HW 9.2 # 5, 7, 11, 15, 19, 25, 31, 37, 45

5. $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$ $\det(A-\lambda I) = \det\begin{pmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{pmatrix} = (1-\lambda)(4-\lambda)+2 = \lambda^2-5\lambda+6 = (\lambda-2)(\lambda-3)$

$\lambda_1 = 2$ $A-\lambda_1 I = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\lambda_2 = 3$ $A-\lambda_2 I = \begin{pmatrix} -2 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-x_1 + 2x_2 = 0$ $x_1 = 2x_2$ $-2x_1+2x_2 = 0$ $x_1 = x_2$ $\vec{v}_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$-x_1 + 2x_2 = 0$ $-x_1 + x_2 = 0$

$\vec{v}_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_2 \\ x_2 \end{pmatrix} = x_2\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\boxed{y(t) = C_1 e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

7. $A = \begin{pmatrix} 2 & -6 \\ 0 & -1 \end{pmatrix}$ $\det(A-\lambda I) = \det\begin{pmatrix} 2-\lambda & -6 \\ 0 & -1-\lambda \end{pmatrix} = (2-\lambda)(-1-\lambda) = \lambda^2-\lambda-2 = (\lambda-2)(\lambda+1)$ $\lambda = -1, 2$

$\lambda_1 = -1$ $A+I = \begin{pmatrix} 3 & -6 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\lambda_2 = 2$ $A-2I = \begin{pmatrix} 0 & -6 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$3x_1 = 6x_2$ $-6x_1 = 0$

$x_1 = 2x_2$

$\vec{v}_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $-3x_2 = 0$

$\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\boxed{y(t) = e^{-t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} - 2e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$y(t) = C_1 e^{-t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $2C_1 + C_2 = 0$

$y(0) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $C_1 = 1$ $C_2 = -2$

11. We know $y(t) = C_1 e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$y(0) = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $2C_1 + C_2 = 3$ $C_1 + C_2 = 2$ $C_1 = 1$ $C_2 = 1$

$$\boxed{y(t) = e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

15. $z(t) = e^{3it}\begin{pmatrix} -1-i \\ 2 \end{pmatrix} = (\cos 3t + i\sin 3t)\left[\begin{pmatrix} -1 \\ 2 \end{pmatrix} + i\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right] = \cos 3t\begin{pmatrix} -1 \\ 2 \end{pmatrix} - \sin 3t\begin{pmatrix} -1 \\ 0 \end{pmatrix} + i\left(\cos 3t\begin{pmatrix} -1 \\ 0 \end{pmatrix} + \sin 3t\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right)$

$\vec{y}_1(t) = Re(z(t)) = \begin{pmatrix} -\cos 3t + \sin 3t \\ 2\cos 3t \end{pmatrix}$ $\vec{y}_2(t) = Im(z(t)) = \begin{pmatrix} -\cos 3t - \sin 3t \\ 2\sin 3t \end{pmatrix}$

$\vec{y}_1'(t) = \begin{pmatrix} +3\sin 3t + 3\cos 3t \\ -6\sin 3t \end{pmatrix}$ $\vec{y}_2'(t) = \begin{pmatrix} 3\sin 3t - 3\cos 3t \\ 6\cos 3t \end{pmatrix}$

$\boxed{\begin{pmatrix} 3 & 3 \\ -6 & -3 \end{pmatrix}\begin{pmatrix} -\cos 3t + \sin 3t \\ 2\cos 3t \end{pmatrix} = \begin{pmatrix} 3\sin 3t + 3\cos 3t \\ -6\sin t \end{pmatrix}}$ $\boxed{\begin{pmatrix} 3 & 3 \\ -6 & -3 \end{pmatrix}\begin{pmatrix} -\cos 3t - \sin 3t \\ 2\sin 3t \end{pmatrix} = \begin{pmatrix} 3\sin 3t - 3\cos 3t \\ 6\cos 3t \end{pmatrix}}$

$y_1(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $y_2(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\det\begin{vmatrix} -1 & -1 \\ 2 & 0 \end{vmatrix} = -1+2 = 1$, so linearly ind. Since $y_1(0)$ and $y_1(0)$ are linearly independent, so is $y_1(t)$ and $y_2(t)$ & form fundamental set

19. $A = \begin{pmatrix} 0 & 4 \\ -2 & -4 \end{pmatrix}$ $\det(A-\lambda I) = \begin{vmatrix} -\lambda & 4 \\ -2 & -4-\lambda \end{vmatrix} = \lambda^2+4\lambda+8$ $\lambda = -2+2i$, $\bar{\lambda} = -2-2i$

$(A-\lambda I) = \begin{pmatrix} 2-2i & 4 \\ -2 & 2-2i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $x_1 = -1-i$, $x_2 = 1$ $z(t) = e^{(-2+2i)t}\begin{pmatrix} -1-i \\ 1 \end{pmatrix}$

