Arthur Zhou
11/1/22

HW 4.3 # 12, 15, 19, 22, 29, 31, 34, 38

$y = e^{\lambda t}$

$y' = \lambda e^{\lambda t}$

$y'' = \lambda^2 e^{\lambda t}$

12. $y'' + 2y' + 17y = 0$

$\lambda^2 + 2\lambda + 17 = 0$

$\lambda = \frac{-2 \pm \sqrt{4 - 68}}{2} = -1 \pm 4i$ $\quad a=-1 \quad b=4$ $\quad \lambda_1 = -1 - 4i, \ \lambda_2 = -1 + 4i$

$z(t) = e^{at}(\cos(bt) + i\sin(bt))$

$\bar{z}(t) = e^{at}(\cos(bt) - i\sin(bt))$

$\boxed{y(t) = A_1 e^{-t}\cos(4t) - A_2 e^{-t}\sin(4t)}$

15. $y'' - 2y' + 4y = 0$

$\lambda^2 - 2\lambda + 4 = 0$

$\lambda = \frac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm i\sqrt{3}$ $\quad a=1 \quad b=\sqrt{3}$

$\boxed{y(t) = A_1 e^{t}\cos t\sqrt{3} - A_2 e^{t}\sin t\sqrt{3}}$

19. $4y'' + 4y' + y = 0$

$4\lambda^2 + 4\lambda + 1 = 0$

$(2\lambda + 1)^2 = 0, \ \lambda = -\frac{1}{2}$

$\boxed{y(t) = (C_1 + C_2 t)e^{-\frac{t}{2}}}$

22. $y'' + 4y' + 4y = 0$

$\lambda^2 + 4\lambda + 4 = (\lambda + 2)^2 = 0, \ \lambda = -2$

$\boxed{y(t) = (C_1 + C_2)e^{-2t}}$

29. $y'' + 10y' + 25y = 0$

$\lambda^2 + 10\lambda + 25 = (\lambda + 5)^2 = 0$

$\lambda = -5$

$y(t) = (C_1 + C_2 t)e^{-5t}$ $\quad y(0) = C_1 = 2$

$y'(t) = C_2 e^{-5t} - 5(C_1 + C_2 t)e^{-5t}$ $\quad y'(0) = C_2 - 5C_1 = -1$

$C_1 = 2, \ C_2 = 9$

$\boxed{y(t) = (2 + 9t)e^{-5t}}$

$y(0) = 1$

$y'(0) = 0$

31. $y'' + 2y' + 3y = 0$

$\lambda^2 + 2\lambda + 3 = 0$

$\lambda = \frac{-2 \pm \sqrt{4 - 12}}{2} = -1 \pm i\sqrt{2}$ $\quad a=-1 \quad b=\sqrt{2}$

$y(t) = A_1 e^{-t}\cos t\sqrt{2} + A_2 e^{-t}\sin t\sqrt{2}$

$y'(t) = e^{-t}(-A_1\sqrt{2}\cos t\sqrt{2} + A_2\sqrt{2}\sin t\sqrt{2}) - e^{-t}(A_1 \sin t\sqrt{2} + A_2 \cos t\sqrt{2})$

$y(0) = A_1 = 1 \qquad A_2 = \frac{1}{\sqrt{2}}$

$y'(0) = -A_1 + \sqrt{2}A_2 = 0$

$\boxed{y(t) = e^{-t}\cos t\sqrt{2} + \frac{1}{\sqrt{2}}e^{-t}\sin t\sqrt{2}}$

$y(1) = 0$

$y'(1) = -2$

34. $4y'' + y = 0$

$4\lambda^2 + 1 = 0$

$\lambda = \pm \frac{1}{2}$

$y(t) = C_1 \cancel{e^0} \cos\frac{t}{2} + C_2 \cancel{e^0} \sin\frac{t}{2} = C_1 \cos\frac{t}{2} + C_2 \sin\frac{t}{2}$

$y'(t) = -\frac{C_1}{2}\sin\frac{t}{2} + \frac{C_2}{2}\cos\frac{t}{2}$

$y(1) = C_1 \cos\frac{1}{2} + C_2 \sin\frac{1}{2} = 0$

$y'(1) = -\frac{C_1}{\cancel{2}}\sin\frac{1}{2} + \frac{C_2}{\cancel{2}}\cos\frac{1}{2} = \cancel{-2}\ -4$

$C_1 \cos\frac{1}{2} - C_1 \sin\frac{1}{2} + C_2 \sin\frac{1}{2} + C_2 \cos\frac{1}{2} = -4$

