Winter 2024
ECE 102
Christina Fragouli
TA: Semira Galijasevic

Systems and Signals

Homework 3
Due 1 PM Friday, Feb 2, 2024
Submit your solutions on Gradescope.

Note: Answers without justification will not be awarded any marks.

Problem 1 (8 points)

Consider the following linear time-invariant (LTI) system described by the following input/output relation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau. \tag{1}$$

(a) Find the impulse response $h(t)$.

(b) Find the response of the system when the input $x(t) = u(t+1) - u(t-2)$.

Problem 2 (12 points)

Consider the following two signals:

- $x(t) = e^{4t} u(-t)$,
- $h(t) = u(t) - u(t-2) + u(t-4) - u(t-6)$.

(a) Compute the signal $y(t)$ as the convolution of $x(t)$ and $h(t)$.

(b) Plot signals $x(t), h(t)$ and $y(t)$ in the time interval $-10 \leq t \leq 10$ at the increments of 0.01.

(c) Plot $x(t_0 - t)$ and $h(t)$ in the same figure for $t_0 = -1, 0, 1, 2$ and explain the region $y(t) = 0$ through the figure.

Problem 3 (16 points)

A system with input signal $x(t)$ and output signal $y(t)$ can be: (1) Linear; (2) Time invariant; (3) Causal; (4) Stable. Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answer.

(a) $y(t) = \frac{1}{(x(t-1))^2}$

(b) $y(t) = \tan(t) x(t+1)$

(c) $y(t) = x(t^4 - 10)$

(d) $y(t) = 1 + e^{x(t)}$

Problem 4 (8 points)

A number of systems are specified below in terms of their input-output relationships. For each of the following systems, determine if the system is (1) linear (2) time-invariant.

(a) $y(t) = |x(t)| + x(t)$

(b) $\int_{-\infty}^{t} x(\lambda)d\lambda$

(c) $(t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

(d) $y(t) = \frac{1}{1+x^2(t)}$

Problem 5 (12 points)

Impulse response and LTI systems Consider the following three LTI systems:

- The first system S_1 is given by its input-output relationship: $y(t) = \int_{-\infty}^{t} e^{-3(t-\tau)}x(\tau)d\tau$;
- The second system S_2 is given by its input-output relationship: $y(t) = \int_{-\infty}^{t-2} x(\tau)d\tau$;
- The third system S_3 is given by its impulse response: $h_3(t) = \delta(t-3)$

(a) (4 points) Compute the impulse response $h_1(t)$ of system S_1

(b) (2 points) Let's define the output signal $w(t)$ of the cascade of three systems as follows:

$$w(t) = S_1[x(t)] - S_3[S_2[x(t)]]$$

Draw a block diagram representation of the above cascade transforming $x(t)$ to $w(t)$.

(c) (2 points) Determine the impulse response $h_{eq}(t)$ of the equivalent system.

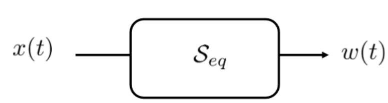

Figure 1: Problem 5 (c)

(d) (4 points) Determine the response of the overall system to the input $x(t) = \delta(t) + 2\delta(t-3)$.

Problem 6 (17 points) **MATLAB**

To complete the following MATLAB tasks, we will provide you with a MATLAB function **(nconv())**, which numerically evaluates the convolution of two continuous-time functions. Make sure to download it from BruinLearn and save it in your working directory in order to use it.
The function syntax is as follows:

```
[y, ty] = nconv(x,tx,h,th)
```

where the inputs are:
x : input signal vector
tx: times over which x is defined **h** : impulse response vector
th: times over which h is defined
and the outputs are: **y** : output signal vector
ty: times over which y is defined.
The function is implemented with the MATLAB's **conv()** function. You are encouraged to look at the implementation of the function provided (the explanations are included as comments in the code).

(a) (7 points) **Task 1**
Use the **nconv()** function to perform convolution $f(t) * g(t)$ where:

$$f(t) = 2\ rect\left(t - \frac{3}{2}\right).$$
$$g(t) = 2\ r(t-1)\ rect\left(t - \frac{3}{2}\right)$$

Plot the output for each problem (you can consider either function to be the input). Properly label the axes of the plots. Make sure to use the same step size for **tx** and **th**.

(b) (5 points) **Task 2**
Using the **nconv()** function, perform the convolution of two unit rect functions: $rect(t) * rect(t)$. Plot and label the result.

(c) (5 points) **Task 3**
Using the result of Task 2 and the same MATLAB function, calculate $y(t) = rect(t) * rect(t) * rect(t)$. Plot and label the result.

(d) (Optional) **Task 4**
Now, what happens if we consider $rect(t) * rect(t) * \cdots * rect(t) = rect^{(N)}(t)$? Using for loop, calculate the result of convolving N rect(t) functions together. Plot and label the results (use N = 100).