HW 9.2 # 5, 7, 11, 15, 19, 25, 31, 37, 45

5. $A=\begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix}$ $\det(A-\lambda I)=\det\begin{pmatrix}1-\lambda & 2\\ -1 & 4-\lambda\end{pmatrix} = (1-\lambda)(4-\lambda)+2 = \lambda^2-5\lambda+6 = (\lambda-2)(\lambda-3)$

$\lambda_1=2$ $A-\lambda_1 I=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $\lambda_2=3$ $A-\lambda_2 I=\begin{pmatrix}-2 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$

$-x_1+2x_2=0$ $x_1=2x_2$ $-2x_1+2x_2=0$ $x_1=x_2$ $\vec{v}_2=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}x_1\\ x_1\end{pmatrix}=x_1\begin{pmatrix}1\\ 1\end{pmatrix}$

$-x_1+2x_2=0$ $-x_1+x_2=0$

$\vec{v}_1=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}2x_2\\ x_2\end{pmatrix}=x_2\begin{pmatrix}2\\ 1\end{pmatrix}=\begin{pmatrix}2\\ 1\end{pmatrix}$ $\vec{v}_2=\begin{pmatrix}1\\ 1\end{pmatrix}$

$\boxed{y(t)=C_1e^{2t}\begin{pmatrix}2\\ 1\end{pmatrix}+C_2e^{3t}\begin{pmatrix}1\\ 1\end{pmatrix}}$

7. $A=\begin{pmatrix}2 & -6\\ 0 & -1\end{pmatrix}$ $\det(A-\lambda I)=\det\begin{pmatrix}2-\lambda & -6\\ 0 & -1-\lambda\end{pmatrix}=(2-\lambda)(-1-\lambda)=\lambda^2-\lambda-2=(\lambda-2)(\lambda+1)$ $\lambda=-1,2$

$\lambda_1=-1$ $A+I=\begin{pmatrix}3 & -6\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $\lambda_2=2$ $A-2I=\begin{pmatrix}0 & -6\\ 0 & -3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$

$3x_1=6x_2$ $-6x_1=0$

$x_1=2x_2$

$\vec{v}_1=\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}2x_2\\ x_2\end{pmatrix}=\begin{pmatrix}2\\ 1\end{pmatrix}$ $-3x_2=0$

$\vec{v}_2=\begin{pmatrix}1\\ 0\end{pmatrix}$ $\boxed{y(t)=e^{-t}\begin{pmatrix}2\\ 1\end{pmatrix}-2e^{2t}\begin{pmatrix}1\\ 0\end{pmatrix}}$

$y(t)=C_1e^{-t}\begin{pmatrix}2\\ 1\end{pmatrix}+C_2e^{2t}\begin{pmatrix}1\\ 0\end{pmatrix}$ $2C_1+C_2=0$

$y(0)=\begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}C_1\\ C_2\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}$ $C_1=1$ $C_2=-2$

11. We know $y(t)=C_1e^{2t}\begin{pmatrix}2\\ 1\end{pmatrix}+C_2e^{3t}\begin{pmatrix}1\\ 1\end{pmatrix}$

$y(0)=\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}C_1\\ C_2\end{pmatrix}=\begin{pmatrix}3\\ 2\end{pmatrix}$ $2C_1+C_2=3$ $C_1+C_2=2$ $C_1=1$ $C_2=1$

$\boxed{y(t)=e^{2t}\begin{pmatrix}2\\ 1\end{pmatrix}+e^{3t}\begin{pmatrix}1\\ 1\end{pmatrix}}$

15. $z(t)=e^{3it}\begin{pmatrix}-1-i\\ 2\end{pmatrix}=(\cos 3t+i\sin 3t)\left[\begin{pmatrix}-1\\ 2\end{pmatrix}+i\begin{pmatrix}-1\\ 0\end{pmatrix}\right]=\cos 3t\begin{pmatrix}-1\\ 2\end{pmatrix}-\sin 3t\begin{pmatrix}-1\\ 0\end{pmatrix}+i\left(\cos 3t\begin{pmatrix}-1\\ 0\end{pmatrix}+\sin 3t\begin{pmatrix}-1\\ 2\end{pmatrix}\right)$

