11/7/22

Ch. 9 Linear Systems w/ Constant Coefficients

Review: Matrices, Vectors, & Determinants

Ex: Solve system of linear equations:

$$x_1 + 2x_2 + 3x_3 = 5$$
$$-4x_1 + 4x_2 = -7$$

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ -4 & 4 & 0 \end{pmatrix}}_{\text{coeff matrix } A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x \text{ column vectors}} = \underbrace{\begin{pmatrix} 5 \\ -7 \end{pmatrix}}_{b}$$

$\vec{x}$ vector of unknowns

$\vec{b}$ RHS of matrix

General Matrix w/ m rows & n columns:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

a_{2n} is 2nd row and nth column

a_{ij} is ith row and jth column

A is m×n Matrix

↑ rows ↑ columns

notation: $A \in \mathbb{R}^{m \times n}$

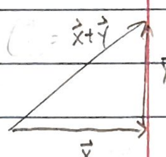
Column Vectors: 1-column matrix $c = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$ is n×1 matrix. Is also called n×1 column vector or n-vector $c \in \mathbb{R}^n$

Row Vector: matrix w/ only one row $c = (c_1, c_2, \cdots c_n)$ is 1×n matrix/row vector

Addition: Literally add up corresponding components

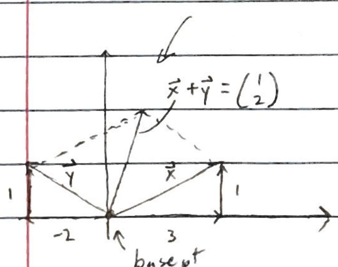
$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 8 & 10 \\ 9 & -10 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 11 & 9 \\ 10 & -10 & 9 \end{pmatrix}$$

If $\vec{x}, \vec{y} \in \mathbb{R}^n$, ← column vectors, then their sum $\vec{x} + \vec{y} \in \mathbb{R}^n$



Ex: $\vec{x} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ then $\vec{x}, \vec{y} \in \mathbb{R}^2$

$\vec{z} = \vec{x} + \vec{y} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in \mathbb{R}^2$



Sum is from basepoint to opposite vertex.

OR YOU CAN TRANSLATE LOL WE J WASTED LIKE 7 MINUTES I LOVE THIS CLASS

Multiplication by #: If A is a matrix & α is a complex # then αA is a matrix in which each component is multiplied

$$\frac{1}{3}\begin{pmatrix} 0 & 1 & 3 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{3} & 1 \\ 1 & 0 & \frac{1}{3} \end{pmatrix}$$

Subtraction: A−B is A+(−B), just multiply B by −1

Multiplication of 2 Matrices: AB is defined by # of columns in A and # of rows in B.

$$A \text{ is } \overbrace{m \times \underbrace{n \quad B \text{ is } n}_{\text{have to be } =} \times r}^{\text{Resulting Matrix}} \Rightarrow C = AB \text{ is } m \times r \text{ matrix}$$

$$\underset{3\times 2}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & & \\ & & \\ & & \end{bmatrix}}$$

Properties:

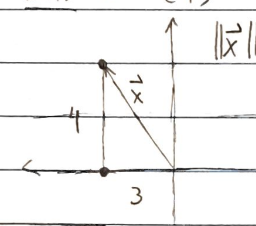
Matrix Multiplication is Associative: $(AB)C = A(BC)$

Matrix Multiplication is Distributive: $A(B+C) = AB + AC \qquad (B+C)A = BA + CA$

Matrix Multiplication is NOT Commutative: $AB \neq BA$ usually

length/magnitude of vector $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$ denote $\boxed{\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$

Ex: $\vec{x} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$



$\|\vec{x}\| = \sqrt{16+9} = 5$

length $= 5$

Identity Matrix: $I \in \mathbb{R}^{n\times n}$

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

If row = column, 1

If row $\neq$ column, 0

Determinant: If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\boxed{\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc}$

For higher order, $\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$

a_{ij} = ith row, jth column

At ith row & jth column, $\boxed{\text{sign is } (-1)^{i+j}}$

Ex: $\begin{array}{c} \\ \\ \\ \text{3rd column} \end{array}$

$$\begin{array}{c} \begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix} \begin{array}{l} \\ \text{2nd row} \\ \\ \end{array} \\ \text{3rd column} \end{array} \qquad a_{23} = 4 \qquad M_{23} = \begin{vmatrix} 1 & -5 \\ 2 & 1 \end{vmatrix} = 1 - (-10) = 11$$

cofactor (or sign) of $a_{23} = (-1)^{2+3} = -1$, $M_{23} = -1 \cdot 11 = -11$

Value: multiply each component of one row/column by its cofactor and add

Ex: If we're using 1st row...

$$\boxed{\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix} = 1\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} + (-1)(-5)\begin{vmatrix} 7 & 4 \\ 2 & 5 \end{vmatrix} + 2\begin{vmatrix} 7 & 3 \\ 2 & 1 \end{vmatrix} = \cancel{11 + 135 + 2 = 148}}$$