10/1/22

Partial Differentiation

$f(x,y,z) = f(x(s,t), y(s,t), z(s,t))$

$x = x(s,t)$
$y = y(s,t)$
$z = z(s,t)$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t}$$

Ex: $R(x,y) = (x-2y)^3$

$x = w^2$, $y = v^w$

$$\frac{\partial R}{\partial v} = \frac{\partial R}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial v}$$

$$= 3(x-2y)^2 \cdot 0 + 3(x-2y)^2(-2)(w v^{w-1})$$

$$= -6(x-2y)^2(w v^{w-1})$$

$$\boxed{= -6(w^2 - 2v^w)^2(w v^{w-1})}$$

$$\frac{\partial R}{\partial w} = \frac{\partial R}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial w}$$

$$= 3(x-2y)^2(2w) + 3(x-2y)^2(-2)(\ln v)v^w$$

$$= 6w(x-2y)^2 - 6(x-2y)^2(\ln v)v^w$$

$$\boxed{= 6w(w^2 - 2v^w)^2 - 6(w^2 - 2v^w)^2(\ln v)v^w}$$