10/19/22

HW 2.7 #5, 6, 9, 25, 27, 28

5. $x' = \frac{t}{x+1}$, $x(0) = 0$ $\qquad \frac{\partial f}{\partial t} = \frac{1}{x+1}$, continuous on R containing $(0,0)$

$f(t,x) = \frac{t}{x+1}$ $\qquad$ but not $x = -1$, unique solution

6. $y' = \frac{1}{x}y + 2$, $y(0) = 1$ $\qquad \frac{\partial f}{\partial x} = -\frac{y}{x^2}$, not unique because R can't contain

$f(x,y) = \frac{y}{x} + 2$ $\qquad (0,1)$ or else $-\frac{y}{0^2}$. Continuous except at $x=0$

9. $y(t) = 0$ & $y(t) = t^3$ are diff. solutions

$y' = 3y^{\frac{2}{3}}$, $y(0) = 0$

$f(t,y) = 3y^{\frac{2}{3}}$

$f'(t,y) = 2y^{-\frac{1}{3}}$ ← not continuous at $y = 0$, so Uniqueness Thm doesn't apply

25. $x_1(t) = t$ $\quad x_1(0) = 0$ $\qquad x_2(t) = \sin t$ $\quad x_2(0) = 0$

$x_1'(t) = f(t, x_1) = 1$ $\qquad x_2'(t) = f(t, x_2) = \cos t$

$\frac{\partial f}{\partial t} = 0$ $\qquad \frac{\partial f}{\partial t} = -\sin t$

No bc $1 \neq \cos t$

27. $x' = x\cos^2 t$ & $x(0) = 1$ $\qquad x' = x\cos^2 t$ is always positive bc

$f(t,x) = x\cos^2 t$ $\quad -\infty < t < \infty$ $\qquad x(t) = 0$ is a constant solution and $x(t)$ can't cross

$\frac{\partial f}{\partial t} = -x\sin(2t)$ $\qquad$ it and also since $x(0)=1$, $x'(0) = 1\cos^2 0$

~~$y(t) = 0$ is a constant solution~~ $\qquad = 1$, we know $x(t)$ is initially positive

and since $\cos^2 t$ is always positive or 0

so x' is always + or 0 and $x(t)$ is always increasing or remaining the same.

28. $y' = (y-3)e^{\cos(ty)}$ & $y(1) = 1$ $\qquad y(t) = 3$ is a constant solution that

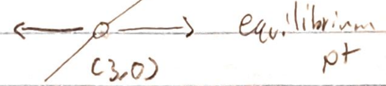
$f(t,y) = (y-3)e^{\cos(ty)}$ $\qquad y(t)$ can't cross or else fails

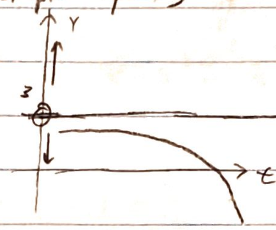
$-\infty < t < \infty$ $\qquad$ Uniqueness Thm. We know $3 > y(t)$

$y(t) = 3$ $\qquad$ because when $t=1$, $y(1) = 1$ so it's below 3 and can't cross above.

HW 2.9 # 7, 10, 12, 18, 21, 23, 27, 29

7. $y' = f(y)$ equilibrium pt: $y = 3$



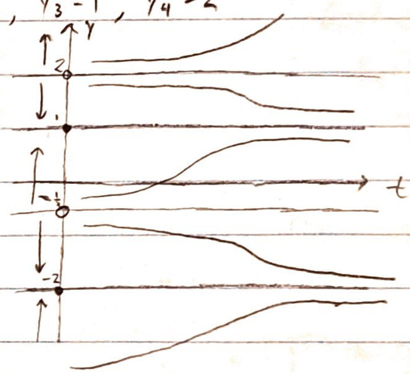


unstable bc slope is $-$ on left, $+$ on right

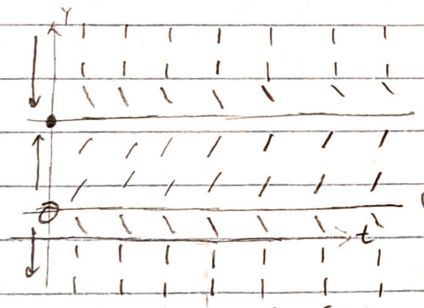
10. $y' = f(y)$ Equilibrium pts: $y_1 = -2$, $y_2 = -\frac{1}{2}$, $y_3 = 1$, $y_4 = 2$

