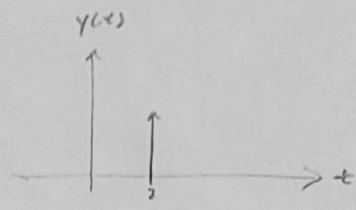
EE 102 HW3

1)

a)

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$



replace x w/ δ : $h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau$

$$= \int_{-\infty}^{t} e^{-(t-2)} \delta(\tau - 2)\, d\tau$$

$$\boxed{h(t) = u(t-2)e^{-(t-2)}} \leftarrow \boxed{= \begin{cases} e^{-(t-2)} & \text{if } t \geq 2 \\ 0 & \text{otherwise} \end{cases}}$$

b) $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

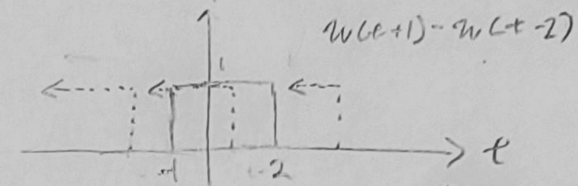
$$= \int_{-\infty}^{\infty} u(t-\tau-2) e^{-(t-\tau-2)} [u(\tau+1) - u(\tau-2)]\, d\tau$$

$x(t) = u(t+1) - u(t-2)$

$h(t) = u(t-2)e^{-(t-2)}$

$u(-\tau+t-2)$ shift

bc tau is negative, this goes backwards



dotted lines are possible values/cases for $u(-\tau+t-2)$

Case 1: $t \leq 1$

$t-2 \leq -1$

$t \leq 1$

$$\int_{-\infty}^{-1} = 0$$

Case 2: $1 < t \leq 4$

$-1 < t-2 \leq 2$

$1 < t \leq 4$

$$\int_{-1}^{t-2} e^{-(t-\tau-2)}\, d\tau = 1 - e^{-t+1}$$

Case 3: $t > 4$

$t-2 > 2$

$t > 4$

$$\int_{-1}^{2} e^{-(t-\tau-2)}\, d\tau = e^{-t+4} - e^{-t+1}$$

$$\boxed{y(t) = \begin{cases} 0 & \text{if } t \leq 1 \\ 1 - e^{-t+1} & \text{if } 1 < t \leq 4 \\ e^{-t+4} - e^{-t+1} & \text{if } 4 < t \end{cases}}$$

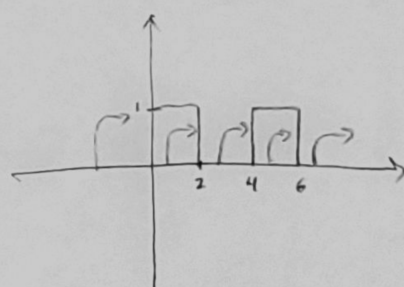
2)

a) $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} e^{4(t-\tau)} u(\tau - t) [u(t) - u(t-2) + u(t-4) - u(t-6)]$$

$u(\tau - t)$ shift to left

since τ is positive, unit step goes forwards



Case 1: $t \leq 0$

$$\int_0^2 e^{4(t-\tau)}\, d\tau + \int_4^6 e^{4(t-\tau)}\, d\tau$$

Case 2: $0 < t \leq 2$

$$\int_t^2 e^{4(t-\tau)}\, d\tau + \int_4^6 e^{4(t-\tau)}\, d\tau$$

Case 3: $2 < t \leq 4$

$$\int_4^6 e^{4(t-\tau)}\, d\tau$$

Case 4: $4 < t \leq 6$

$$\int_t^6 e^{4(t-\tau)}\, d\tau$$

Case 5: $6 < t$

$$\int = 0$$

$= -\frac{1}{4}e^{4(t-2)} + \frac{1}{4}e^{4t} - \frac{1}{4}e^{4(t-6)} + \frac{1}{4}e^{4(t-4)}$	$= -\frac{1}{4}e^{4(t-2)} + \frac{1}{4} - \frac{1}{4}e^{4(t-6)} + \frac{1}{4}e^{4(t-4)}$	$= -\frac{1}{4}e^{4(t-6)} + \frac{1}{4}e^{4(t-4)}$	$= -\frac{1}{4}e^{4(t-6)} + \frac{1}{4}$	$= 0$
if $t \leq 0$	if $0 < t \leq 2$	if $2 < t \leq 4$	if $4 < t \leq 6$	if $6 < t$

3)

a) $y(t) = \frac{1}{(x(t-1))^2}$

$\boxed{\text{Not linear}}$ since squaring isn't a linear system, $2x_1(t)$ doesn't turn into $2y_1(t)$

Is $\boxed{\text{time invariant}}$ bc squaring is time invariant.