$= e^{-2t}(\cos 2t + i\sin 2t)\left(\begin{pmatrix} -1 \\ 1 \end{pmatrix} + i\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right)$

$(2-2i)x_1 + 4x_2 = 0$ $= e^{-2t}\left(\cos 2t\begin{pmatrix} -1 \\ 1 \end{pmatrix} - \sin 2t\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) + ie^{-2t}\left(\cos 2t\begin{pmatrix} -1 \\ 0 \end{pmatrix} + \sin 2t\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)$

$(-2x_1 + (2-2i)x_2 = 0)(-1+i)$

$(4-4i)x_1 + (-2+2i+2i+2)x_2 = 0$

$$\boxed{\begin{aligned} y_1(t) &= e^{-2t}\begin{pmatrix} -\cos 2t + \sin 2t \\ \cos 2t \end{pmatrix} \\ y_2(t) &= e^{-2t}\begin{pmatrix} -\cos 2t - \sin 2t \\ \sin 2t \end{pmatrix} \end{aligned}}$$

25. we know $y_1(t) = e^{-2t}\begin{pmatrix} -\cos 2t + \sin 2t \\ \cos 2t \end{pmatrix}$ $\quad y_2(t) = e^{-2t}\begin{pmatrix} -\cos 2t - \sin 2t \\ \sin 2t \end{pmatrix}$

$$y(t) = C_1 e^{-2t}\begin{pmatrix} -\cos 2t + \sin 2t \\ \cos 2t \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} -\cos 2t - \sin 2t \\ \sin 2t \end{pmatrix}$$

$$y(0) = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \qquad -C_1 - C_2 = -1 \qquad C_1 = 2 \qquad C_2 = -1$$

$$\boxed{y(t) = 2e^{-2t}\begin{pmatrix} -\cos 2t + \sin 2t \\ \cos 2t \end{pmatrix} - e^{-2t}\begin{pmatrix} -\cos 2t - \sin 2t \\ \sin 2t \end{pmatrix}}$$

31. $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ $\quad \det(A - \lambda I) = \det\begin{pmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{pmatrix} = (3-\lambda)(1-\lambda) + 1 = \lambda^2 - 4\lambda + 4$, $\lambda = 2$

$A - 2I = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad y_1(t) = e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Use $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to satisfy $(A - 2I)v_2 = v_1$

$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \checkmark$

$y_2(t) = e^{t}(v_2 + t v_1) = e^{t}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$

$$\boxed{y(t) = C_1 e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 t e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

37. $y(0) = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\qquad C_1 + C_2 = 2 \qquad C_1 = -1 \qquad C_2 = 3$

$$\boxed{y(t) = -e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3e^{t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3te^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

45. $A = \begin{pmatrix} -4 & -5 \\ 2 & 2 \end{pmatrix}$, $\det(A - \lambda I) = \det\begin{pmatrix} -4-\lambda & -5 \\ 2 & 2-\lambda \end{pmatrix} = \lambda^2 + 2\lambda + 2$, $\lambda = -1 + i \quad \bar{\lambda} = -1 - i$

$A - \lambda I = \begin{pmatrix} -3-i & 5 \\ 2 & 3-i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad x_1 = 5 \quad x_2 = -3-i$

$$z(t) = e^{(-1+i)t}\begin{pmatrix} 5 \\ -3-i \end{pmatrix} = e^{-t}(\cos t + i\sin t)\left[\begin{pmatrix} 5 \\ -3 \end{pmatrix} + i\begin{pmatrix} 0 \\ -1 \end{pmatrix}\right]$$

$$= e^{-t}\left(\cos t\begin{pmatrix} 5 \\ -3 \end{pmatrix} - \sin t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + i\cos t\begin{pmatrix} 0 \\ -1 \end{pmatrix} + i\sin t\begin{pmatrix} 5 \\ -3 \end{pmatrix}\right)$$

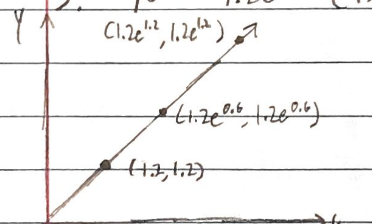
$$\boxed{y(t) = C_1 e^{-t}\left(\cos t\begin{pmatrix} 5 \\ -3 \end{pmatrix} - \sin t\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) + C_2 e^{-t}\left(\cos t\begin{pmatrix} 0 \\ -1 \end{pmatrix} + \sin t\begin{pmatrix} 5 \\ -3 \end{pmatrix}\right)}$$

