EE 102 HW1

1.

i) $\boxed{\text{Even}}$, because if you square $x(t)$, since $x(t)$ is an even signal we are guaranteed $x^2(t)$ to be even as $x^2(t) = x^2(-t)$. On the other hand $x^2(t)$ can never be negative so it isn't odd

Even : $x(t) = x(-t)$
odd : $x(t) = -x(t)$

ii) $\boxed{\text{Neither}}$. If n is even, then $y^n(t) = y^n(-t)$ as the negatives will cancel. If n is odd then $y^n(-t) = -y^n(t)$ as $y(t)$ is odd.

iii) $\boxed{\text{Neither}}$. Deriving an even signal gets you an odd signal and vice versa. $\frac{dx}{dt} - \frac{dy}{dt} \Rightarrow$ odd - even = is neither unless one of $x(t)$ or $y(t)$ is zero.

iv) $\boxed{\text{Even}}$, If $y(t)$ is odd, then $4y(2t)$ cancels $4y(-2t)$ leaving us with $x(3t)$, an even signal

2.

i)

a) $x(t) = e^{j4\pi t}\cos(7\pi t) + e^{j3t}$

$= e^{j4\pi t}\frac{1}{2}\left(e^{j7\pi t} + e^{-j7\pi t}\right) + e^{j3t}$

$x(t) = \frac{1}{2}\left(e^{j11\pi t} + e^{-3j\pi t}\right) + e^{j3t}$

$T_1 = \frac{2\pi}{11\pi} = \frac{2}{11}$ $T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$ $T_3 = \frac{2\pi}{3}$

$\frac{T_1}{T_2} = \frac{\frac{2}{11}}{\frac{2}{3}} = \frac{3}{11}$ rational $\frac{T_1}{T_3} = \frac{\frac{2}{11}}{\frac{2\pi}{3}}$ irrational

$\boxed{\text{NOT PERIODIC}}$

$\cos(7\pi t) = \frac{e^{j7\pi t} + e^{-j7\pi t}}{2}$

because
$e^{j7\pi t} = \cos(7\pi t) + i\sin(7\pi t)$
$e^{-j7\pi t} = \cos(7\pi t) - i\sin(7\pi t)$

b) $x(-t) = e^{-j4\pi t}\cos(-7\pi t) + e^{-j3t} = e^{-j4\pi t}\cos(7\pi t) + e^{-j3t}$

$x_e = \frac{x(t) + x(-t)}{2} = \boxed{\frac{1}{2}\left(e^{j4\pi t}\cos(7\pi t) + e^{j3t} + e^{-j4\pi t}\cos(7\pi t) + e^{-j3t}\right)}$

$x_o = \frac{x(t) - x(-t)}{2} = \boxed{\frac{1}{2}\left(e^{j4\pi t}\cos(7\pi t) + e^{j3t} - e^{-j4\pi t}\cos(7\pi t) - e^{-j3t}\right)}$

c) $x(t) = \frac{1}{2}\left(e^{j11\pi t} + e^{-3j\pi t}\right) + e^{j3t}$

$x(-t) = \frac{1}{2}\left(e^{-j11\pi t} + e^{3j\pi t}\right) + e^{-j3t}$

$-x(t) = -\frac{1}{2}\left(e^{j11\pi t} + e^{-3j\pi t}\right) - e^{j3t}$ $\boxed{\text{Neither}}$

$x(t) \neq x(-t)$ and $x(-t) \neq -x(t)$

ii)

a) $x(t) = (u(t) - \sin(3t))^2$

$= u^2(t) - 2\sin(3t)u(t) + \sin^2(3t)$

$= u^2(t) - 2\sin(3t)u(t) + \frac{1-\cos(6t)}{2}$

$\cos(2\theta) = 1 - 2\sin^2(\theta)$

$\boxed{\text{Non-periodic}}$ $u^2(t)$ is only positive/single sided while $\frac{1-\cos(6t)}{2}$ is double-sided and if the domain is all real #'s single + double sided function isn't periodic.

c) $-x(t) = -\sin(4t) - \cos(3t) + i\sin(3t)$
$x(t) \neq x(-t)$ $-x(t) \neq x(-t)$
$\boxed{\text{neither}}$

b. $x_e(t) = \frac{1}{2}\left(u^2(t) - 2\sin(3t)u(t) + \frac{1-\cos(6t)}{2} + u^2(-t) + 2\sin(3t)u(-t) + \frac{1-\cos(6t)}{2}\right)$