$\frac{1}{(x_1(t-1-k))^2} = y_1(t-k)$

$\boxed{\text{Causal}}$ since $y(t)$ depends on $t-1$ which is in the past

$\boxed{\text{Not stable}}$ since $y(t)$ is undefined if $x(t-1)$ is 0

b) $y(t) = \tan(t)\, x(t+1)$

$y_1(t+1) = \tan(t) x_1(t+1)$

$y_2(t+1) = \tan(t) x_2(t+1)$

$y_3(t+1) = \tan(t) x_3(t+1)$

$= \tan(t)(a x_1(t+1) + b x_2(t+1))$

$= a\tan(t) x_1(t+1) + b\tan(t) x_2(t+1)$

$= a y_1(t+1) + b y_2(t+1)$

$\boxed{\text{Linear}}$

$\boxed{\text{Not time Invariant}}$ bc $\tan(t)$ won't change if we multiply it by $x(t-t_0)$

$\boxed{\text{Not Causal}}$ since $y(t)$ depends on $x(t+1)$ which is in the future

$\boxed{\text{Not stable}}$ since $\tan(t)$ is undefined at $\frac{\pi}{2}$

c) $y(t) = x(t^4 - 10)$

$y_1(t) = x_1(t^4-10)$ $\quad x_3(t^4-10) = a x_1(t^4-10) + b x_2(t^4-10)$

$y_2(t) = x_2(t^4-10)$ $\quad \boxed{\text{Linear}}$

$y_3(t) = x_3(t^4-10)$

$= a x_1(t^4-10) + b x_2(t-10)$ ✓

$\boxed{\text{Not time invariant}}$ since $y(t-t_0) = x_1((t-t_0)^4 - 10)$ not $x_1((t^4-10)-t_0)$

$\boxed{\text{Not causal}}$ since t^4 can be infinite if an int. greater than 1 is put in

$\boxed{\text{Stable}}$ bc there are no holes

d) $y(t) = 1 + e^{x(t)}$

$y_1(t) = 1 + e^{x_1(t)}$ $\quad x_3(t) = a x_1(t) + b x_2(t)$

$y_2(t) = 1 + e^{x_2(t)}$ $\quad \boxed{\text{Not linear}}$

$y_3(t) = 1 + e^{x_3(t)}$

$= 1 + e^{a x_1(t) + b x_2(t)}$

$= 1 + e^{a x_1(t)} e^{b x_2(t)}$

$\neq a y_1(t) + b y_2(t)$

$\boxed{\text{Is time invariant}}$, y will change as x does

$\boxed{\text{Causal}}$ since $y(t)$ depends on t

$\boxed{\text{Stable}}$ bc no holes

4)

a) $y(t) = |x(t)| + x(t)$

$\boxed{\text{Not linear}}$ bc if $x_1(t) = -1$, $y_1(t) = 0$. If $x_2(t) = -x_1(t)$, then $x_2(t) = 1$ so $y_2(t) = 2$. $y_1(t) \neq y_2(t)$

$\boxed{\text{Time Invariant}}$ since $y(t)$ will change as $x(t)$ changes (time-wise)

b) $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$

$\boxed{\text{Linear}}$, $a x_1(t) + b x_2(t)$

$= \int_{-\infty}^{t} a x_1(\lambda) + b x_2(\lambda)\, d\lambda$

$= a\int_{-\infty}^{t} x_1(\lambda)\, d\lambda + b\int_{-\infty}^{t} x_2(\lambda)\, d\lambda$

$\boxed{\text{Time Invariant}}$

$x_1(t-t_0) = x_2(t)$

$y_2(t) = \int_{-\infty}^{t} x_2(\lambda)\, d\lambda = \int_{-\infty}^{t} x_1(\lambda - t_0)\, d\lambda$

