Winter 2024
ECE 102
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Systems and Signals

Homework 5
Due 1 PM Friday, Mar 1, 2024
Submit your solutions on Gradescope.

Note: Answers without justification will not be awarded any marks.

Problem 1 (15 points)

A periodic real signal $x(t)$ has the following Fourier Series coefficients:
$X_k = \frac{A}{|k|} + jB|k|$ for $k \neq 0$
$X_0 = C$.
$x(t)$ has the following properties:

1. $x(t)$ is even.
2. $\int_{-2}^{2} |x(t)|^2 dt = 2$.
3. $\int_0^2 x(t)dt = 1$.
4. The period of $x(t)$ is 4.

Determine the values of the constants A, B and C. (Hint: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ might be a useful sum to solve this problem).

Problem 2 (15 points)

Calculate the Fourier Transforms for the following signals:

(a) $x_1(t) = e^{(6+4j)t}u(-t)$;

(b) $x_2(t) = 2\sin(3t)\,[u(t-1) - u(t-3)]$;

(c) $x_3(t) = \begin{cases} 1 + \cos(\pi t), & |t| < 2 \\ 0, & \text{otherwise} \end{cases}$

Problem 3 (18 points)

Calculate the inverse Fourier Transforms for the following signals:

(a) $X(\omega)$ is given in figure 1. Note: for the figure "Phase on $X(\omega)$" , the phase continues linearly

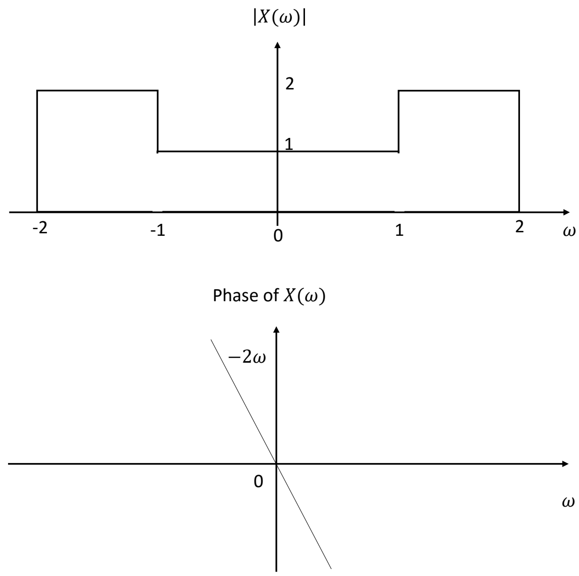


Figure 1: $X(\omega)$ for problem 2 part a

for non-depicted values (i.e. the line extends all the way from $\omega = -\infty$ to $\omega = \infty$)

(b) $X(\omega) = \omega^2 \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right] + \frac{2}{-j(\omega - \omega_0) + 1}$;

Problem 4 (20 points)

Consider a real signal $x(t)$. Let $x_e(t)$ denote the even component of $x(t)$ and let $x_o(t)$ be its odd component.

(a) Show that the Fourier Transform of $x_e(t)$ is found from the cosine transform:

$$X_1(\omega) = \int_{-\infty}^{\infty} x_e(t)\cos(\omega t)dt$$

Is $X_1(\omega)$ real? Is $X_1(\omega)$ even or odd?

(b) Show that the Fourier Transform of $x_o(t)$ is found from the sine transform:

$$X_2(\omega) = -j\int_{-\infty}^{\infty} x_o(t)\sin(\omega t)dt$$

Is $X_2(\omega)$ real? Is $X_2(\omega)$ even or odd?

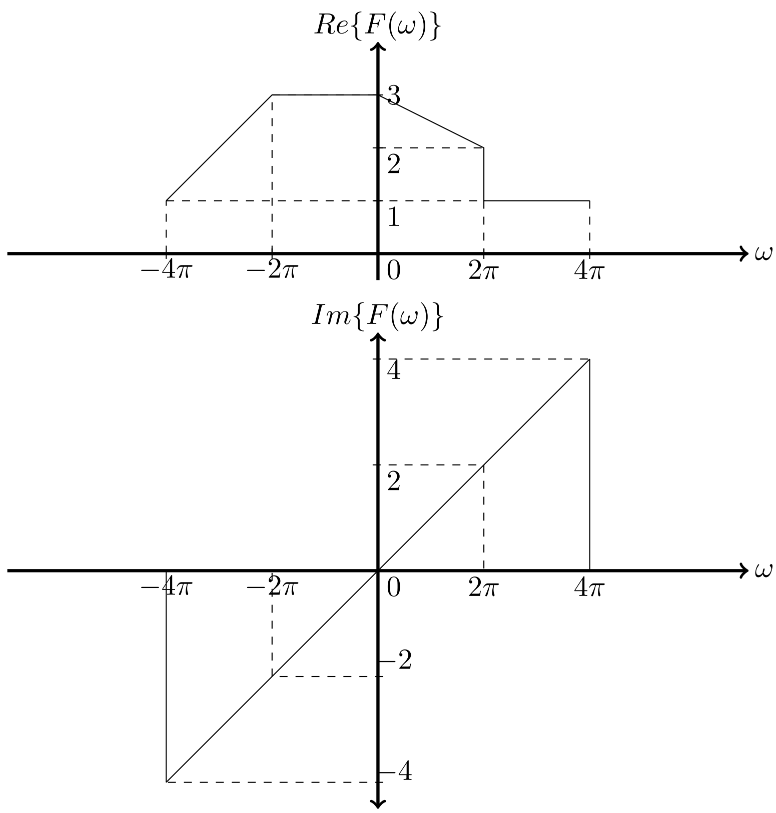
(c) Conclude that $\mathrm{Re}\{X(\omega)\} = X_1(\omega)$ and $j\mathrm{Im}\{X(\omega)\} = X_2(\omega)$, i.e. the real part of the Fourier Transform of $x(t)$ is the Fourier Transform of its even component and the imaginary part of the Fourier Transform is the Fourier Transform of its odd component.

(d) If $x(t)$ is real and even, what are the properties of $X(\omega)$? If $x(t)$ is real and odd, what are the properties of $X(\omega)$?

Problem 5 (20 points)

The function $F(\omega)$ sketched below is the Fourier Transform of an unknown function $f(t)$:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt$$



Evaluate the following:

1. $\int_{-\infty}^{\infty} f(t) \, dt$
2. $\int_{-\infty}^{\infty} f(t) e^{j2\pi t} \, dt$
3. $\int_{-\infty}^{\infty} f(t) \cos(2\pi t) e^{j2\pi t} \, dt$
4. $f(0)$

Is $f(t)$ a real signal?

Problem 6 (12 points)

Given that $x(t)$ has the Fourier Transform $X(\omega)$, express the Fourier Transforms of the following signals in terms of $X(\omega)$:

(a) $x_1(t) = x(2-t) + x(-2-t)$;

(b) $x_2(t) = x(2t-4)$;

(c) $x_3(t) = \frac{d^2}{dt^2}x(t-2)$.