$\boxed{= \frac{u^2(t) + u^2(-t)}{2} + \frac{1-\cos(6t)}{2} - \sin(3t)(u(t) - u(-t))}$

$x_o(t) = \frac{1}{2}\left(u^2(t) - 2\sin(3t)u(t) + \frac{1-\cos(6t)}{2} - u^2(-t) - 2\sin(3t)u(-t) - \frac{1-\cos(6t)}{2}\right)$

$\boxed{= \frac{u^2 - u^2(-t)}{2} - 2\sin(3t)(u(t) + u(-t))}$

c. $x(-t) = u^2(-t) + 2\sin(3t)u(-t) + \frac{1-\cos(6t)}{2}$

$-x(t) = -u^2(t) + 2\sin(3t)u(t) - \frac{1-\cos(6t)}{2}$

$x(-t) \neq x(t)$ $-x(t) \neq x(-t)$

$\boxed{\text{Neither}}$ Also neither x_e or x_o is equal to $x(t)$

iii)

a) $x(t) = \sin(2\pi t + \theta) + \sin(9\pi t)$

$T_1 = \frac{2\pi}{2\pi} = 1$ $T_2 = \frac{2\pi}{9\pi} = \frac{2}{9}$

$\frac{T_1}{T_2} = \frac{9}{2}$ rational

$\boxed{\text{fundamental}: 2}$ since T_1 and T_2's LCM is 2

b) $x(-t) = \sin(-2\pi t + \theta) - \sin(9\pi t)$

$x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$= \frac{1}{2}(\sin(2\pi t + \theta) + \sin(9\pi t) + \sin(-2\pi t + \theta) - \sin(9\pi t))$

$\boxed{= \frac{1}{2}(\sin(2\pi t + \theta) + \sin(-2\pi t + \theta))}$

$x_o(t) = \frac{1}{2}(x(t) - x(-t))$

$= \frac{1}{2}(\sin(2\pi t + \theta) + \sin(9\pi t) - \sin(-2\pi t + \theta) + \sin(9\pi t))$

$\boxed{= \frac{1}{2}(\sin(2\pi t + \theta) - \sin(-2\pi t + \theta) + 2\sin(9\pi t))}$

c) $\boxed{\text{Neither}}$ since neither $x_e(t)$ or $x_o(t)$ is 0.

iv)

a) $x(t) = \sin(4t) + e^{-j3t} = \sin(4t) + \cos(3t) - i\sin(3t)$

$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$ $T_2 = \frac{2\pi}{3}$

$\frac{T_1}{T_2} = \frac{\frac{\pi}{2}}{\frac{2\pi}{3}} = \frac{3}{4}$ rational

$\boxed{\text{fundamental}: 2\pi}$ since 2π is LCM of T_1 & T_2

b) $x(-t) = -\sin(4t) + e^{j3t} = -\sin(4t) + \cos(3t) + i\sin(3t)$

$x_e(t) = \frac{\sin(4t) + e^{-j3t} - \sin(4t) + e^{j3t}}{2}$ $\boxed{= \cos(3t)}$

$x_o(t) = \frac{\sin(4t) + e^{-j3t} + \sin(4t) - e^{j3t}}{2}$

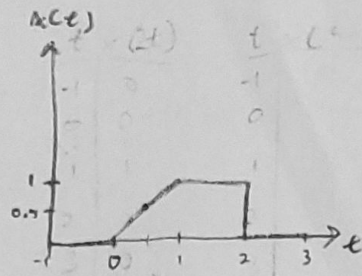
$= \frac{1}{2}(2\sin(4t) + \cos(3t) - i\sin(3t) - \cos(3t) - i\sin(3t))$

$\boxed{= \sin(4t) - i\sin(3t)}$

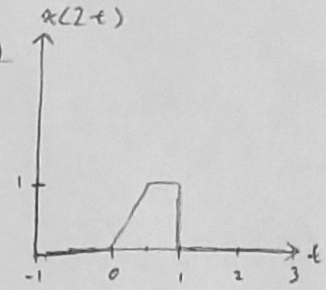
3)

a)