$\vec{y}_1(t)=Re(z(t))=\begin{pmatrix}-\cos 3t+\sin 3t\\ 2\cos 3t\end{pmatrix}$ $\vec{y}_2(t)=Im(z(t))=\begin{pmatrix}-\cos 3t-\sin 3t\\ 2\sin 3t\end{pmatrix}$

$\vec{y}_1'(t)=\begin{pmatrix}+3\sin 3t+3\cos 3t\\ -6\sin 3t\end{pmatrix}$ $\vec{y}_2'(t)=\begin{pmatrix}3\sin 3t-3\cos 3t\\ 6\cos 3t\end{pmatrix}$

$\boxed{\begin{pmatrix}3 & 3\\ -6 & -3\end{pmatrix}\begin{pmatrix}-\cos 3t+\sin 3t\\ 2\cos 3t\end{pmatrix}=\begin{pmatrix}3\sin 3t+3\cos 3t\\ -6\sin t\end{pmatrix}}$ $\boxed{\begin{pmatrix}3 & 3\\ -6 & -3\end{pmatrix}\begin{pmatrix}-\cos 3t-\sin 3t\\ 2\sin 3t\end{pmatrix}=\begin{pmatrix}3\sin 3t-3\cos 3t\\ 6\cos 3t\end{pmatrix}}$

$y_1(0)=\begin{pmatrix}-1\\ 2\end{pmatrix}$ $y_2(0)=\begin{pmatrix}-1\\ 0\end{pmatrix}$

$\det\begin{vmatrix}-1 & -1\\ 2 & 0\end{vmatrix}=-1+2=1$, so linearly ind. Since $y_1(0)$ and $y_1(0)$ are linearly independent, so is $y_1(t)$ and $y_2(t)$ & form fundamental set

19. $A=\begin{pmatrix}0 & 4\\ -2 & -4\end{pmatrix}$ $\det(A-\lambda I)=\begin{vmatrix}-\lambda & 4\\ -2 & -4-\lambda\end{vmatrix}=\lambda^2+4\lambda+8$ $\lambda=-2+2i$ $\bar{\lambda}=-2-2i$

$(A-\lambda I)=\begin{pmatrix}2-2i & 4\\ -2 & -2-2i\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $x_1=-1-i$ $x_2=1$ $z(t)=e^{(-2+2i)t}\begin{pmatrix}-1-i\\ 1\end{pmatrix}$

$=e^{-2t}(\cos 2t+i\sin 2t)\left(\begin{pmatrix}-1\\ 1\end{pmatrix}+i\begin{pmatrix}-1\\ 0\end{pmatrix}\right)$

$(2-2i)x_1+4x_2=0$ $=e^{-2t}\left(\cos 2t\begin{pmatrix}-1\\ 1\end{pmatrix}-\sin 2t\begin{pmatrix}-1\\ 0\end{pmatrix}\right)+ie^{-2t}\left(\cos 2t\begin{pmatrix}-1\\ 0\end{pmatrix}+\sin 2t\begin{pmatrix}-1\\ 1\end{pmatrix}\right)$

$(-2x_1+(2-2i)x_2=0)\,\Xi(1+i)$ $\boxed{y_1(t)=e^{-2t}\begin{pmatrix}-\cos 2t+\sin 2t\\ \cos 2t\end{pmatrix}}$

$(4-4i)x_1+(-2+2i+2i+2)x_2=0$ $\boxed{y_2(t)=e^{-2t}\begin{pmatrix}-\cos 2t-\sin 2t\\ \sin 2t\end{pmatrix}}$

