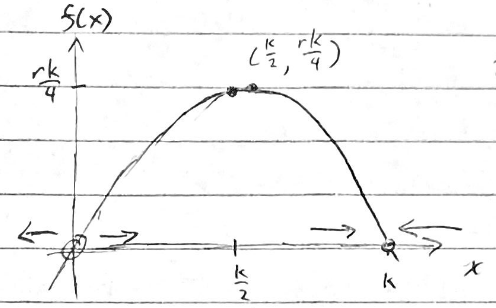
- solutions in this case w/ $0 < x_0 < k$ approach k but don't exceed it, hence it being the carrying capacity.

$\lim_{t \to \infty} x(t) = k$ & $\lim_{t \to -\infty} x(t) = 0$



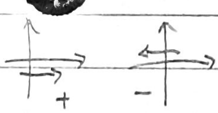
$f(x) = r(1 - \frac{x}{k})x$

Phase Line: consider $y' = f(y)$ and y as "distance" from 0 along number line (y-axis)

- $y' = f(y)$ describes motion dynamics modeled by $y(t)$ along the x-axis line which is called the phase line
- Equilibrium point types:

$\leftarrow o \rightarrow$ $\quad$ $\rightarrow \bullet \leftarrow$

- A + pointing arrow means solution increases around equilibrium points, − arrow means decreasing

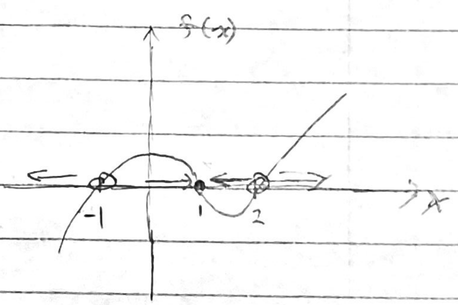


- To left of k but right of 0, $f(x) > 0$ so solution $x(t)$ is increasing denoted w/ $\longrightarrow$
- To left of 0 $f(x) < 0$, $x(t)$ decreasing denoted by $\longleftarrow$

Ex: Describe behavior as $t \to \infty$ for all solutions to

$$x' = f(x) = (x^2 - 1)(x - 2)$$
$$= (x+1)(x-1)(x-2)$$

Graph $f(x)$ RHS & phase line info



Equilibrium pts: $x_1 = -1$, $x_2 = 1$, $x_3 = 2$

$-\infty \quad -1 \quad 1 \quad 2 \quad \infty$

$- \quad + \quad - \quad +$

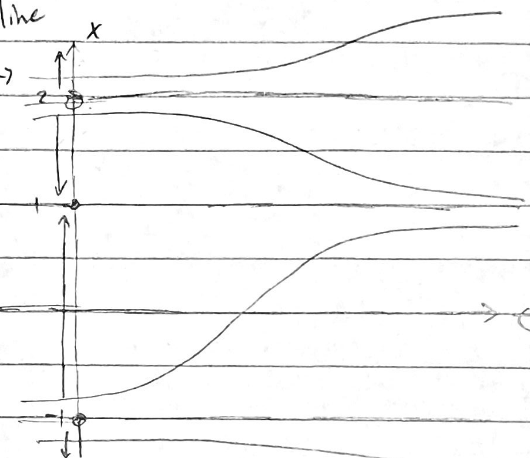
- In intervals limited by equilibrium points, use sign of f to draw arrows:

Consider $x(t)$ that is a equilibrium solution

- If $f(x) > 0$, arrow is $\rightarrow$ bc $x'(t) = f(x) > 0$ so $x(t)$ increases
- If $f(x) < 0$, arrow is $\leftarrow$ bc $x'(t) = f(x) < 0$ so $x(t)$ decreases

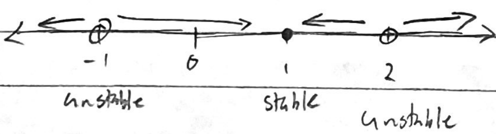
• check f, & $\frac{\partial f}{\partial x}$ is continuous on $\mathbb{R}^2$, ads any $(t_0, x_0) \in R \in \mathbb{R}^2$
so Uniqueness Thm applies & solution curves can't cross equilibrium points, basically limited by the lines
- If solution starts in one of the intervals, then $x(t)$ is in that interval for all t

Create tx-plane
- transfer plane to x-axis
- draw equilibrium solutions
- use phase line info to sketch nonequilibrium solutions in interval limited by equilibrium points



Equilibrium points $x_1 = -1$, $x_3 = 2$ unstable bc solutions move away from $x_1 = -1$ & $x_3 = 2$ as $t \to \infty$

Equilibrium point $x_2 = 1$ stable bc solutions approach $x_2 = 1$ as $t \to \infty$



← also a phase line

So

$\leftarrow \circ \rightarrow$ unstable — solutions move away from equilibrium point as $t \to \infty$

$\rightarrow \bullet \leftarrow$ stable — solutions approach equilibrium point as $t \to \infty$

At equilibrium point x_0 for $x' = f(x)$ asymptotically stable only if f is DECREASING at x_0

use same test

First Derivative Test for Stability

basically a 2nd derivative test for $x(t)$

Thm 9.8: Suppose x_0 is an equilibrium point for $x' = f(x)$ where f is a differentiable function
1) If $f'(x_0) < 0$, then f decreases at x_0 & x_0 is asymptotically STABLE
2) If $f'(x_0) > 0$, f is increasing at x_0 and is UNSTABLE
3) If $f'(x_0) = 0$ no conclusion