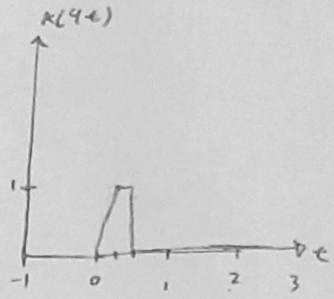
t	x(t)
-1	0
0	0
1	1
2	1
3	0



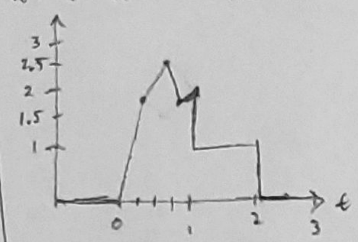
t	x(2t)
-1	0
0	0
0.5	1
1	1
2	0
3	0



t	x(4t)
-1	0
0	0
0.25	1
0.5	1
1	0
2	0
3	0



$x(t)+x(2t)+x(4t)$



t	sum
0	0
0.25	0.25+0.5+1 = 1.75
0.5	0.5+1+1 = 2.5
0.75	0.75+1 = 1.75
1	2
2	1
3	0

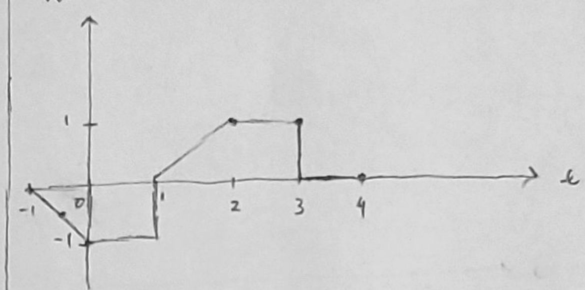
b)

t	x(t-1)
-1	0
0	0
1	0
2	1
3	1
4	0

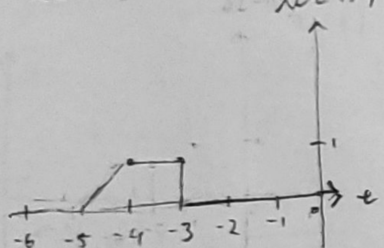
t	x(t+1)
-1	0
0	1
1	1
2	0
3	0

t	subtract
-1	0
-0.5	-0.5
0	-1
1	-1
2	1
3	1
4	0

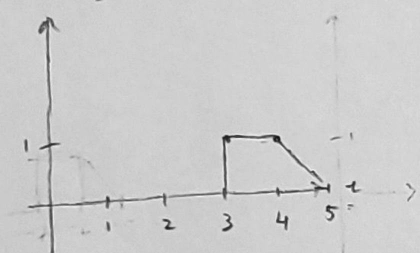
$x(t-1)-x(t+1)$



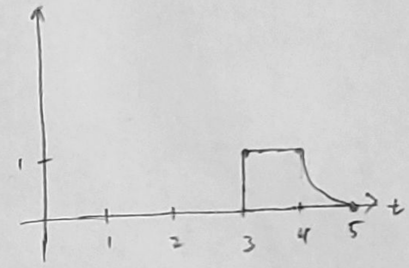
c) $x(t+5)$



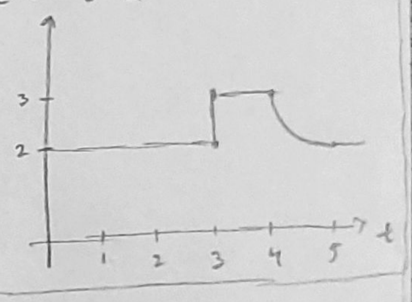
$x(-t+5)$



$x^2(-t+5)$



$x^2(-t+5)+2$



4)

i) Time dilation & time shift doesn't change that $\Pi(4t+5)$ is still a finite signal, $\boxed{\text{energy signal}}$

ii) $\boxed{\text{Power signal}}$, the infinite sigma makes energy infinite, and it has a finite avg power.

iii) e^{-3t} $\boxed{\text{Neither}}$

$$E_x = \int_{-\infty}^{\infty} |e^{-3t}|^2 dt = \underbrace{\int_{-\infty}^{0} e^{-6t} dt}_{\text{infinite}} + \underbrace{\int_{0}^{\infty} e^{-6t} dt}_{\text{finite}}$$

