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HW 4.5 # 17, 22, 26, 29, 33, 37, 41

17. $y'' + 3y' + 4y = t^3$

$y(t) = a + bt + ct^2 + dt^3$

$y'(t) = b + 2ct + 3dt^2$

$y''(t) = 2c + 6dt$

$(2c + 6dt) + 3(b + 2ct + 3dt^2) + 4(a + bt + ct^2 + dt^3) = t^3$

$(4a + 3b + 2c) + (4b + 6c + 6d)t + (4c + 9d)t^2 + (4d)t^3 = t^3$

$4a + 3b + 2c = 0 \quad 4b + 6c + 6d = 0 \quad 4c + 9d = 0 \quad d = \frac{1}{4}$

$a = -\frac{9}{128} \quad b = \frac{15}{32} \quad c = -\frac{9}{16}$

$$\boxed{y(t) = -\frac{9}{128} + \frac{15}{32}t - \frac{9}{16}t^2 + \frac{1}{4}t^3}$$

22. $y'' + 4y' + 4y = 4 - t \qquad y(0) = -1, \; y'(0) = 0$

$\lambda^2 + 4\lambda + 4 = (\lambda + 2)^2 = 0$

$\lambda = -2$

$y(t) = y_p + y_h$

$y_p = at + b$

$y_p' = a$

$y_p'' = 0$

$0 + 4a + 4at + 4b = 4 - t$

$4a + 4b = 4$

$4at = -t$

$a = -\frac{1}{4}, \quad b = \frac{5}{4}$

$y_p = -\frac{t}{4} + \frac{5}{4}$

$y_h = C_1 y_1 + C_2 y_2 = (C_1 + C_2 t)e^{-2t}$

$y_1 = e^{\lambda t} = e^{-2t}$

$y_2 = te^{-2t}$

$y(t) = y_p + y_h = -\frac{t}{4} + \frac{5}{4} + (C_1 + C_2 t)e^{-2t}$

$y(0) = \frac{5}{4} + C_1 e^0 = -1, \quad C_1 = -\frac{9}{4}$

$y'(t) = -\frac{1}{4} + C_2 e^{-2t} - 2e^{-2t}(C_1 + C_2 t)$

$y'(0) = -\frac{1}{4} + C_2 - 2C_1, \quad C_2 = -\frac{17}{4}$

$$\boxed{y(t) = -\frac{t}{4} + \frac{5}{4} + \left(-\frac{9}{4} - \frac{17}{4}t\right)e^{-2t}}$$

26. $y'' + 4y = 4\cos 2t$ = Re($4e^{2it}$)

$4e^{2it} = (4\cos 2t) + i(4\sin 2t)$

$z'' + 4z = 4e^{2it}$

$z = ae^{2it}$

$z' = 2ai e^{2it}$

$z'' = -4a e^{2it}$

$(-4ae^{2it}) + 4(ae^{2it}) = 4e^{2it}$

DOESN'T WORK

$z = ate^{2it}$

$z' = ae^{2it}(1 + 2it)$

$z'' = 4ae^{2it}(i - t)$

$(4aie^{2it} - 4at e^{2it}) + 4ate^{2it} = 4e^{2it}$

$4ai = 4, \quad a = \frac{1}{i} = -i$

$z = -ite^{2it} = -it(\cos 2t + i\sin 2t) = t\sin 2t - it\cos 2t$

$\boxed{y = t\sin 2t}$, bc it's only real part

2a. $y'' + 6y' + 9y = 5e^{-3t}$

$\lambda^2 + 6\lambda + 9 = 0$

$(\lambda+3)^2 = 0$

$\lambda = -3$

$y_p = at^2e^{-3t}$

$y_p' = -3at^2e^{-3t} + 2ate^{-3t}$

$y_p'' = 9at^2e^{-3t} - 12ate^{-3t} + 2ae^{-3t}$

$9at^2e^{-3t} - 12ate^{-3t} + 2ae^{-3t} - 18at^2e^{-3t} + 12ate^{-3t} + 9at^2e^{-3t} = 5e^{-3t}$

$a = 2.5$

$\boxed{y_p(t) = 2.5t^2e^{-3t}}$

33. $y'' + 25y = 2 + 3t + \cos 5t$

$y_1 = at + b$

$y_1' = a$

$y_1'' = 0$

$25at + 25b = 2 + 3t$

$a = \frac{3}{25}, \quad b = \frac{2}{25}$

$y_1 = \frac{3}{25}t + \frac{2}{25}$

$\boxed{y(t) = \frac{3}{25}t + \frac{2}{25} + \frac{t}{10}\sin 5t}$

$z'' + 25z = 0$

$\lambda^2 + 25 = 0$

$\lambda = \pm 5i$

$z = Ate^{5it}$

$z' = 5Aite^{5it} + Ae^{5it}$

$z'' = -25Ate^{5it} + 10iAe^{5it}$

$-25Ate^{5it} + 10iAe^{5it} + 25Ate^{5it} = e^{5it}$

$A = -\frac{1}{10}i$

$z = -\frac{i}{10}te^{5it} = -\frac{it}{10}(\cos 5t + i\sin 5t)$

$y_2 = \frac{t}{10}\sin 5t$

37. $y'' + 4y' + 4y = e^{-2t} + \sin 2t$

$y_1 = \frac{t^2}{2}e^{-2t}$

$\boxed{y(t) = \frac{t^2}{2}e^{-2t} - \frac{1}{8}\cos 2t}$

$z'' + 4z' + 4z = 0$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda = -2$

