Winter 2024
ECE 102
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Systems and Signals

Homework 2
Due 1 PM Friday, Jan. 26, 2024
Submit your solutions on Gradescope.

Note: Answers without justification will not be awarded any marks.

Problem 1 (4 points)

1. Express the triangle $\Lambda(t)$ in Fig. 1 as a function of $r(t)$ signals (appropriately shifted and weighted)
2. Calculate the first and second derivative of $\Lambda(t)$.

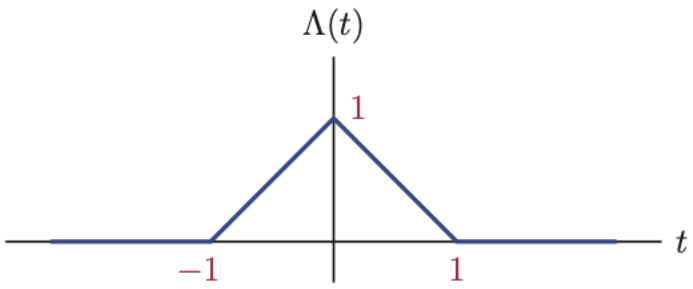

Figure 1: Problem 1

Problem 2 (8 points)
Simplify the following expressions as much as possible. Provide an explanation or intermediate steps.

(a) $\cos\left(\frac{\pi}{6}t\right)\delta(t-3) - \int_{-\infty}^{3} \sin\left(\frac{\pi}{4}\tau\right)\delta(\tau-2)d\tau$

(b) $e^{-t^2}[u(t-4)-u(t-5)]\delta(t+5)$

(c) $\int_{t}^{\infty} e^{-3\tau}u(\tau-1)d\tau$

(d) $\int_{-\infty}^{\infty} \cos\left(\frac{\pi}{2}\tau\right)(u(\tau-2)-u(\tau-6))d\tau$

Problem 3 (12 points) [MATLAB]

(a) (4 points) Task 1
A complex sinusoid is denoted:

$$y(t) = e^{(\sigma + j\omega)t}$$

First compute a vector representing time from 0 to 10 seconds in about 500 steps (You can use *linspace*). Use this vector to compute a complex sinusoid with a period of 2 seconds, and a decay rate that reduces the signal level at 10 seconds to 1/3 its original value. What σ and ω did you choose? If your complex exponential is y, plot:

```
>> plot(y);
```

What is MATLAB doing here?

(b) (4 points) Task 2
Use the **real()** and **imag()** MATLAB functions to extract the real and imaginary parts of the complex exponential, and plot them as a function of time (plot them separately, you can use subplot for this task). This should look more reasonable. Label your axes, and check that your signal has the required period and decay rate.

(c) (4 points) Task 3
Use the **abs()** and **angle()** functions to plot the magnitude and phase angle of the complex exponential (plot them in the same figure). Scale the **angle()** plot by dividing it by 2π so that it fits well on the same plot as the **abs()** plot (i.e. plot the angle in cycles, instead of radians, the function **angle(x)** returns the angle in radians).

Problem 4 (4 points)

(a) Let $g(t) = \sin\left(\frac{2\pi m}{r}t\right) + \cos\left(\frac{2\pi}{r}t\right) + \sin\left(\frac{2\pi}{k}t\right)$ where m, r and k are positive integers. Is $g(t)$ periodic? If so, what is its period?

(b) Consider a time-invariant system $\mathcal{S}$. Let $q(t)$ be the output of $\mathcal{S}$ when the input is a periodic signal $x(t)$ with period T, i.e., $q(t) = \mathcal{S}[x(t)]$. Is $q(t)$ periodic? If so, what is its period?

Problem 5 (6 points)

1. An LTI system is defined by the following input-output relation:

$$y(t) = \int_{t-2}^{t+2} x(\tau)d\tau$$

   (a) (2 Points) Determine the impulse response of this system.

   (b) (2 Points) Calculate the output of the system when the input is $x(t) = u(t+1)$ and plot your answer.

2. (2 Points) The unit-step response of an LTI system is $y(t) = -e^{-t}\cos(5t) - (0.2)e^{-t}\sin(5t) + 1$ for $t \geq 0$. Calculate the impulse response $h(t)$ for this system. Using MATLAB plot impulse response $h(t)$ on time interval from $t = 0$ to $t = 5$ (CTLTI: continuous-time linear and time-invariant systems).