$= \int_{-\infty}^{t-t_0} x_1(\tau)\, d\tau = y_1(t-t_0)$

c) $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)\, d\lambda$

$\boxed{\text{Linear}}$

$a x_1(t) + b x_2(t) =$

$= (t+1)\int_{-\infty}^{t} (a x_1(\lambda) + b x_2(\lambda))\, d\lambda$

$= a(t+1)\int_{-\infty}^{t} x_1(\lambda)\, d\lambda + b(t+1)\int_{-\infty}^{t} x_2(\lambda)\, d\lambda$

$= a y_1(t) + b y_2(t)$ ✓

$\boxed{\text{Not time invariant}}$

If t is alone by itself it's not time invariant

d) $y(t) = \frac{1}{1 + x^2(t)}$

$\boxed{\text{Not Linear}}$ since

$2x_1(t) = \frac{1}{1 + 4x_1^2(t)} \neq 2y_1(t)$ ✗

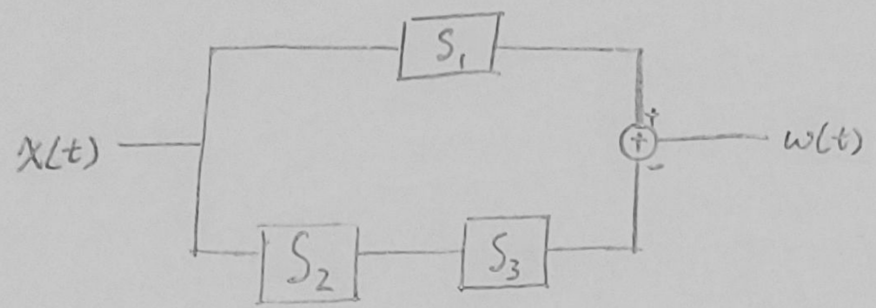
$\boxed{\text{Time invariant}}$ since

$x_1(t-t_0) = \frac{1}{1 + x^2(t-t_0)} = y_1(t-t_0)$ ✓

5)

a) Replace x w/ δ: $\int_{-\infty}^{t} e^{-3(t-\tau)}\delta(\tau)\,d\tau = \int_{-\infty}^{t} e^{-3t}\delta(\tau)\,d\tau = e^{-3t}\int_{-\infty}^{t}\delta(\tau)\,d\tau = \boxed{e^{-3t}u(t)}$

b)



c) $h_{eq}(t) = h_1(t) - h_2(t) * h_3(t)$

$= e^{-3t}u(t) - u(t-2) * \delta(t-3) = \boxed{e^{-3t}u(t) - u(t-5)}$

$h_1(t) = e^{-3t}u(t)$

$h_2(t) = \int_{-\infty}^{t-2}\delta(\tau)\,d\tau = u(t-2)$

$h_3(t) = \delta(t-3)$

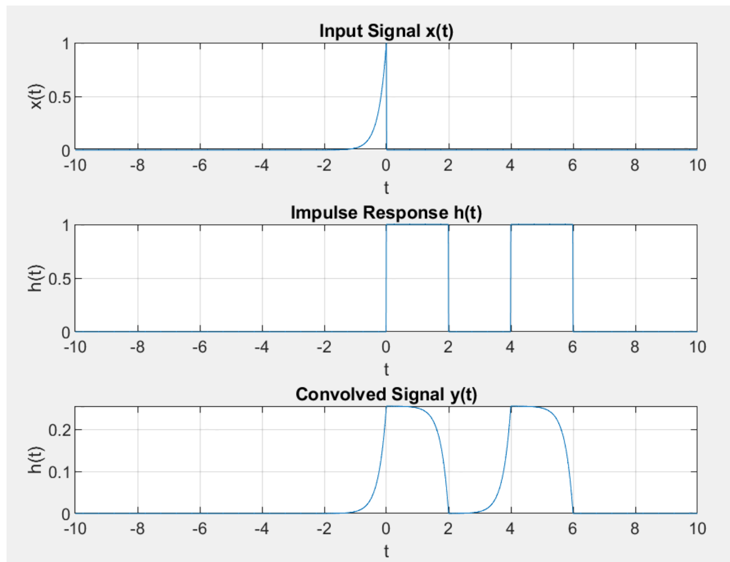
d) $x(t) = \delta(t) + 2\delta(t-3)$ can be rewritten as $y(t) = w(t) + 2w(t-3)$

