9/23/22

Diff Equations

equations ↑

Diff Equations: Principles / laws underlying natural world are relations involving rates at which things happen.

rates ↓ derivatives

Equations w/ derivatives are diff. equations.

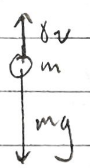
Ex: Newton's 2nd law

$$F = ma \Rightarrow F = m\frac{dv}{dt}$$

F: net force (N), m: mass (kg), a: acceleration (m/s^2)

- F = net force (N)
- m = mass (kg)
- a = acceleration ($\frac{m}{s^2}$)
- v = velocity ($\frac{m}{s}$)
- $\frac{dv}{dt} = a$

force due to air resistance (proportional to velocity)



$g = 9.81 \frac{m}{s^2}$

$F = mg - \gamma v = ma$

$$\boxed{mg - \gamma v = m\frac{dv}{dt}}$$

← mathematical model of object falling; m, g, γ constants

Ordinary Differential Equations (ODE)

involves unknown function of one variable w/ one or more of its derivatives **ex:** $\frac{dy}{dt} = y - t$ $y(t)$ is unknown function and t is ind. variable

other examples:

first order – $[\, y' = y - t \quad ty' = y$
$y' + 4y = e^{-3t} \quad y' = \cos(ty) \,]$

2nd order – $[\, yy'' + t^2 y = \cos t \quad y'' = y^2 \,]$

3rd order – $[\, y''' + 2e^t y'' + yy' = t^4 \,]$

- The order of a diff. equation is of highest derivative

Partial Differential Equations (PDE)

an equation w/ more than one unknown variable

ex: $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$ unknown function w is dependent on both t and x.

Any first order equation can be put in this form:

$$\phi(t, y, y') = 0$$

where ϕ is a function of 3 variables

General equation of order n written as:
$\phi(t, y, y', \cdots, y^{(n)}) = 0$
ϕ is a function of $n+2$ variables.

ex: $y' = y - t \Rightarrow y' - y + t = 0$

$$\phi(t, y, z) = z - y + t = 0$$

Useful to solve for highest derivative:

A first-order DE $y' = f(t, y)$ is said to be in normal form
Similarly, $y^{(n)} = f(t, y, y', \cdots, y^{(n-1)})$ is also in normal form

ex: Put $t + 4yy' = 0$ into normal form

solve for $\boxed{y' = -\frac{t}{4y}}$ - function of t and y
so $y' = f(t, y)$

Solutions

A solution of a first-order ODE $\phi(t, y, y') = 0$ is a diff. function $y(t)$ such that

$\phi(t, y(t), y'(t)) = 0$ for all t in the interval where $y(t)$ is defined

To determine if given func. is solution to a DE, just sub the function & its derivative into the equation