HW 9.3 #3, 7, 9, 12, 15, 16, 19, 20, 21, 23

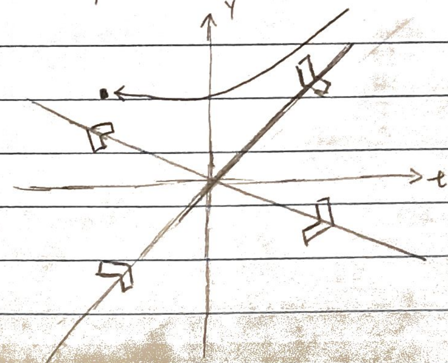
3. $y(t) = 1.2e^{0.6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad y(0) = 1.2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$y(1) = 1.2e^{0.6}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$y(2) = 1.2e^{1.2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



7. $y(t) = 0.8e^{0.4t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} + 1.2e^{-0.2t}\begin{pmatrix} 4 \\ 4 \end{pmatrix}$



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9.2 HW # 29, 33

29. $A = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ $A - \lambda I = \begin{pmatrix} -2-\lambda & 0 \\ 0 & -2-\lambda \end{pmatrix} = \lambda^2 + 4\lambda + 4$ $\lambda = -2$

$A - (-2)I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ choose $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as eigenvectors

$$\boxed{x(t) = C_1 e^{-2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

33. $A = \begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}$ $A - \lambda I = \begin{pmatrix} -2-\lambda & 1 \\ -9 & 4-\lambda \end{pmatrix} = \lambda^2 - 2\lambda - 8 + 9 = \lambda^2 - 2\lambda + 1$ $\lambda = 1$

$A - I = \begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix}$ only gives $\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\vec{y}_1(t) = e^t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

find v_2 such that $(A - I)v_2 = v_1$

$\begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $x_1 = 1$, $x_2 = 0$ $v_2 = -\frac{1}{3} w = \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix}$

$= -3v_1$

$\vec{y}_2(t) = e^t \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix} + e^t t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\boxed{\vec{y}(t) = C_1 e^t \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^t \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix} + C_2 e^t t \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$$

9.4 HW # 3, 5, 7, 11, 17, 19, 23

3.

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HOMEWORK 10

9.2 HW # 29, 33

29. $A = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ $\quad A - \lambda I = \begin{pmatrix} -2-\lambda & 0 \\ 0 & -2-\lambda \end{pmatrix} = \lambda^2 + 4\lambda + 4 \quad | \quad \lambda = -2$

$A - (-2)I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad$ choose $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as eigenvectors

$$\boxed{x(t) = C_1 e^{-2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

33. $A = \begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}$ $\quad A - \lambda I = \begin{pmatrix} -2-\lambda & 1 \\ -9 & 4-\lambda \end{pmatrix} = \lambda^2 - 2\lambda - 8 + 9 = \lambda^2 - 2\lambda + 1 \quad \lambda = 1$

$A - I = \begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix}$ only gives $\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\vec{y}_1(t) = e^t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

find v_2 such that $\quad (A - I)v_2 = v_1 \quad x_1 = 1$

$\begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad x_2 = 0 \qquad v_2 = -\frac{1}{3} w = \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix}$

$= -3v_1$

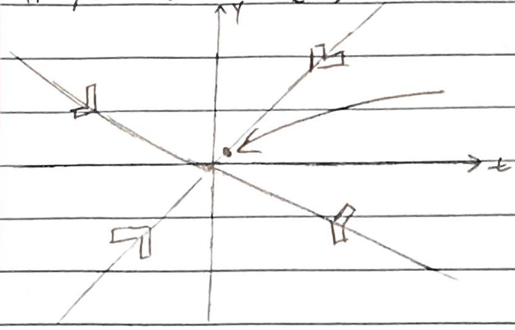
$\vec{y}_2(t) = e^t \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix} + e^t t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\boxed{\vec{y}(t) = C_1 e^t \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^t \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix} + C_2 e^t t \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$$

9.4 HW # 3, 5, 7, 11, 17, 19, 23

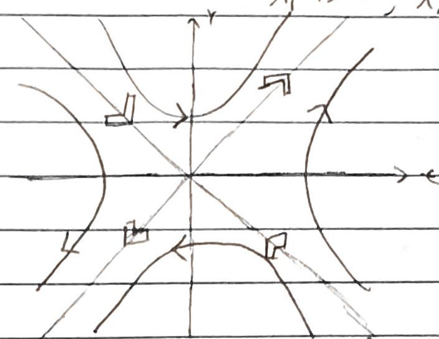
3.

