9/26/22

Homework 1

HW 2.1: #1, 7, 13, 14, 17

1. $\phi(x,y,z) = x^2 z + (1+x)y$

$\phi(t,y,y') = t^2 y' + (1+t)y = 0$

$y' = \frac{-(1+t)y}{t^2}$

7. $y(t) = 0 \qquad y' = y(4-y) = 0$

$\frac{d}{dt}(1+Ce^{-4t})$
$= C\frac{d}{dt}(e^{-4t})$
$= Ce^{-4t} \cdot -4$

But $y(t) = \frac{4}{1+Ce^{-4t}}$, $y'(t) = 4\frac{d}{dt}\left((1+Ce^{-4t})^{-1}\right) = 4\left(-\frac{1}{(1+Ce^{-4t})^2} \cdot \frac{d}{dt}(1+Ce^{-4t})\right)$

$= 4\left(-\frac{1}{(1+Ce^{-4t})^2} \cdot -4Ce^{-4t}\right) = \boxed{\frac{16Ce^{-4t}}{(1+Ce^{-4t})^2}}$ general solution

If you make $C = [1,5]$, can never be 0, so no C value will produce this

13. $y(1) = 2$ or $(1,2)$ initial condition

$y(1) = \frac{1}{3}(1)^2 + \frac{C}{1}$
$= \frac{1}{3} + C = 2$
$\boxed{C = \frac{5}{3}}$

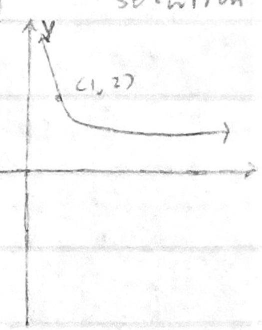
$ty' + y = t^2 \Rightarrow ty' = t^2 - y \Rightarrow y' = t - \frac{y}{t}$

LHS: $\frac{d}{dt}\left(\frac{1}{3}t^2 + Ct^{-1}\right) = \boxed{\frac{2}{3}t - \frac{C}{t^2}}$

RHS: $t - \frac{y}{t} = t - \left(\frac{\frac{t^2}{3} + \frac{C}{t}}{t}\right) = t - \frac{t}{3} - \frac{C}{t^2} \boxed{= \frac{2}{3}t - \frac{C}{t^2}}$

Same, verifies solution

$\boxed{y' = \frac{2}{3}t - \frac{5}{3t^2}, \quad y = \frac{1}{3}t^2 + \frac{5}{3t}}$



$\boxed{\text{I of E is } (0, \infty)}$

14. $y(1) = \frac{1}{e}$ or $(1, \frac{1}{e})$ IC

$y(1) = e^{-1}(1 + \frac{C}{1}) = \frac{1+C}{e} = \frac{1}{e}$, $\boxed{C = 0}$

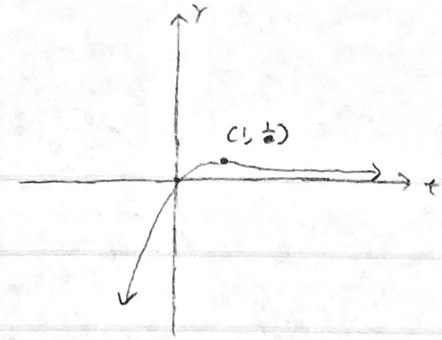
$ty' + (t+1)y = 2te^{-t} \Rightarrow ty' = 2te^{-t} - ty - y \Rightarrow y' = 2e^{-t} - y - \frac{y}{t}$

LHS: $\frac{d}{dt}(e^{-t}(t + \frac{C}{t})) = \frac{d}{dt}(te^{-t} + \frac{Ce^{-t}}{t}) = e^{-t} - te^{-t} + \frac{d}{dt}\left(\frac{Ce^{-t}}{t}\right)$

$= e^{-t} - te^{-t} + \cancel{C}\left(\frac{-e^{-t} \cdot t - e^{-t}}{t^2}\right) = \boxed{e^{-t} - te^{-t}}$

Same, verified

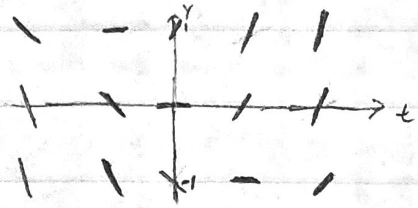
RHS: $2e^{-t} - e^{-t}(t + \cancel{\frac{C}{t}}) - \frac{e^{-t}(t + \frac{C}{t})}{t} = 2e^{-t} - e^{-t}t - e^{-t} = \boxed{e^{-t} - te^{-t}}$



$y' = e^{-t} - te^{-t}$, $y = e^{-t}t$

$\boxed{\text{I of E is } (-\infty, \infty)}$

17.



HW 2.2: #3, 8

3. $y' = e^{x-y} \Rightarrow \frac{e^x}{e^y}$

$\frac{dy}{dx} = \frac{e^x}{e^y} \Rightarrow e^y dy = e^x dx$

$\Rightarrow \int e^y dy = \int e^x dx \Rightarrow e^y = e^x + C$

$\boxed{\Rightarrow y = \ln(e^x + C)}$ ← implicit, depending on what C is graph changes

8. $y' = \frac{xy}{x-1} \Rightarrow \frac{dy}{dx} = \frac{xy}{x-1}$ $\quad w = x-1 \quad \frac{dw}{dx} = 1$

$\frac{1}{xy}dy = \frac{1}{x-1}dx \Rightarrow \frac{1}{y}dy = \frac{x}{x-1}dx \Rightarrow \int \frac{1}{y}dy = \int \frac{x}{x-1}dx \quad \int \frac{w+1}{w}dw$

$\Rightarrow \ln|y| + C = \int \frac{w+1}{w}dw = \int dw + \int \frac{1}{w}dw = w + \ln|w| = x - 1 + \ln|x-1|$

$\Rightarrow \ln|y| + C = x - 1 + \ln|x-1|$

$\Rightarrow \ln|y| = x - 1 + \ln|x-1| + C = \ln(e^{x-1+c}) + \ln|x-1| = \ln(e^{x-1+c} \cdot |x-1|)$

$\Rightarrow |y| = e^{x-1+c} \cdot |x-1| = e^{c-1}e^x|x-1|$

Since $|y|$ can't be negative, $y = e^{c-1}e^x|x-1|$

$A = e^{c-1}$, $\boxed{y = A|x-1|e^x}$