10/28/22

HW 4.1 # 1, 7, 15, 17, 19, 22, 24, 26, 29

1. $y'' + 3y' + 5y = 3\cos 2t$

$p(t) \quad q(t) \quad g(t)$

$\boxed{\text{linear and homogeneous}}$

7. $y'' + 3y' + 4\sin y = 0$

$\sin y$ makes it $\boxed{\text{nonlinear}}$

15. $y'' - 2y' + 2y = 0$

$y_1(t) = e^t \cos t$

$y_1'(t) = e^t(\cos t - \sin t)$

$y_1''(t) = -2e^t \sin t$

$-2e^t \sin t - 2e^t(\cos t - \sin t) + 2e^t \cos t$

$\boxed{= 0}$

$y_2(t) = e^t \sin t$

$y_2'(t) = e^t(\sin t + \cos t)$

$y_2''(t) = 2e^t \cos t$

$2e^t \cos t - 2e^t(\sin t + \cos t) + 2e^t \sin t$

$\boxed{= 0}$

Any combination of $y(t) = C_1 y_1(t) + C_2 y_2(t)$ will work

$y(t) = C_1 y_1(t) + C_2 y_2(t) = C_1 e^t \cos t + C_2 e^t \sin t$

$y'(t) = C_1 e^t(\cos t - \sin t) + C_2 e^t(\sin t + \cos t)$

$y''(t) = C_1 \cdot -2e^t \sin t + C_2 \cdot 2e^t \cos t$

$y'' - 2y' + 2y = -2C_1 e^t \sin t + 2C_2 e^t \cos t - 2C_1 e^t(\cos t - \sin t) - 2C_2 e^t(\sin t + \cos t) + 2C_1 e^t \cos t + 2C_2 e^t \sin t$

$\boxed{= 0}$

17. $\dfrac{y_1(t)}{y_2(t)} = \dfrac{e^{-t}}{e^{2t}} = \boxed{e^{-3t}, \text{ not constant! So they are linearly independent}}$

$W(t) = \det \begin{vmatrix} e^{-t} & e^{2t} \\ -e^{-t} & 2e^{2t} \end{vmatrix} = 2e^t + e^t = \boxed{3e^t \neq 0, \text{ so linearly ind.}}$

19. $\dfrac{y_1(t)}{y_2(t)} = \dfrac{e^{-2t}\cos 3t}{e^{-2t}\sin 3t} = \boxed{\cot 3t, \text{ not constant. So they are linearly ind.}}$

$W(t) = \det \begin{vmatrix} e^{-2t}\cos 3t & e^{-2t}\sin 3t \\ -e^{-2t}(2\cos 3t + 3\sin 3t) & -e^{-2t}(2\sin 3t - 3\cos 3t) \end{vmatrix} = -e^{-4t}\cos 3t(2\sin 3t - 3\cos 3t)$

$+ e^{-4t}\sin 3t(2\cos 3t + 3\sin 3t) = e^{-4t}(3\cos^2 3t + 3\sin^2 3t) \boxed{= 3e^{-4t} \neq 0, \text{ linearly ind.}}$

22. $y'' + 2y' - 3y = 0$

$\dfrac{y_1(t)}{y_2(t)} = \dfrac{e^t}{e^{-3t}}$

$\boxed{= e^{4t}, \text{ not constant!}}$

$y_1(t) = e^t \qquad y_2(t) = e^{-3t}$

$y_1'(t) = e^t \qquad y_2'(t) = -3e^{-3t}$

$y_1''(t) = e^t \qquad y_2''(t) = 9e^{-3t}$

$e^t + 2e^t - 3e^t = 0$

$9e^{-3t} - 6e^{-3t} - 3e^{-3t} = 0$

$y(t) = C_1 y_1(t) + C_2 y_2(t) = C_1 e^t + C_2 e^{-3t}$

$y(0) = C_1 + C_2 = 1$

$y'(t) = C_1 y_1'(t) + C_2 y_2'(t) = C_1 e^t - 3C_2 e^{-3t}$

$y'(0) = C_1 - 3C_2 = -2$

$C_1 + C_2 = 1$

$-(C_1 - 3C_2 = -2)$

$4C_2 = 3$

$C_1 = \frac{1}{4}, C_2 = \frac{3}{4}$

$\boxed{y(t) = \frac{e^t}{4} + \frac{3e^{-3t}}{4}}$

$$\frac{y_1(t)}{y_2(t)} = \frac{e^{-t}\cos 2t}{e^{-t}\sin 2t} = \boxed{\cot 2t, \text{ not constant!}}$$

24. $y'' + 2y' + 5y = 0$

$y_1(t) = e^{-t}\cos 2t$

$y_1'(t) = -e^{-t}\cos 2t - 2e^{-t}\sin 2t$

$y_1''(t) = -3e^{-t}\cos 2t + 4e^{-t}\sin 2t$

$y_1'' + 2y_1' + 5y_1 = -3e^{-t}\cos 2t + 4e^{-t}\sin 2t$
$-2e^{-t}\cos 2t - 4e^{-t}\sin 2t + 5e^{-t}\cos 2t$
$= 0$

$y_2(t) = e^{-t}\sin 2t$

$y_2'(t) = -e^{-t}\sin 2t + 2e^{-t}\cos 2t$

$y_2''(t) = -3e^{-t}\sin 2t - 4e^{-t}\cos 2t$

$y_2'' + 2y_2' + 5y_2 = -3e^{-t}\sin 2t - 4e^{-t}\cos 2t$
$-2e^{-t}\sin 2t + 4e^{-t}\cos 2t + 5e^{-t}\sin 2t$
$= 0$

