Winter 2024
ECE 102
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Systems and Signals

Homework 4
Due 1 PM Friday, Feb 16, 2024
Submit your solutions on Gradescope.

Note: Answers without justification will not be awarded any marks.

Problem 1 (3 points)

Derive the expression for the exponential Fourier Series representation for the following periodic signals:

(a) $x_1(t) = \sin(2\pi 4t) + \cos(2\pi 5t)$.

(b) $x_3(t) = \sum_{k=-\infty}^{\infty} f(t-k)$, where

$$f(t) = \begin{cases} |t| & -0.25 < t < 0.25 \\ 0 & \text{otherwise.} \end{cases}$$

Hint: You may need to use the Euler's equation and the formula for the following integral

$$\int_{t_1}^{t_2} te^{at}dt = [\frac{t}{a}e^{at} - \frac{1}{a^2}e^{at}]\Big|_{t_1}^{t_2}.$$

Problem 2 (4 points)

Take function $x_3(t)$ from Problem 1. Plot the function using MATLAB in the time interval $-10 \leq t \leq 10$ using time increments of 0.01. Plot the power spectrum of the Fourier Series representation of $x_3(t)$ using only the components with $|k| \leq 5, 10$ and 20, respectively. Comment on the plotted signals in each case, in comparison to the plotted $x_3(t)$.

Problem 3 (6 points)

Let $x_f(t)$ be a periodic signal with period $T = 1$, where

$$x_f(t) = \sin(\pi t), \quad 0 \leq t \leq 1.$$

(a) Compute the Fourier Series coefficients of $x_f(t)$ (first sketch $x_f(t)$ to understand what kind of signals you have).

(b) Using result from part (a), deduce the Fourier Series coefficients of the following signals:

  (i) $y_1(t) = x_f(-t)$

(ii) $y_2(t) = 2x_f(t-1) + 3$

(iii) $y_3(t) = 4x_f(t - 1/2)$

(iv) $y_4(t)$ is given as a periodic signal with period $T_y = 2$ such that

$$y_4(t) = \begin{cases} \sin(\pi t), & 0 \leq t \leq 1 \\ 0, & 1 < t \leq 2. \end{cases}$$

Problem 4 (8 points)

(a) Find the Fourier series coefficients for each of the following periodic signals:

i. $f(t) = sin(5\pi t) + \frac{1}{2}cos(4\pi t)$

ii. $f(t)$ is a periodic signal with period $T = 1\ s$, where one period of the signal is defined as e^{-t} for $0 < t < 1\ s$, as shown below

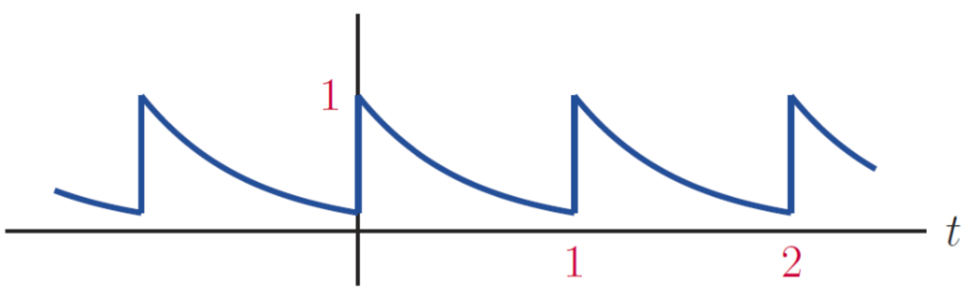

Figure 1: Problem 4 (a) ii.

(b) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

i. If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = x(t) - y(t)$ in terms of x_k and y_k.

ii. If $T_1 = \frac{1}{2}T_2$, express the Fourier series coefficients of $w(t) = x(t) - y(t)$ in terms of x_k and y_k.

Problem 5 (8 points) **Fourier series of transformation of signals**

Suppose that $f(t)$ is a periodic signal with period T_0, with the following Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Determine the period of each of the following signals, then express its Fourier series in terms of c_k:

(a) $g(t) = 2f(t)$

(b) $g(t) = f(-2t)$

(c) $g(t) = f(t - t_0)$

(d) $g(t) = f(t/a)$, where a is positive real number.

Problem 6 (5 points) Determine the trigonometric Fourier series (TFS) coefficients for the periodic signal in Figure 2.

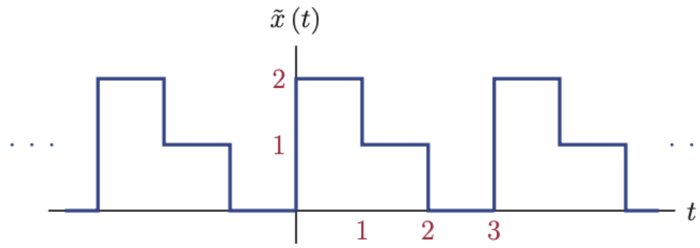

Figure 2: Problem 6