10/7/22

HW 2.2 #7, 15, 17, 18, 33

7. $\frac{dy}{dx} = \frac{x}{y+2}$

$(y+2)dy = x\,dx \Rightarrow \int (y+2)dy = \int x\,dx$

$\Rightarrow \frac{y^2}{2} + 2y = \frac{x^2}{2} + C \Rightarrow y^2 + 4y = x^2 + C$

$\Rightarrow y^2 + 4y - (x^2 + C) = 0$

$C = C + 4$

$\frac{-4 \pm \sqrt{16 + 4(x^2+C)}}{2} = \frac{-4 \pm \sqrt{16 + 4x^2 + C}}{2} = \frac{-4 \pm 2\sqrt{4 + x^2 + C}}{2} = \boxed{-2 \pm \sqrt{x^2 + C}}$

15. $y' = \frac{\sin x}{y}, \quad y\left(\frac{\pi}{2}\right) = 1$

$\frac{dy}{dx} = \frac{\sin x}{y}$

$y\left(\frac{\pi}{2}\right) = \sqrt{C - 2\cos\frac{\pi}{2}} = 1$

$\int y\,dy = \int \sin x\,dx$

$C = 1$

$\frac{y^2}{2} = -\cos x + C$

$\boxed{y(t) = \sqrt{1 - 2\cos t}}$ IofE $\boxed{\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)}$

or else $\sqrt{\ }$ will be $-$

$y^2 = -2\cos x + C$

$y = \sqrt{C - 2\cos x}$

17. $y' = 1 + y^2, \quad y(0) = 1$

$\frac{dy}{dx} = 1 + y^2$

$y(0) = \tan(0 + C) = 1$

$\int \frac{1}{1+y^2} dy = \int dx$

$C = \frac{\pi}{4}$

$\tan^{-1} y = x + C$

$\boxed{y(t) = \tan\left(x + \frac{\pi}{4}\right)}$

$y = \tan(x + C)$

$\boxed{\text{I of E} \quad \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)}$ So not divide by 0

18. $y' = \frac{x}{1+2y}, \quad y(-1) = 0$

$\frac{dy}{dx} = \frac{x}{1+2y}$

$y(-1) = \frac{-1 + \sqrt{2(-1)^2 + C}}{2} = 0$

$\int 1 + 2y\,dy = \int x\,dx$

$1 = \sqrt{2(-1)^2 + C}$

$y + y^2 = \frac{x^2}{2} + C$

$1 = C + 2 \Rightarrow C = -1$

$y^2 + y - \left(\frac{x^2}{2} + C\right) = 0$

$y = \frac{-1 \pm \sqrt{1 + 4\left(\frac{x^2}{2} + C\right)}}{2} = \frac{-1 \pm \sqrt{1 + 2x^2 + C}}{2} = \frac{-1 + \sqrt{2x^2 + C}}{2}$

$\boxed{y(x) = \frac{-1 + \sqrt{2x^2 - 1}}{2}}$

$T_2 = 29°C$ } 60
$T_1 = 31°C$
$T_0 = 37°C$ } $t = ?$
$A = 21°C$

33. $T(t) = A + (T_0 - A)e^{-kt}$ $\qquad \frac{dT}{dt} = -k(T-A)$

$$\frac{dT}{dt} = -k(T-A)$$
$$\frac{dT}{T-A} = -k\,dt$$
$$\int_{31}^{29} \frac{dT}{T-A} = \int_0^{60} -k\,dt$$
$$\ln|T-A|\Big|_{31}^{29} = -kt\Big|_0^{60}$$
$$\ln\frac{|29-21|}{|31-21|} = -k\cdot 60$$
$$k \approx 0.00372$$

$$T(t) = 21 + (37-21)e^{-0.00372t}$$
$$31 = 21 + (37-21)e^{-0.00372t}$$
$$10 = 16e^{-0.00372t}$$
$$\frac{5}{8} = e^{-0.00372t}$$
$$\ln\frac{5}{8} = -0.00372t$$
$$t \approx 126.35$$