$z = At^2e^{-2t}$

$z' = -2At^2e^{-2t} + 2Ate^{-2t}$

$z'' = 4At^2e^{-2t} - 8Ate^{-2t} + 2Ae^{-2t}$

$4At^2e^{-2t} - 8Ate^{-2t} + 2Ae^{-2t} - 8At^2e^{-2t} + 8Ate^{-2t} + 4At^2e^{-2t} = e^{-2t}$

$2A = 1, \quad A = \frac{1}{2}$

$\sin 2t = \mathrm{Im}(e^{2it})$

$z'' + 4z' + 4z = \sin 2t = e^{2it}$

$z = ae^{2it}$

$z' = 2aie^{2it}$

$z'' = -4ae^{2it}$

$-4ae^{2it} + 8aie^{2it} + 4ae^{2it} = e^{2it}$

$a = -\frac{i}{8}$

$z = -\frac{i}{8}e^{2it} = -\frac{i}{8}(\cos 2t + i\sin 2t) = -\frac{i}{8}\cos 2t + \cdots$ ↓

$y_2 = -\frac{1}{8}\cos 2t$

41. $y'' + 2y' + y = t^2e^{-2t}$

$y = (at^2 + bt + c)e^{-2t}$

$y' = (-2at^2 + (2a - 2b)t + (b - 2c))e^{-2t}$

$y'' = (4at^2 + (-8a + 4b)t + (2a - 4b + 4c))e^{-2t}$

Sub In: $4at^2 - 8at + 4bt + 2a - 4b + 4c - 4at^2 + 4at - 4bt + 2b - 4c + at^2 + bt + c =$

$at^2 - 4at + bt + 2a - 2b + c = t^2$

$a = 1$

$-4a + b = 0, \quad b = 4$

$2a - 2b + c = 0, \quad c = 6$

$\boxed{y = e^{-2t}(t^2 + 4t + 6)}$

HW 4.6 # 4, 11, 13

4. $x'' - 2x' - 3x = 4e^{3t}$

$(\lambda - 3)(\lambda + 1) = 0$

$x_1 = e^{-t}$

$x_2 = e^{3t}$

$x = v_1 x_1 + v_2 x_2 = v_1 e^{-t} + v_2 e^{3t}$

$x' = v_1' e^{-t} + v_2' e^{3t} - v_1 e^{-t} + 3v_2 e^{3t}$

$v_1' e^{-t} + v_2' e^{3t} = 0$

$x' = -v_1 e^{-t} + 3v_2 e^{3t}$

$x'' = -v_1' e^{-t} + 3v_2' e^{3t} + v_1 e^{-t} + 9v_2 e^{3t}$

$x'' - 2x' - 3x = -v_1' e^{-t} + 3v_2' e^{3t} = 4e^{3t}$

$v_1' e^{-t} + v_2' e^{3t} = 0$

$v_1' = -e^{4t}$ $\quad v_2' = 1$ $\quad v_1 = -\frac{e^{4t}}{4}$ $\quad v_2 = t$

$$\boxed{x(t) = -\frac{e^{3t}}{4} + te^{3t}}$$

11. $y'' + y = \tan t + \sin t + 1$

$y'' + y = 0$

$y_1 = \cos t$

$y_2 = \sin t$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = 1$$

$y_p = v_1 y_1 + v_2 y_2$

$= v_1 \cos t + v_2 \sin t$

$y_p' = \underbrace{(v_1' \cos t + v_2' \sin t)}_{=0} + (-v_1 \sin t + v_2 \cos t)$

$$v_1' = \frac{-\sin t(\tan t + \sin t + 1)}{1} = \cancel{\cos t \sec t} - \frac{1}{2} + \frac{1}{2}\cos 2t - \sin t$$

$$v_1 = \sin t - \ln|\sec t + \tan t| - \frac{1}{2}t + \frac{1}{4}\sin 2t + \cos t$$

$$v_2' = \frac{\cos t(\tan t + \sin t + 1)}{1} = \sin t + \frac{1}{2}\cancel{\sin 2t}\cos t$$

$$v_2 = \sin t - \cos t - \frac{1}{4}\cos 2t$$

$y_p = v_1 y_1 + v_2 y_2$

$$= \frac{1}{4}\sin t - \frac{1}{2}t\cos t - \cos t \ln|\sec t + \tan t| + 1$$

13. $t^2 y''(t) + 3t y'(t) - 3y(t) = 0$

$y''(t) + \frac{3}{t} y'(t) - \frac{3}{t^2} y(t) = 0$

$y_1 = t$

$y_2 = t^{-3}$

$y_1 = t$ $\quad y_2 = t^{-3}$

$y_1' = 1$ $\quad y_2' = -3t^{-4}$

$y_1'' = 0$ $\quad y_2'' = 12t^{-5}$

$y_1'' + \frac{3}{t} y_1' - \frac{3}{t^2} y_1 = 0 + \frac{3}{t} - \frac{3}{t} = 0$

$y_2'' + \frac{3}{t} y_2' - \frac{3}{t^2} y_2 = 12t^{-5} - 9t^{-5} - 3t^{-5} = 0$ ✓