HW 9.1 # 3, 9, 12, 20, 25, 48

3. $A=\begin{pmatrix}-2 & 3\\ 0 & -5\end{pmatrix}$ $\quad A-\lambda I=\begin{vmatrix}-2-\lambda & 3\\ 0 & -5-\lambda\end{vmatrix}=(-2-\lambda)(-5-\lambda)=\boxed{(\lambda+5)(\lambda+2) \quad \lambda_1=-5 \quad \lambda_2=-2}$

9. $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 0 & 2\\ 0 & 3 & 1\end{pmatrix}$ $\quad A-\lambda I=\begin{vmatrix}1-\lambda & 2 & 3\\ 0 & -\lambda & 2\\ 0 & 3 & 1-\lambda\end{vmatrix}=1-\lambda\begin{vmatrix}-\lambda & 2\\ 3 & 1-\lambda\end{vmatrix}-2\begin{vmatrix}0 & 2\\ 0 & 1-\lambda\end{vmatrix}+3\begin{vmatrix}0 & -\lambda\\ 0 & 3\end{vmatrix}=(-1-\lambda)(-\lambda+\lambda^2)(6)$

$\boxed{=-6\lambda(\lambda+1)(\lambda-1) \quad \lambda=-1,0,1}$

12. $A=\begin{pmatrix}1 & 0 & -1\\ -2 & -1 & 3\\ -4 & 0 & 4\end{pmatrix}$ $\quad A-\lambda I=\begin{vmatrix}1-\lambda & 0 & -1\\ -2 & -1-\lambda & 3\\ -4 & 0 & 4-\lambda\end{vmatrix}=(1-\lambda)\begin{vmatrix}-1-\lambda & 3\\ 0 & 4-\lambda\end{vmatrix}-0\begin{vmatrix}-2 & 3\\ -4 & 4-\lambda\end{vmatrix}-\begin{vmatrix}-2 & -1-\lambda\\ -4 & 0\end{vmatrix}=\boxed{\lambda(\lambda+1)(\lambda-5) \quad \lambda=-1,0,5}$

20. $A=\begin{pmatrix}3 & -2\\ 4 & -3\end{pmatrix}$ $\quad A-\lambda I=\begin{vmatrix}3-\lambda & -2\\ 4 & -3-\lambda\end{vmatrix}=(-9+\lambda^2)+8=\lambda^2-1 \quad \lambda_1=-1,\ \lambda_2=1$

$A-\lambda_1 I=\begin{pmatrix}4 & -2\\ 4 & -2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $\quad 4x_1-2x_2=0$, $4x_1-2x_2=0$ $\quad 2x_1=x_2$ $\quad \vec{v}_1=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}x_1\\ 2x_1\end{pmatrix}=x_1\begin{pmatrix}1\\ 2\end{pmatrix}=\begin{pmatrix}1\\ 2\end{pmatrix}$

$A-\lambda_2 I=\begin{pmatrix}2 & -2\\ 4 & -4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $\quad 2x_1-2x_2=0$, $4x_1-4x_2=0$ $\quad x_1=x_2$ $\quad \vec{v}_2=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}x_1\\ x_1\end{pmatrix}=x_1\begin{pmatrix}1\\ 1\end{pmatrix}=\begin{pmatrix}1\\ 1\end{pmatrix}$

$\boxed{\vec{y}_1(t)=e^{-t}\begin{pmatrix}1\\ 2\end{pmatrix} \qquad \vec{y}_2(t)=e^{t}\begin{pmatrix}1\\ 1\end{pmatrix}}$

25. $A=\begin{pmatrix}-1 & 0 & 0\\ 2 & -5 & -6\\ -2 & 3 & 4\end{pmatrix}$ $\quad A-\lambda I=\begin{vmatrix}-1-\lambda & 0 & 0\\ 2 & -5-\lambda & -6\\ -2 & 3 & 4-\lambda\end{vmatrix}=-1-\lambda\begin{vmatrix}-5-\lambda & -6\\ 3 & 4-\lambda\end{vmatrix}=(-1-\lambda)((-5-\lambda)(4-\lambda)+18)=-(\lambda+1)(-20+5\lambda-4\lambda+\lambda^2+18)$

$=-(\lambda+1)(\lambda^2+\lambda-2)=-(\lambda+1)(\lambda+2)(\lambda-1) \quad \lambda_1=-2,\ \lambda_2=-1,\ \lambda_3=1$

$A-\lambda_1 I=\begin{pmatrix}1 & 0 & 0\\ 2 & -3 & -6\\ -2 & 3 & 6\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$ $\quad x_1=0$, $x_2+2x_3=0$, $x_2=-2$, $x_3=1$ $\quad \boxed{y_1(t)=e^{-2t}\begin{pmatrix}0\\ -2\\ 1\end{pmatrix}}$

$A-\lambda_2 I=\begin{pmatrix}0 & 0 & 0\\ 2 & -4 & -6\\ -2 & 3 & 5\end{pmatrix}=\begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$ $\quad x_1-x_3=0$, $x_2+x_3=0$ $\quad x_1=x_3$, $x_2=-x_3$ $\quad \vec{v}_2=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}x_3\\ -x_3\\ x_3\end{pmatrix}=\begin{pmatrix}1\\ -1\\ 1\end{pmatrix}$ $\quad \boxed{\vec{y}_2(t)=e^{-t}\begin{pmatrix}1\\ -1\\ 1\end{pmatrix}}$

$A-\lambda_3 I=\begin{pmatrix}-2 & 0 & 0\\ 2 & -6 & -6\\ -2 & 3 & 3\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$ $\quad x_1=0$, $x_2+x_3=0$ $\quad \vec{v}_3=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ x_2\\ -x_2\end{pmatrix}=x_2\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}$ $\quad \boxed{\vec{y}_3(t)=e^{t}\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}}$

48. If A is the matrix,

$Av=\lambda v$ and $Aw=\lambda w$

And if $y=av+bw$

$Ay=A(av+bw)$

$=(aAv+bAw)$

$=a(Av)+b(Aw)$

$=a(\lambda v)+b(\lambda w)$

$=\lambda(av+bw)=\lambda y$