Not energy

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{-3t}|^2 dt = \text{"} \int_{-T}^{T} e^{-6t} = \lim_{T\to\infty} \frac{1}{2T} \left(\frac{e^{-6t}}{-6}\right)\Big|_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{-e^{-6T} + e^{6T}}{6}\right) = \infty$$

L'hopital's to ∞

Not power

Same

iv) e^{j3t} is periodic w/ $T = \frac{2\pi}{3}$. Energy is ∞ while avg power is finite,

$\boxed{\text{power signal}}$

v) $e^{-4|t|+j2t} = e^{-4|t|} e^{j2t}$ $\boxed{\text{Energy signal}}$

$$E_x = \int_{-\infty}^{\infty} |e^{-4|t|} \cancel{e^{j2t}}|^2 dt = \int_{-\infty}^{\infty} e^{-8|t|} dt = \int_{-\infty}^{0} e^{8t} dt + \int_{0}^{\infty} e^{-8t} dt = \frac{1}{8} e^{8t}\Big|_{-\infty}^{0} - \frac{1}{8} e^{-8t}\Big|_{0}^{\infty}$$

1 power signal, modulus is 1

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

finite, so

$\boxed{\text{Energy Signal}}$

5)

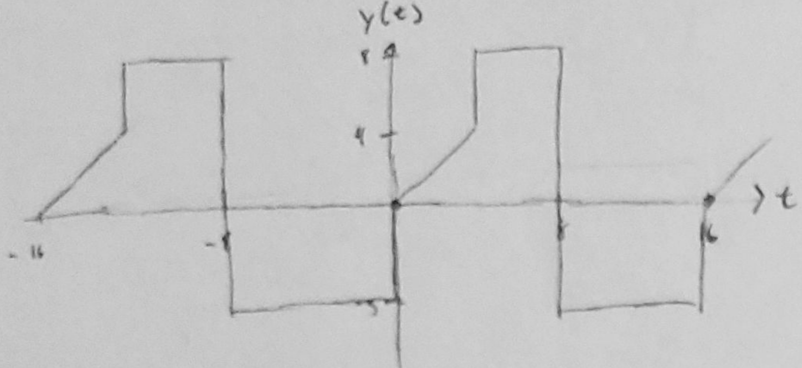
a) $y(t) = \sum_{n=-\infty}^{\infty} x(t-16n)$

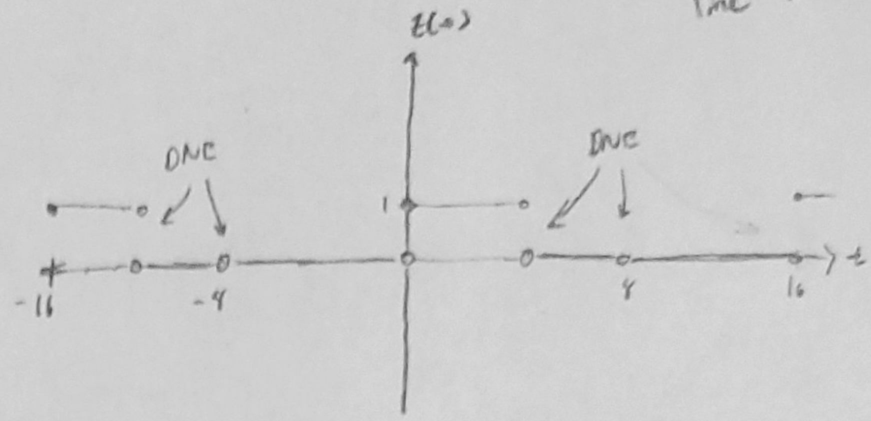
periodic as every multiple of 16 the graph repeats. Period is $\boxed{16}$

$x(t) = x(t-16) = x(t-32) \cdots$

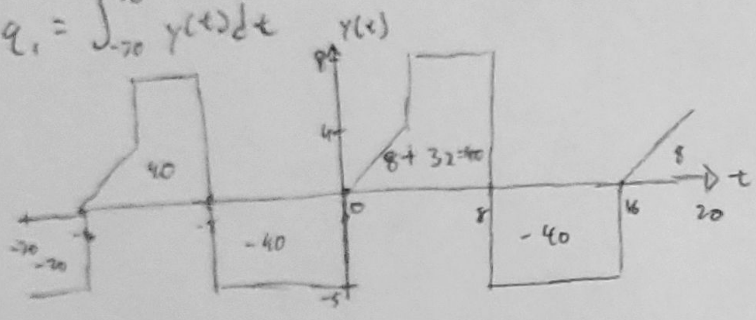
Derivative of vertical line DNE

b)