9. $y(t) = -0.4e^{-0.4t}\begin{pmatrix}-2\\1\end{pmatrix} + 0.4e^{-0.2t}\begin{pmatrix}4\\4\end{pmatrix}$



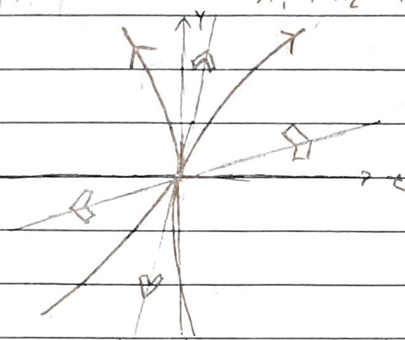
12. $Y(t) = C_1e^{t}\begin{pmatrix}1\\1\end{pmatrix} + C_2e^{-2t}\begin{pmatrix}1\\-1\end{pmatrix}$

λ_1 is +, λ_2 is −, $\boxed{\text{saddle}}$



15. $y(t) = C_1e^{3t}\begin{pmatrix}4\\1\end{pmatrix} + C_2e^{t}\begin{pmatrix}1\\5\end{pmatrix}$

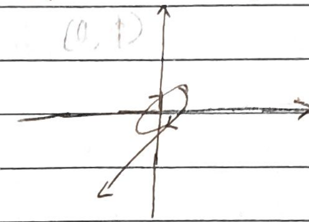
λ_1 & λ_2 +, $\boxed{\text{source}}$



16. $A = \begin{pmatrix}-4 & 8\\-4 & 4\end{pmatrix}$ $A-\lambda I = \begin{pmatrix}-4-\lambda & 8\\-4 & 4-\lambda\end{pmatrix} = \lambda^2+16$, $\lambda = \pm 4i$.

Since α in $\alpha \pm ib$ is 0, equilibrium at origin is a center.

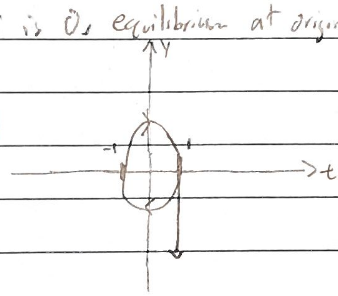
$\begin{pmatrix}-4 & 8\\-4 & 4\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-4\\-4\end{pmatrix}$,



19. $A = \begin{pmatrix}0 & 1\\-4 & 0\end{pmatrix}$ $A-\lambda I = \begin{pmatrix}-\lambda & 1\\-4 & -\lambda\end{pmatrix} = \lambda^2+4$, $\lambda = \pm 2i$

Since α in $\alpha \pm bi$ is 0, equilibrium at origin is a center

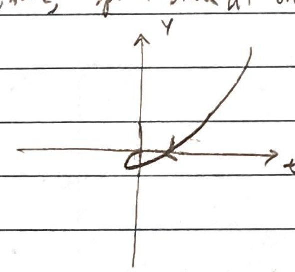
$\begin{pmatrix}0 & 1\\-4 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-4\end{pmatrix}$



20. $A = \begin{pmatrix}-2 & 2\\-1 & 0\end{pmatrix}$ $A-\lambda I = \begin{pmatrix}-2-\lambda & 2\\-1 & -\lambda\end{pmatrix} = \lambda^2+2\lambda+2$, $\lambda = -1 \pm i$

Bc α in $\alpha \pm ib$ is negative, spiral sink at origin aka equilibrium point.

$\begin{pmatrix}-2 & 2\\-1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix}$

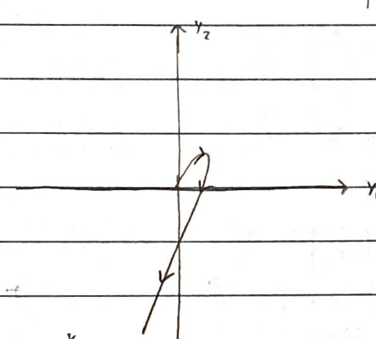


21. $A = \begin{pmatrix}-1 & 1\\-5 & 3\end{pmatrix}$ $A-\lambda I = \begin{pmatrix}-1-\lambda & 1\\-5 & 3-\lambda\end{pmatrix} = \lambda^2-2\lambda+2$

$\lambda_1 = 1+i$ $\lambda_2 = 1-i$

Bc α in $\lambda = \alpha \pm ib$ is positive, we have a source

$\begin{pmatrix}-1 & 1\\-5 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-1\\-5\end{pmatrix}$



23. $A = \begin{pmatrix}-3 & 2\\-4 & 1\end{pmatrix}$ $A-\lambda I = \begin{pmatrix}-3-\lambda & 2\\-4 & 1-\lambda\end{pmatrix} = \lambda^2+2\lambda+5$

$\lambda_1 = -1+2i$ $\lambda_2 = -1-2i$

Since α in $\lambda = \alpha \pm ib$ is negative, we get a sink to origin

$\begin{pmatrix}-3 & 2\\-4 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-3\\-4\end{pmatrix}$