$C_1(\cos\frac{1}{2} - \sin\frac{1}{2}) + C_2(\sin\frac{1}{2} + \cos\frac{1}{2}) = -4$

$C_1 = 4\sin\frac{1}{2} \qquad C_2 = -4\cos\frac{1}{2}$

$y(t) = 4\sin\frac{1}{2}\cos\frac{t}{2} - 4\cos\frac{1}{2}\sin\frac{t}{2}$

$\boxed{= 4\sin\left(\frac{1}{2} - \frac{t}{2}\right)}$

$\lambda^2 + p\lambda + q = 0$ $\quad p = -2\lambda_1$

$(\lambda - \lambda_1)(\lambda - \lambda_1) = 0$ $\quad q = \lambda_1^2$

$\lambda^2 - 2\lambda_1\lambda + \lambda_1^2 = 0$

38. $y = te^{\lambda_1 t}$

$y' = \lambda_1 te^{\lambda_1 t} + e^{\lambda_1 t}$

$y'' = \lambda_1^2 te^{\lambda_1 t} + \lambda_1 e^{\lambda_1 t} + \lambda_1 e^{\lambda_1 t}$

$= \lambda_1^2 te^{\lambda_1 t} + 2\lambda_1 e^{\lambda_1 t}$

$y'' + py' + qy = \lambda_1^2 te^{\lambda_1 t} + 2\lambda_1 e^{\lambda_1 t} + p(\lambda_1 te^{\lambda_1 t} + e^{\lambda_1 t}) + q(te^{\lambda_1 t})$

$= \lambda_1^2 te^{\lambda_1 t} + 2\lambda_1 e^{\lambda_1 t} - 2\lambda_1^2 te^{\lambda_1 t} - 2\lambda_1 e^{\lambda_1 t} + \lambda_1^2 te^{\lambda_1 t}$ $\boxed{= 0}$

HW 4.4 # 8, 9, 11, 13, 20

8. $y = e^{-\frac{t}{4}}(\sqrt{3}\cos 4t - \sin 4t)$ $\qquad A: \sqrt{3+1} = 2$

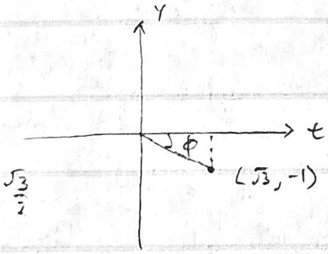
$y = 2e^{-\frac{t}{4}}\left(\frac{\sqrt{3}}{2}\cos 4t - \frac{1}{2}\sin 4t\right)$ $\qquad \phi: -\frac{\pi}{6}$

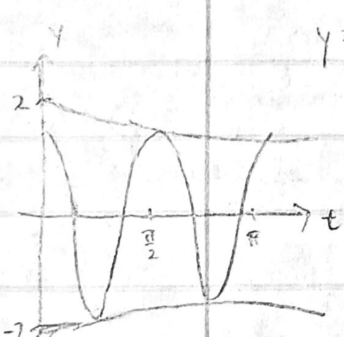
$\sin\phi = -\frac{1}{2} \quad \cos\phi = \frac{\sqrt{3}}{2}$

$= 2e^{-\frac{t}{4}}(\cos\phi\cos 4t + \sin\phi\sin 4t)$

$= 2e^{-\frac{t}{4}}\cos(4t - \phi)$ $\boxed{= 2e^{-\frac{t}{4}}\cos\left(4t + \frac{\pi}{6}\right)}$

$T = \frac{2\pi}{4} = \frac{\pi}{2}$





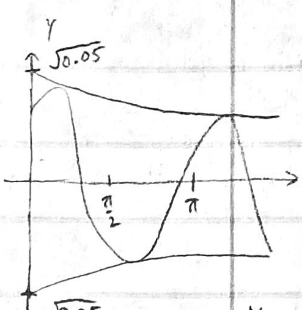
9. $y = e^{-0.1t}(0.2\cos 2t + 0.1\sin 2t)$ $\qquad A = \sqrt{0.2^2 + 0.1^2} = \sqrt{0.05}$

$= \sqrt{0.05}\, e^{-0.1t}\left(\frac{0.2}{\sqrt{0.05}}\cos 2t + \frac{0.1}{\sqrt{0.05}}\sin 2t\right)$ $\qquad \phi = \frac{0.1}{0.2} = 0.46$

$= \sqrt{0.05}\, e^{-0.1t}(\cos\phi\cos 2t + \sin\phi\sin 2t)$

$= \sqrt{0.05}\, e^{-0.1t}\cos(2t - \phi)$ $\boxed{= \sqrt{0.05}\, e^{-0.1t}\cos(2t - 0.46)}$ $\qquad T = \frac{2\pi}{\omega} = \pi$