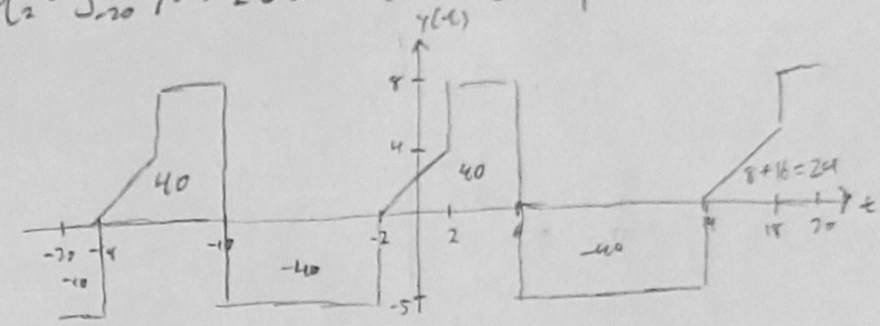
c) $q_1 = \int_{-20}^{20} y(t)\,dt$



$40+40+8-20-40-40$

$q_1 \boxed{= -12}$

$q_2 = \int_{-20}^{20} y(t+2)\,dt$ shift left by 2



$40+40+24-10-40-40 = \boxed{14 = q_2}$

Just add sums of the area under the graph

d) y(t) is an infinite energy signal as the absolute values of the sum under each period's graph is nonzero and y(t) iterates to infinity. z(t) is also non-finite as it has a sum under each period and it also iterates to ∞. Only x(t) is finite as it only has one period's graph/iteration. However, y(t) is a finite power signal as it has a finite average power throughout all the periods.

e)

$$g(t) = \sum_{n=-\infty}^{\infty} y(t-16n) = \sum_{n=-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty} x((t-16n)-16n)\right) = \sum_{n=-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty} x(t-32n)\right)$$

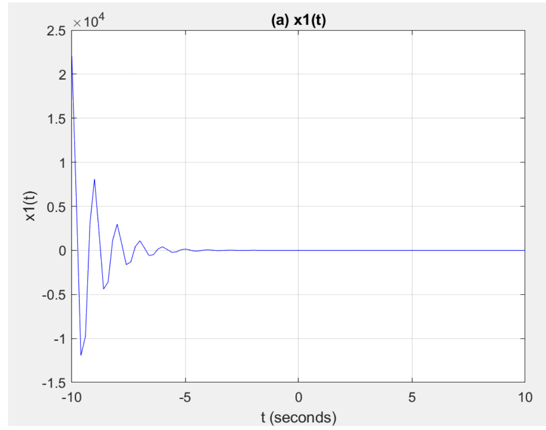
$\boxed{\text{g(t) has double y(t)'s period as g(t) has } x(t-32n)}$

Problem 6

a) Code and graph:

Code in main.m:

```
t = -10:0.2:10;
x1 = @(t) exp(-t) .* cos(2*pi*t);
plot(t, x1(t), 'b');
xlabel("t (seconds)");
ylabel("x1(t)");
title("(a) x1(t)");
grid on;
```



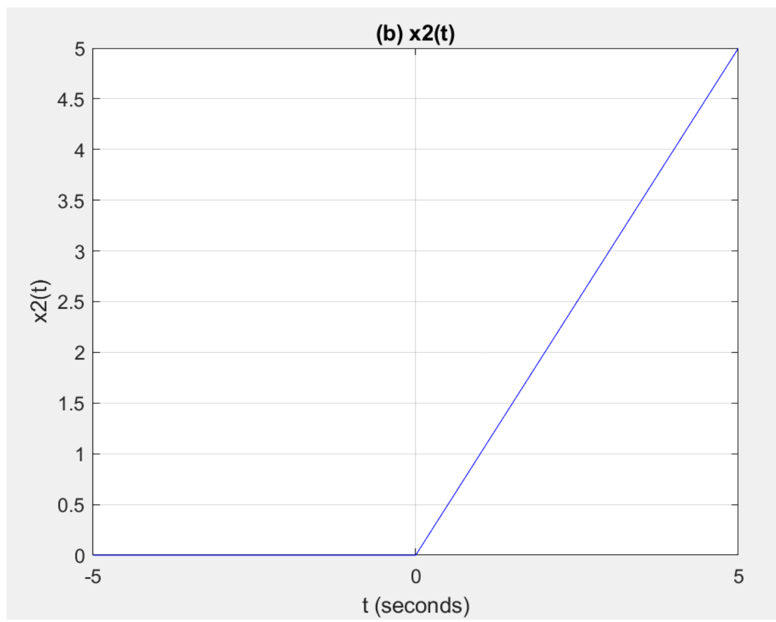
b) Code and graph:

