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Math 33B Midterm 2

1. $y'' - 2y' + 10y = 0 \qquad y(0) = 2 \qquad y'(0) = 3$

$\lambda^2 - 2\lambda + 10 = 0$

$\lambda = \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i \qquad y_1(t) = e^{at}\cos bt$

$a = 1 \qquad b = 3 \qquad y_2(t) = e^{at}\sin bt$

~~$y(t) = C_1 e^t \cos 3t + C_2 e^t \sin 3t$~~

$y'(t) = C_1 e^t \cos 3t + C_2 e^t \sin 3t - 3C_1 e^t \sin 3t + 3C_2 e^t \cos 3t$

$= C_1 e^t (\cos 3t - 3\sin 3t) + C_2 e^t (\sin 3t + 3\cos 3t)$

$y(0) = C_1 e^0 \cos 0 + C_2 e^0 \sin 0 = C_1 = 2$

$y'(0) = C_1 e^0(\cos 0 - 3\sin 0) + C_2 e^0(\sin 0 + 3\cos 0) = C_1 + 3C_2 = 3, \quad C_2 = \frac{1}{3}$

$\boxed{y(t) = 2e^t \cos 3t + \frac{1}{3} e^t \sin 3t}$

2. a. $y_1(t) = t \qquad y_2(t) = t^3 \qquad$ ~~$w(t)$~~ $= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t & t^3 \\ 1 & 3t^2 \end{vmatrix} = 3t^3 - t^3 = 2t^3$

$y_1'(t) = 1 \qquad y_2'(t) = 3t^2$

From the interval $0 < t < \infty$, $2t^3$ cannot be 0. If $w(t) \neq 0$, it means y_1 & y_2 are linearly independent solutions. Since y_1 & y_2 are linearly independent, they make a fundamental set of solutions.

b. $y'' - \frac{3}{t}y' + \frac{3}{t^2}y = t^2 \qquad y_p = v_1 y_1 + v_2 y_2 = -\frac{t^3}{6} \cdot t + t^3 \cdot \frac{t}{2}$

$y_1(t) = t, \quad y_2(t) = t^3$

$v_1' = \frac{-y_2 t^2}{w(t)} = \frac{-t^5}{2t^3} = \frac{-t^2}{2} \qquad v_1 = -\frac{t^3}{6} \qquad \Rightarrow \boxed{y_p = \frac{t^4}{3}}$

$v_2' = \frac{y_1 t^2}{w(t)} = \frac{t^3}{2t^3} = \frac{1}{2} \qquad v_2 = \frac{t}{2}$

3. $y'' + y' - 2y = e^t + 2t + \cos 3t$

$y_1'' + y_1' - 2y_1 = e^t \qquad y_2'' + y_2' - 2y_2 = 2t \qquad y_3'' + y_3' - 2y_3 = \cos 3t$

$y_1 = ate^t \qquad \boxed{y_1 = \frac{te^t}{3}} \qquad y_2 = at + b \qquad a - 2at + 2b = 2t \qquad y_3 = a\cos 3t + b\sin 3t$

$y_1' = ate^t + ae^t \qquad y_2' = a \qquad -2at = 2t \qquad a = -1 \qquad y_3' = -3a\sin 3t + 3b\cos 3t$

$y_1'' = ate^t + 2ae^t \qquad y_2'' = 0 \qquad a - 2b = 0 \qquad b = -\frac{1}{2} \qquad y_3'' = -9a\cos 3t - 9b\sin 3t$

$\boxed{y_2 = -t - \frac{1}{2}} \qquad -9a\cos 3t - 9b\sin 3t - 3a\sin 3t + 3b\cos 3t$

$ate^t + 2ae^t + ate^t + ae^t - 2ate^t = e^t \qquad -2a\cos 3t - 2b\sin 3t = \cos 3t$

$3ae^t = e^t \qquad (-11a + 3b)\cos 3t + (-3a - 11b)\sin 3t = \cos 3t$

$a = \frac{1}{3} \qquad -11a + 3b = 1 \qquad a = \frac{-11}{130}$

$\boxed{y(t) = \frac{te^t}{3} - t - \frac{1}{2} - \frac{11}{130}\cos 3t + \frac{3}{130}\sin 3t} \qquad -3a - 11b = 0 \qquad y_3 = \frac{-11}{130}\cos 3t + \frac{3}{130}\sin 3t$

$b = \frac{3}{130}$

4. $\boxed{b, e}$

$n = 0, 2, 4, 5$

5. a. underdamped $c < \omega_0$ $\boxed{n = 2}$ $y'' + 2cy' + \omega_0^2 y = 0$ $\omega_0 = \frac{1}{2}$

b. critical $c = \omega_0$ $\boxed{n = 4}$ $4y'' + ny' + y = 0$ $2c = \frac{n}{4}$

c. overdamped $c > \omega_0$ $\boxed{n = 5}$ $y'' + \frac{n}{4}y' + \frac{y}{4} = 0$ $c = \frac{n}{8}$

6. a. $\boxed{\text{True}}$

b. $\boxed{\text{False}}$

c. $\boxed{\text{True}}$