$\boxed{9{:}54\,\text{pm}}$

HW 2.4 #3, 4, 10, 15, 19, 22, 37

3. $y' + \frac{2y}{x} = \frac{\cos x}{x^2}$ $\qquad a = -\frac{2}{x},\ f = \frac{\cos x}{x^2}$

$$y'x^2 + 2xy = \cos x \qquad u(t) = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = x^2$$
$$(x^2y)' = \cos x$$
$$x^2 y = \int \cos x\,dx = \sin x + C$$
$$\boxed{y = \frac{\sin x + C}{x^2}}$$

4. $y' + 2ty = 5t$ $\qquad a = -2t,\ f = 5t$

$$y'e^{t^2} + 2tye^{t^2} = 5te^{t^2} \qquad u(t) = e^{+\int 2t\,dt} = e^{+t^2} = e^{t^2}$$
$$(ye^{t^2})' = 5te^{t^2}$$
$$u = t^2 \quad \frac{du}{dt} = 2t$$
$$ye^{t^2} = \int 5te^{t^2}dt = 5\cdot\frac{1}{2}\int e^u du = \frac{5}{2}e^u + C = \frac{5}{2}e^{t^2} + C$$
$$ye^{t^2} = \frac{5}{2}e^{t^2} + C \Rightarrow \boxed{y = \frac{5}{2} + Ce^{-t^2}}$$

10. $y' - my = C_1e^{mx}$ $\qquad a = m,\ f = C_1e^{mx}$

$$y'e^{-mx} - mye^{-mx} = C_1 \qquad u(x) = e^{-\int m\,dx} = e^{-mx}$$
$$(ye^{-mx})' = C_1$$
$$ye^{-mx} = C_1x + C \Rightarrow \boxed{y = e^{mx}(C_1x + C)}$$

$$a = \frac{3x}{x^2+1}, \quad f = \frac{6x}{x^2+1}$$

15. $(x^2+1)y' + 3xy = 6x, \quad y(0) = -1$

$$y' + \frac{3x}{(x^2+1)} y = \frac{6x}{x^2+1}$$

$$w(x) = e^{\int \frac{3x}{x^2+1} dx} = e^{\frac{3}{2}\ln(x^2+1)} = (x^2+1)^{\frac{3}{2}}$$

$$y'(x^2+1)^{\frac{3}{2}} + 3xy\sqrt{x^2+1} = 6x\sqrt{x^2+1}$$

$$\left(y(x^2+1)^{\frac{3}{2}}\right)' = 6x\sqrt{x^2+1}$$

$$y(x^2+1)^{\frac{3}{2}} = \int 6x\sqrt{x^2+1}\,dx = 2(x^2+1)^{\frac{3}{2}} + C$$

$$y = 2 + C(x^2+1)^{-\frac{3}{2}}$$

$$y(0) = 2 + C(0+1)^{-\frac{3}{2}} = -1$$

$$C = -3$$

$$\boxed{y(x) = 2 - 3(x^2+1)^{-\frac{3}{2}}}$$

19. $(2x+3)y' = y + (2x+3)^{\frac{1}{2}}, \quad y(-1) = 0$

$$(2x+3)y' - y = \sqrt{2x+3}$$

$$a = \frac{1}{(2x+3)}, \quad f = \frac{1}{\sqrt{2x+3}}$$

$$y' - \frac{y}{(2x+3)} = \frac{1}{\sqrt{2x+3}}$$

$$w(x) = e^{-\int \frac{1}{2x+3} dx} = e^{-\frac{1}{2}\ln|2x+3|} = \frac{1}{\sqrt{2x+3}}$$

$$y' \cdot \frac{1}{\sqrt{2x+3}} - \frac{y}{(2x+3)^{\frac{3}{2}}} = \frac{1}{2x+3}$$

$$\left(\frac{y}{\sqrt{2x+3}}\right)' = \frac{1}{2x+3} \qquad u = 2x+3 \quad du = 2dx$$

$$\frac{y}{\sqrt{2x+3}} = \int \frac{1}{2x+3} dx = \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|2x+3| + C$$

$$y = \frac{1}{2}\sqrt{2x+3}\ln(2x+3) + C$$

$$y(-1) = \frac{1}{2}\sqrt{1}\,\cancel{\ln 1}^{0} + C = 0, \quad C = 0$$

$$\boxed{y(t) = \frac{1}{2}\sqrt{2x+3}\ln(2x+3)} \qquad \boxed{\text{I of E is } (-\tfrac{3}{2}, \infty)}$$

because less than $-\frac{3}{2}$ will make $\sqrt{\ }$ and ln undefined and $-\frac{3}{2}$ itself will also make ln undefined