Code in main.m:

```
t = -5:0.1:5;
x1 = arrayfun(@relu, t);
x2 = @(t) x1;
plot(t, x2(t), 'b');
xlabel("t (seconds)");
ylabel("x2(t)");
title("(b) x2(t)");
grid on;
```

Code in relu.m:

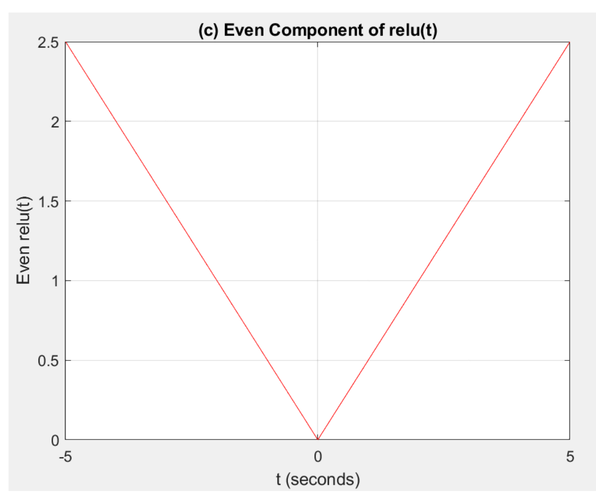
```
function out = relu(t)
   if t < 0
       %fprintf("t: %.2f \n", t);
       out = 0;
   else
       %fprintf("t: %.2f \n", t);
       out = t;
   end
end
```



c) Code and graph:

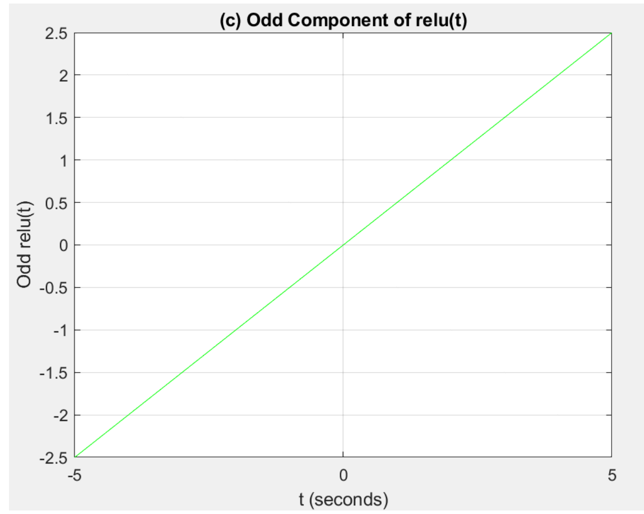
Code in even.m:

```
function output = even(t, f)
   return_array = zeros(size(t));

   for i = 1:length(t)
       return_array(i) = f(t(i)) + f(-t(i));
   end
   output = return_array * 0.5;
end
```



Code in `odd.m`:

```
function output = odd(t, f)
   return_array = zeros(size(t));

   for i = 1:length(t)
       return_array(i) = f(t(i)) - f(-t(i));
   end
   output = return_array * 0.5;
end
```



Code in `main.m`:

```
t = -5:0.1:5;
xEven = even(t, @relu);
xOdd = odd(t, @relu);
figure(1);
plot(t, xOdd, 'g');
xlabel("t (seconds)");
ylabel("Odd relu(t)");
title("(c) Odd Component of relu(t)");
grid on;
figure(2);
plot(t, xEven, 'r');
xlabel("t (seconds)");
ylabel("Even relu(t)");
title("(c) Even Component of relu(t)");
grid on;
```