-t}\sin(5t)}$$