$y(t) = C_1 y_1(t) + C_2 y_2(t) = C_1 e^{-t}\cos 2t + C_2 e^{-t}\sin 2t$

$y'(t) = -C_1 e^{-t}\cos 2t - 2C_1 e^{-t}\sin 2t - C_2 e^{-t}\sin 2t + 2C_2 e^{-t}\cos 2t$

$y(0) = -1 = C_1$

$y'(0) = 0 = -C_1 + 2C_2$

$C_1 = -1, \quad C_2 = -\frac{1}{2}$

$$\boxed{y(t) = -e^{-t}\cos 2t - \frac{e^{-t}}{2}\sin 2t}$$

26. a. $t^2y'' + ty' - 4y = 0$

$y_1(t) = t^2$

$y_1'(t) = 2t$

$y_1''(t) = 2$

$2t^2 + 2t^2 - 4t^2 = 0$

b. $y_2(t) = v y_1(t) = vt^2$

$y_2'(t) = v't^2 + 2vt$

$y_2''(t) = v''t^2 + 4v't + 2v$

$t^2(v''t^2 + 4v't + 2v) + t(v't^2 + 2vt)$
$-4(vt^2) = t^4v'' + 5t^3v' = 0$

$$\boxed{tv'' + 5v' = 0}$$

29. $t^2y'' - 3ty' + 3y = 0, \quad y_1(t) = t$ ;

$y_1(t) = t$

$y_1'(t) = 1$

$y_1''(t) = 0$

$t^2(0) - 3t(1) + 3t = 0$

$$\boxed{y(t) = C_1 t + C_2 t^3}$$

$y_2(t) = v y_1(t) = vt$

$y_2'(t) = v't + v$

$y_2''(t) = v''t + 2v'$

$t^2(v''t + 2v') - 3t(v't + v) + 3vt = v''t^3 - v't^2$

$v''t^3 = v't^2 \Rightarrow v''t = v'$

$\frac{v''}{v'} = \frac{1}{t}$

$\ln v' = \ln t$

$v' = t$

$v = \frac{1}{2}t^2$

$y_2(t) = vt = \frac{1}{2}t^3$

HW 4.3 # 3, 6, 25, 37

3. $y''+5y'+6y=0$ , $y=e^{\lambda t}$

$\lambda^2 e^{\lambda t}+5\lambda e^{\lambda t}+6e^{\lambda t}$ $\quad y'=\lambda e^{\lambda t}$

$=e^{\lambda t}(\lambda^2+5\lambda+6)=0$ $\quad y''=\lambda^2 e^{\lambda t}$

$\boxed{y(t)=C_1e^{-3t}+C_2e^{-2t}}$

$(\lambda+3)(\lambda+2)=0$

$\lambda_1=-3,\ \lambda_2=-2$ so $y_1(t)=e^{-3t}$ and $y_2(t)=e^{-2t}$

6. $6y''+y'-y=0$ , $y=e^{\lambda t}$

$6\lambda^2e^{\lambda t}+\lambda e^{\lambda t}-e^{\lambda t}$ $\quad y'=\lambda e^{\lambda t}$

$=e^{\lambda t}(6\lambda^2+\lambda-1)=0$ $\quad y''=\lambda^2e^{\lambda t}$

$\boxed{y(t)=C_1e^{-\frac{t}{2}}+C_2e^{\frac{t}{3}}}$

$(3\lambda-1)(2\lambda+1)$

$\lambda_1=-\frac{1}{2},\ \lambda_2=\frac{1}{3}$ so $y_1(t)=e^{-\frac{t}{2}}$ and $y_2(t)=e^{\frac{t}{3}}$

25. $y''-y'-2y=0$ , $y=e^{\lambda t}$ , $y(0)=-1=C_1e^{-(0)}+C_2e^{2(0)} \Rightarrow C_1+C_2=-1$

$\lambda^2e^{\lambda t}-\lambda e^{\lambda t}-2e^{\lambda t}$ $\quad y'=\lambda e^{\lambda t}$ $\quad y'(0)=2=-C_1e^{-(0)}+2C_2e^{2(0)} \Rightarrow -C_1+2C_2=2$

$=e^{\lambda t}(\lambda^2-\lambda-2)=0$ $\quad y''=\lambda^2e^{\lambda t}$ $\quad 3C_2=1$

$(\lambda-2)(\lambda+1)$ $\quad y(t)=C_1e^{-t}+C_2e^{2t}$ $\quad \boxed{y(t)=-\frac{4}{3}e^{-t}+\frac{1}{3}e^{2t}}$ $\quad C_2=\frac{1}{3}$

$y'(t)=-C_1e^{-t}+2C_2e^{2t}$

$\lambda_1=-1,\ \lambda_2=2$ so $y_1(t)=e^{-t}$ and $y_2(t)=e^{2t}$ $\quad C_1=-\frac{4}{3}$

37. a) If $\lambda_1$ & $\lambda_2$ roots,

$\lambda^2+p\lambda+q=(\lambda-\lambda_1)(\lambda-\lambda_2)$

$=\lambda^2-\lambda\overset{p}{(\lambda_1+\lambda_2)}+\overset{q}{\lambda_1\lambda_2}=0$

$p=(\lambda_1+\lambda_2)$

$\boxed{q=\lambda_1\lambda_2}$

b) Basically $\lambda^2-\lambda(\lambda_1+\lambda_2)+\lambda_1\lambda_2=0$

$\lambda^2+p\lambda+q=0$

$\boxed{p=-(\lambda_1+\lambda_2) \text{ also } -p=\lambda_1+\lambda_2}$