$\boxed{y(t) = e^{-3t}u(t) - u(t-5) + 2(e^{-3(t-3)}u(t-3) - u(t-8))}$

Problem 2

b) x(t), h(t), and y(t)

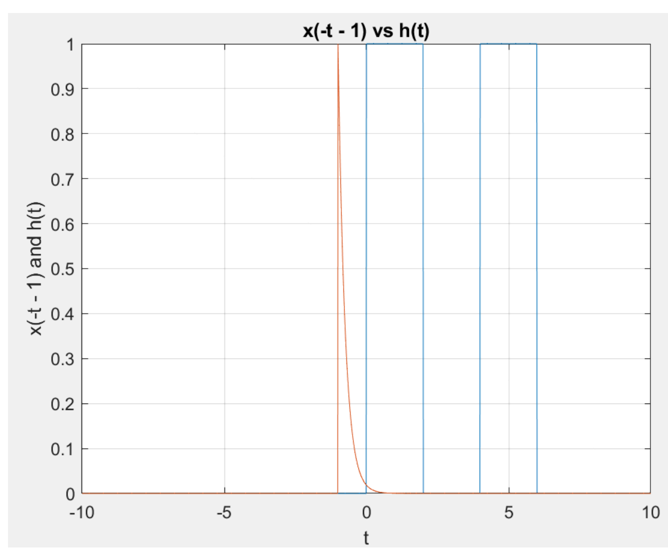
```
u_step = @(t) (t >= 0);
t = -10:0.01:10;
x_t = exp(t*4) .* u_step(-t);
h_t = u_step(t) - u_step(t-2) + u_step(t-4) - u_step(t-6);
y = conv(x_t, h_t, "same") * 0.01; %normalize step size
ty = linspace(-10, 10, length(y));
subplot(3, 1, 1);
plot(t, x_t);
title("Input Signal x(t)");
xlabel("t");
ylabel("x(t)");
grid on;
subplot(3,1,2);
plot(t, h_t);
title('Impulse Response h(t)');
xlabel("t");
ylabel("h(t)");
grid on;
subplot(3,1,3);
plot(t, y);
title('Convolved Signal y(t)');
xlabel("t");
ylabel("h(t)");
grid on;
```



c) Orange = x(t), blue = h(t)

For t = -1

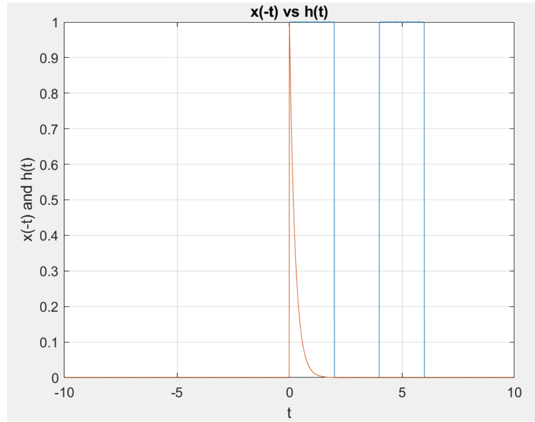
```
t_variable = -1;
x_t = exp(4 * (-t + t_variable)) .* u_step(-(-t + t_variable));
hold off;
plot(t, h_t);
hold on;
plot(t, x_t);
title("x(-t - 1) vs h(t)");
xlabel("t");
ylabel("x(-t - 1) and h(t)");
grid on;
```



For t = 0

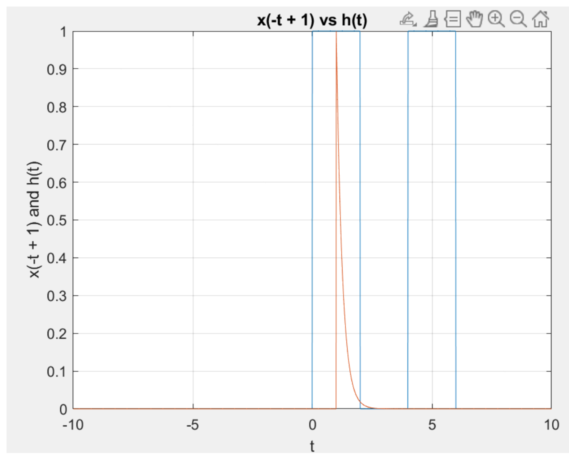
```
t_variable = 0;
```

```
x_t = exp(4 * (-t + t_variable)) .* u_step(-(-t + t_variable));
hold off;
plot(t, h_t);
hold on;
plot(t, x_t);
title("x(-t) vs h(t)");
xlabel("t");
ylabel("x(-t) and h(t)");
grid on;
```