25. we know $y_1(t) = e^{-2t}\begin{pmatrix}-\cos 2t + \sin 2t \\ \cos 2t\end{pmatrix}$ $\quad y_2(t) = e^{-2t}\begin{pmatrix}-\cos 2t - \sin 2t \\ \sin 2t\end{pmatrix}$

$$y(t) = C_1 e^{-2t}\begin{pmatrix}-\cos 2t + \sin 2t \\ \cos 2t\end{pmatrix} + C_2 e^{-2t}\begin{pmatrix}-\cos 2t - \sin 2t \\ \sin 2t\end{pmatrix}$$

$$y(0) = \begin{pmatrix}-1 & -1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}C_1 \\ C_2\end{pmatrix} = \begin{pmatrix}-1 \\ 2\end{pmatrix} \qquad -C_1 - C_2 = -1 \qquad C_1 = 2 \qquad C_2 = -1$$

$$\boxed{y(t) = 2e^{-2t}\begin{pmatrix}-\cos 2t + \sin 2t \\ \cos 2t\end{pmatrix} - e^{-2t}\begin{pmatrix}-\cos 2t - \sin 2t \\ \sin 2t\end{pmatrix}}$$

31. $A = \begin{pmatrix}3 & -1 \\ 1 & 1\end{pmatrix}$ $\quad \det(A - \lambda I) = \det\begin{pmatrix}3-\lambda & -1 \\ 1 & 1-\lambda\end{pmatrix} = (3-\lambda)(1-\lambda) + 1 = \lambda^2 - 4\lambda + 4, \quad \lambda = 2$

$A - 2I = \begin{pmatrix}1 & -1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}x_1 \\ x_2\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix}$ $\quad v_1 = \begin{pmatrix}1 \\ 1\end{pmatrix}$ $\quad y_1(t) = e^{2t}\begin{pmatrix}1 \\ 1\end{pmatrix}$

Use $\vec{v}_2 = \begin{pmatrix}1 \\ 0\end{pmatrix}$ to satisfy $(A - 2I)v_2 = v_1$

$\begin{pmatrix}1 & -1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix} = \begin{pmatrix}1 \\ 1\end{pmatrix} \checkmark$

$y_2(t) = e^{t}(v_2 + t v_1) = e^{t}\left(\begin{pmatrix}1 \\ 0\end{pmatrix} + t\begin{pmatrix}1 \\ 1\end{pmatrix}\right)$

$$\boxed{y(t) = C_1 e^{2t}\begin{pmatrix}1 \\ 1\end{pmatrix} + C_2 e^{t}\begin{pmatrix}1 \\ 0\end{pmatrix} + C_2 t e^{t}\begin{pmatrix}1 \\ 1\end{pmatrix}}$$

37. $y(0) = C_1\begin{pmatrix}1 \\ 1\end{pmatrix} + C_2\begin{pmatrix}1 \\ 0\end{pmatrix} = \begin{pmatrix}1 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}C_1 \\ C_2\end{pmatrix} = \begin{pmatrix}2 \\ -1\end{pmatrix}$ $\quad C_1 + C_2 = 2 \quad C_1 = -1 \quad C_2 = 3$

$$\boxed{y(t) = -e^{2t}\begin{pmatrix}1 \\ 1\end{pmatrix} + 3e^{t}\begin{pmatrix}1 \\ 0\end{pmatrix} + 3te^{t}\begin{pmatrix}1 \\ 1\end{pmatrix}}$$

45. $A = \begin{pmatrix}-4 & -5 \\ 2 & 2\end{pmatrix}$, $\det(A - \lambda I) = \det\begin{pmatrix}-4-\lambda & -5 \\ 2 & 2-\lambda\end{pmatrix} = \lambda^2 + 2\lambda + 2$, $\lambda = -1 + i$ $\quad \bar{\lambda} = -1 - i$

$A - \lambda I = \begin{pmatrix}-3-i & 5 \\ 2 & 3-i\end{pmatrix}\begin{pmatrix}x_1 \\ x_2\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix}$ $\quad x_1 = 5 \quad x_2 = -3-i$