$-\infty$ $+$ $y_1 = -2$ $-$ $y_2 = -\frac{1}{2}$ $+$ $y_3 = 1$ $-$ $y_4 = 2$ $+$ ∞

stable unstable stable unstable



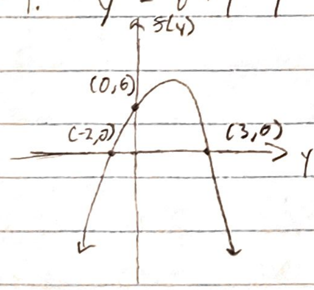
12.



stable

unstable

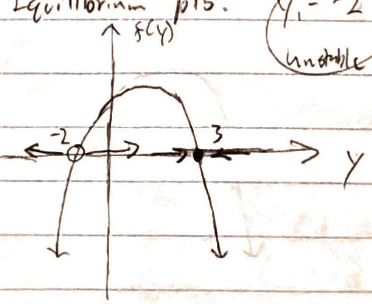
18. i. $y' = 6 + y - y^2 = f(y)$



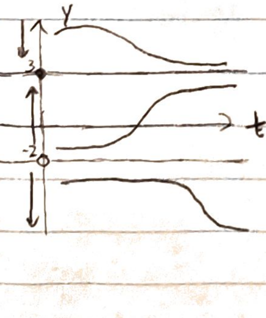
ii. $f(y) = (-y+3)(y+2)$

Equilibrium pts: $y_1 = -2$ $y_2 = 3$

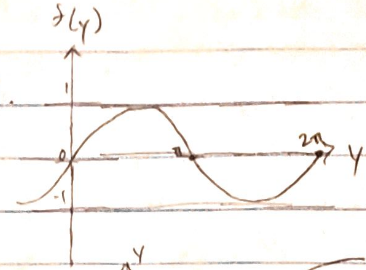
unstable stable



iii.

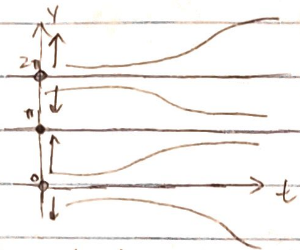


21. i.



ii. Equilibrium pts: $y_1 = 0$, $y_2 = \pi$, $y_3 = 2\pi$

iii.



23. i. $y' = 6 - y$, $y(0) = 2$ $\quad a = -1$

$y'e^t + ye^t = 6e^t$ $\quad u = e^{+\int 1\,dt} = e^t$

$(ye^t)' = 6e^t$

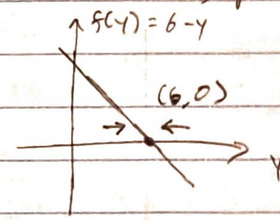
$ye^t = \int 6e^t\,dt = 6e^t + C$

$y = 6 + Ce^{-t}$ $\quad C = -4$ based on $y(0) = 2$

$\boxed{y(t) = 6 - 4e^{-t}}$

ii. $\lim_{t\to\infty} y(t) = 6$

iii. $f(y) = 6 - y$, phase line:



27. $f(x) = x' = 4 - x^2 = (2-x)(2+x)$

equilibrium pts: $x_1 = -2$, $x_2 = 2$

$f'(x) = -2x$

$$\boxed{\begin{array}{l} f(x_1) = f(-2) = 4 \leftarrow \text{unstable} \\ f(x_2) = f(2) = -4 \leftarrow \text{stable} \end{array}}$$

29. a) $f(x) = x^2$ & $f(x) = x^3$

b) $f(x) = -x^3$ & $f(x) = -x^5$