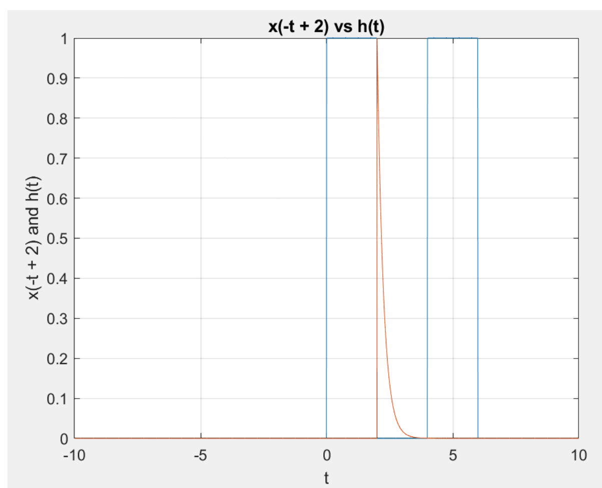
For t = 1

```
t_variable = 1;
x_t = exp(4 * (-t + t_variable)) .* u_step(-(-t + t_variable));
hold off;
plot(t, h_t);
hold on;
plot(t, x_t);
title("x(-t + 1) vs h(t)");
xlabel("t");
ylabel("x(-t + 1) and h(t)");
grid on;
```



For t = 2

```
t_variable = 2;
x_t = exp(4 * (-t + t_variable)) .* u_step(-(-t
+ t_variable));
hold off;
plot(t, h_t);
hold on;
plot(t, x_t);
title("x(-t + 2) vs h(t)");
xlabel("t");
ylabel("x(-t + 2) and h(t)");
grid on;
```



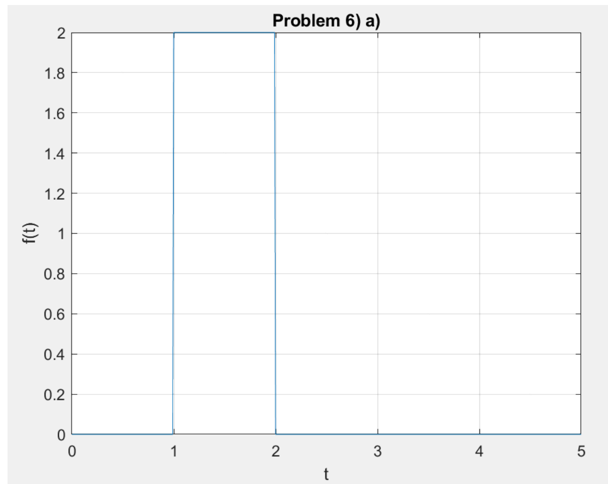
y(t) is approximately 0 when t = 2 since x(2 - τ) is 0 everywhere that h(τ) is nonzero. So the integral of

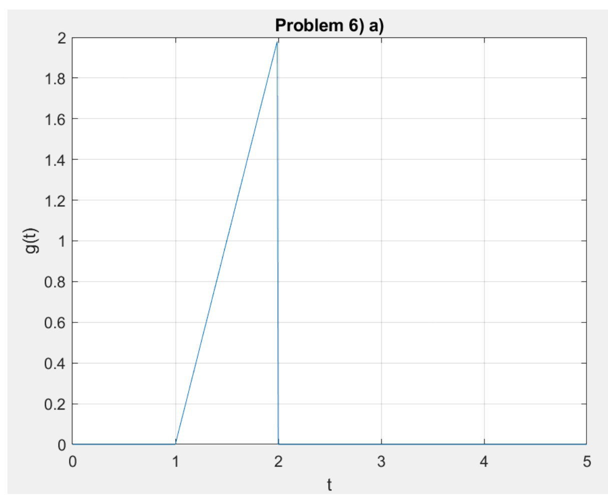
$\int x(2 - \tau)h(\tau)d\tau$ is 0. In all other graphs the orange and blue overlap so the value of the convolution is not zero. If t < -2 or 6 < t, x(t) would be shifted so it doesn't overlap with h(t) and y(t) would be 0 for those intervals.