22. $x' = a(t)x + f(t)x^n \qquad n \neq 0, 1$

$$z = x^{1-n}$$

$$\frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} = (1-n)x^{-n}(a(t)x + f(t)x^n) \boxed{= (1-n)(a(t)x^{1-n} + f(t))}$$

$y' = a(t)y + f(t)$

$a = -\frac{1}{2}, \; f = t$

37. $y' + \frac{1}{2}y = t, \quad y(0) = 1$

$y_h' = a(t)\, y_h = -\frac{1}{2} y_h$

$\hookrightarrow y_h(t) = e^{\int a(t)dt} = e^{\int -\frac{1}{2}dt} = e^{-\frac{t}{2}}$

$v = \frac{y}{y_h} \Rightarrow v y_h = y = ve^{-\frac{t}{2}}$

$y' + \frac{1}{2}y = t$

$(ve^{-\frac{t}{2}})' + \frac{1}{2}(ve^{-\frac{t}{2}}) = t$

$v'e^{-\frac{t}{2}} - \cancel{\frac{1}{2}e^{-\frac{t}{2}}v} + \cancel{\frac{1}{2}(ve^{-\frac{t}{2}})} = t$

$v'e^{-\frac{t}{2}} = t \Rightarrow v' = te^{\frac{t}{2}}$ $\quad w = \frac{t}{2} \quad \frac{dw}{dt} = \frac{1}{2} \Rightarrow 2dw = dt$

$v = \int te^{\frac{t}{2}}dt = 4\int e^{w} w\, dw = 4(e^{w}w - \int e^{w}dw)$

$= 4(e^{w}w - e^{w}) = 4e^{w}(w-1) = 4e^{\frac{t}{2}}(\frac{t}{2} - 1) + C$

$y(t) = v\, y_h = (4e^{\frac{t}{2}}(\frac{t}{2}-1) + C)(e^{-\frac{t}{2}})$

$= 4(\frac{t}{2} - 1) + Ce^{-\frac{t}{2}} = (2t - 4) + Ce^{-\frac{t}{2}}$

$y(0) = (-4) + Ce^{0} = 1 \Rightarrow C = 5$

$\boxed{y(t) = (2t-4) + 5e^{-\frac{t}{2}}}$

HW 2.5 #1

1. a. $V(t) = 100$ $\qquad \frac{dx}{dt} = \text{rate in} - \text{rate out} = 0.6 - \frac{3x}{100}$

$x(0) = 0$

$x(t) = 20 - \frac{20}{e^{\frac{3t}{100}}}$

$a = -\frac{3}{100}, \; f = 0.6$

$x(0) = 20 + C = 0 \Rightarrow C = -20$ $\qquad x' + \frac{3x}{100} = 0.6$ $\qquad w(t) = e^{+\int \frac{3}{100}dt} = e^{\frac{3t}{100}}$

$x(20) = 20 - \frac{20}{e^{\frac{60}{100}}}$ $\boxed{\approx 9.024 \text{ lbs.}}$ $\qquad x'e^{\frac{3t}{100}} + \frac{3x}{100}e^{\frac{3t}{100}} = 0.6e^{\frac{3t}{100}}$

$(xe^{\frac{3t}{100}})' = 0.6e^{\frac{3t}{100}}$

$xe^{\frac{3t}{100}} = \int 0.6e^{\frac{3t}{100}}dt = 0.6 \cdot \frac{100}{3}e^{\frac{3t}{100}} + C = 20e^{\frac{3t}{100}} + C$

b. $15 = 20 - \frac{20}{e^{\frac{3t}{100}}} \Rightarrow \frac{20}{e^{\frac{3t}{100}}} = 5 \Rightarrow e^{\frac{3t}{100}} = 4 \Rightarrow \frac{3t}{100} = \ln 4 \Rightarrow t = \frac{100\ln 4}{3}$

$\boxed{\approx 46.210 \text{ min.}}$

c. $x(\infty) = 20 - \cancel{\frac{20}{e^{\infty}}}$ $\boxed{= 20 \text{ lbs.}}$