$z(t) = e^{(-1+i)t}\begin{pmatrix}5 \\ -3-i\end{pmatrix} = e^{-t}(\cos t + i\sin t)\left[\begin{pmatrix}5 \\ -3\end{pmatrix} + i\begin{pmatrix}0 \\ -1\end{pmatrix}\right]$

$= e^{-t}\left(\cos t\begin{pmatrix}5 \\ -3\end{pmatrix} - \sin t\begin{pmatrix}0 \\ -1\end{pmatrix} + i\cos t\begin{pmatrix}0 \\ -1\end{pmatrix} + i\sin t\begin{pmatrix}5 \\ -3\end{pmatrix}\right)$

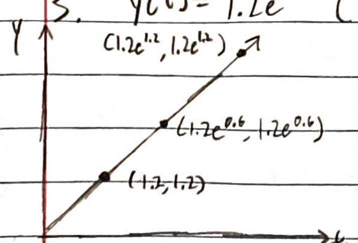
$$\boxed{y(t) = C_1 e^{-t}\left(\cos t\begin{pmatrix}5 \\ -3\end{pmatrix} - \sin t\begin{pmatrix}0 \\ 1\end{pmatrix}\right) + C_2 e^{-t}\left(\cos t\begin{pmatrix}0 \\ -1\end{pmatrix} + \sin t\begin{pmatrix}5 \\ -3\end{pmatrix}\right)}$$

# HW 9.3 #3, 7, 9, 12, 15, 16, 19, 20, 23

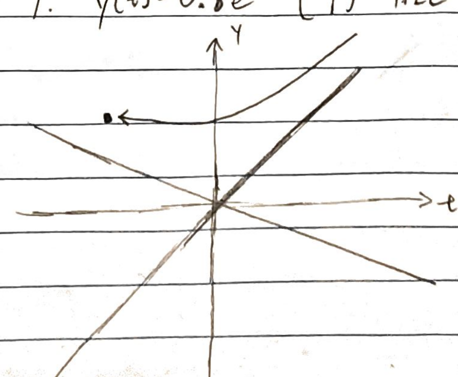
3. $y(t) = 1.2e^{0.6t}\begin{pmatrix}1 \\ 1\end{pmatrix}$ $\qquad y(0) = 1.2\begin{pmatrix}1 \\ 1\end{pmatrix}$

$y(1) = 1.2e^{0.6}\begin{pmatrix}1 \\ 1\end{pmatrix}$

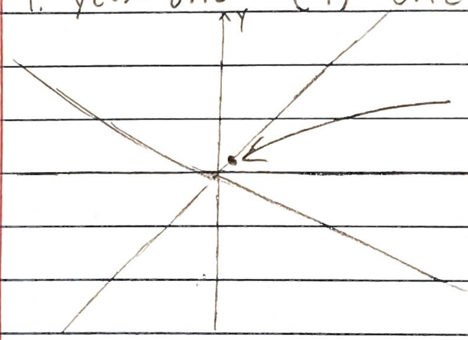
$y(2) = 1.2e^{1.2}\begin{pmatrix}1 \\ 1\end{pmatrix}$




7. $y(t) = 0.8e^{0.4t}\begin{pmatrix}-2 \\ 1\end{pmatrix} + 1.2e^{-0.2t}\begin{pmatrix}4 \\ 4\end{pmatrix}$

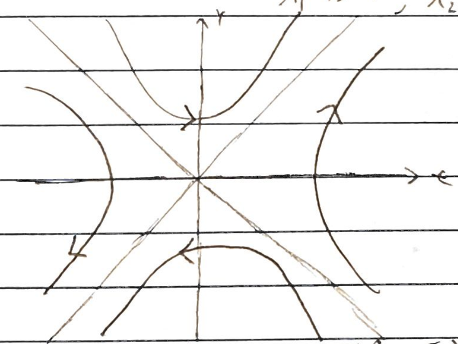


9. $y(t) = -0.4e^{-0.4t}\begin{pmatrix}-2\\1\end{pmatrix} + 0.4e^{-0.2t}\begin{pmatrix}4\\4\end{pmatrix}$