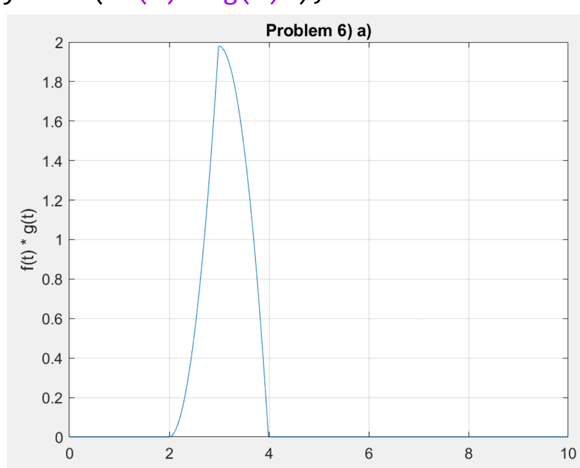
Problem 6

a)

```
u_step = @(t) (t >= 0);
t = 0:0.01:5;
f = 2 * rectpuls(t - 1.5, 1);
g = 2 * (u_step(t - 1) .* (t - 1)) .* rectpuls(t - 1.5, 1);
figure;
plot(t, f);
grid on;
title("Problem 6) a)");
xlabel("t");
ylabel("f(t)");
figure;
plot(t, g);
grid on;
title("Problem 6) a)");
xlabel("t");
ylabel("g(t)");
hold on;
figure;
[y, ty] = nconv(g, t, f, t);
plot(ty, y);
grid on;
title("Problem 6) a)");
xlabel("t");
ylabel("f(t) * g(t)");
```



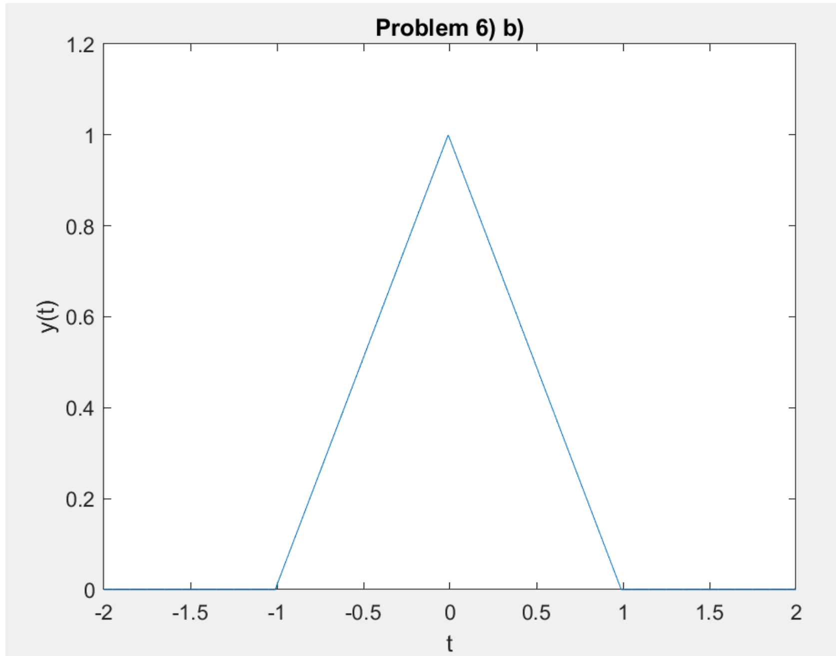




b)

```
t = -1:0.01:1;
r = rectpuls(t, 1);
```

```
[y, ty] = nconv(r, t, r, t);
plot(ty, y);
title("Problem 6) b)");
xlabel("t");
ylabel("y(t)");
```



c)

```
t = -1:0.01:1;
r = rectpuls(t, 1);
[y, ty] = nconv(r, t, r, t);
[y, ty] = nconv(y, ty, r, t);
plot(ty, y);
title("Problem 6) c)");
xlabel("t");
ylabel("y(t)");
```