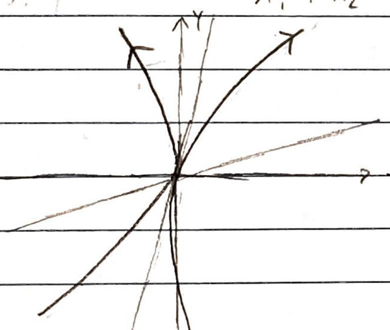

12. $Y(t) = C_1 e^{t}\begin{pmatrix}1\\1\end{pmatrix} + C_2 e^{-2t}\begin{pmatrix}1\\-1\end{pmatrix}$

$\lambda_1$ is +, $\lambda_2$ is −, $\boxed{\text{saddle}}$




15. $y(t) = C_1 e^{3t}\begin{pmatrix}4\\1\end{pmatrix} + C_2 e^{t}\begin{pmatrix}1\\5\end{pmatrix}$

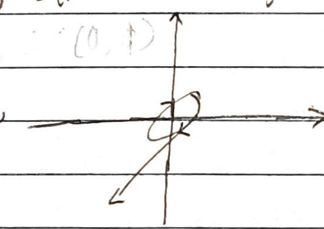
$\lambda_1$ & $\lambda_2$ +, $\boxed{\text{source}}$




16. $A = \begin{pmatrix}-4 & 8\\-4 & 4\end{pmatrix}$ $A - \lambda I = \begin{pmatrix}-4-\lambda & 8\\-4 & 4-\lambda\end{pmatrix} = \lambda^2 + 16$, $\lambda = \pm 4i$.

Since $\alpha$ in $\alpha \pm ib$ is 0, equilibrium at origin is a center.

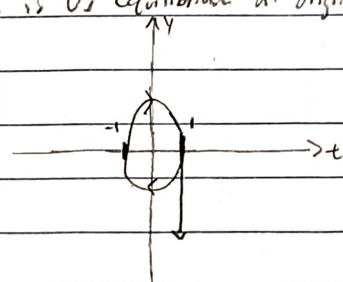
$\begin{pmatrix}-4 & 8\\-4 & 4\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-4\\-4\end{pmatrix}$,



19. $A = \begin{pmatrix}0 & 1\\-4 & 0\end{pmatrix}$ $A - \lambda I = \begin{pmatrix}-\lambda & 1\\-4 & -\lambda\end{pmatrix} = \lambda^2 + 4$, $\lambda = \pm 2i$

Since $\alpha$ in $\alpha \pm bi$ is 0, equilibrium at origin is a center

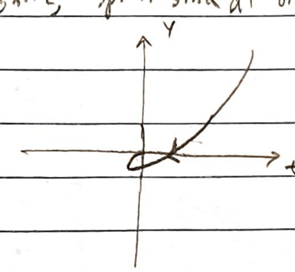
$\begin{pmatrix}0 & 1\\-4 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-4\end{pmatrix}$




20. $A = \begin{pmatrix}-2 & 2\\-1 & 0\end{pmatrix}$ $A - \lambda I = \begin{pmatrix}-2-\lambda & 2\\-1 & -\lambda\end{pmatrix} = \lambda^2 + 2\lambda + 2$, $\lambda = -1 \pm i$

Bc $\alpha$ in $\alpha \pm ib$ is negative, spiral sink at origin aka equilibrium point.

$\begin{pmatrix}-2 & 2\\-1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix}$




23. $A = \begin{pmatrix}-3 & 2\\-4 & 1\end{pmatrix}$ $A - \lambda I = \begin{pmatrix}-3-\lambda & 2\\-4 & 1-\lambda\end{pmatrix} = \lambda^2 + 2\lambda + 5$, $\lambda = -1 \pm 2i$

Bc $\alpha$ is negative, spiral sink to center at origin

$\begin{pmatrix}-3 & 2\\-4 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-3\\-4\end{pmatrix}$