$y = \sqrt{0.05}\, e^{-0.1t}, \quad y = -\sqrt{0.05}\, e^{-0.1t}$



11. $my'' + \cancel{\mu y'}^{\,0} + ky = 0$ $\qquad z(t) = \underset{y_1}{\cos 5t} + i\underset{y_2}{\sin 5t}$ $\qquad m = 0.2\text{ kg}$, $k = 5\frac{\text{kg}}{\text{s}^2}$

$y(0) = 0.5\text{ m}$, $y'(0) = 0\ \frac{\text{m}}{\text{s}}$

$0.2y'' + 5y = 0$ $\qquad y(t) = C_1\cos 5t + C_2\sin 5t$

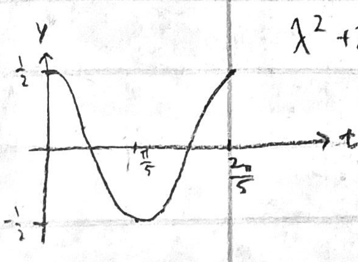
$y'' + 25y = 0$ $\qquad y'(t) = -5C_1\sin 5t + 5C_2\cos 5t$

$\lambda^2 + 25 = 0, \quad \lambda = \pm 5i$
$a = 0, b = 5$

$y(0) = C_1\cos(0) = 0.5, \quad C_1 = 0.5$

$y'(0) = 5C_2\cos(0) = 0, \quad C_2 = 0$

$\boxed{y(t) = 0.5\cos 5t}$ $\qquad T = \frac{2\pi}{5}$



$x(0)=2$
$x'(0)=v_0$

13. $\frac{2}{5}x''+kx=0$ $\quad \omega_0^2=\frac{5k}{2}$ $\quad T=\frac{2\pi}{\omega_0}=\frac{\pi}{2},\ \omega_0=4$

$x''+\frac{5k}{2}x=0$ $\quad 16=\frac{5k}{2}$

$x''+\omega_0^2x=0$ $\quad \boxed{k=6.4\ \frac{kg}{s^2}}$

$x''+2\zeta x'+\omega_0^2x=f(t)$

$x''+16x=0$

$\lambda^2+16=0,\ \lambda=\pm4i$
$a=0,\ b=4$

$z(t)=e^{ai}(\cos bt+i\sin bt)$
$=\cos4t+i\sin4t$

$x(t)=C_1\cos4t+C_2\sin4t$ $\quad x(0)=C_1\cos(0)=2,\ C_1=2$

$x'(t)=-4C_1\sin4t+4C_2\cos4t$ $\quad x'(0)=4C_2\cos(0)=v_0,\ C_2=\frac{v_0}{4}$

$x(t)=2\cos4t+\frac{v_0}{4}\sin4t$ $\quad A=2$ $\quad \sqrt{2^2+(\frac{v_0}{4})^2}=2$

$2^2+(\frac{v_0}{4})^2=2^2$

$\boxed{v_0=0\ \frac{m}{s}}$

20. a. $x''+\mu x'+4x=0$

$x''+4x'+4x=0$

$\lambda^2+4\lambda+4=0$

$(\lambda+2)^2=0$, repeat root so it's crit dampened

$x(0)=2m,\ x'(0)=1\frac{m}{s}$
$m=1kg$
$k=4\frac{kg}{s^2}$

b. $\mu=4$ $\quad x''+4x'+4x=0$

$(\lambda+2)^2=0,\ \lambda=-2$

$x(t)=C_1e^{-2t}+C_2te^{-2t}$

$x'(0)=-2C_1e^{-2t}+C_2e^{-2t}-2C_2te^{-2t}$

$x(0)=C_1=2$

$x'(0)=-2C_1+C_2=1,\ C_2=5$

$\boxed{x(t)=2e^{-2t}+5te^{-2t}}$

$\mu=5$ $\quad x''+5x'+5x=0$ $\quad \boxed{\text{overdampened}}$

$\lambda=\frac{-5\pm\sqrt{25-20}}{2}=\frac{-5\pm\sqrt{5}}{2}$

$\lambda_1\approx-3.62,\ \lambda_2\approx-1.38$

$x(t)=C_1e^{-3.62t}+C_2e^{-1.38t}$

$x'(t)=-3.62C_1e^{-3.62t}-1.38C_2e^{-1.38t}$

$x(0)=C_1+C_2=2$

$x'(0)=-3.62C_1-1.38C_2=1$

$1.38(C_1+C_2=2)$

$-3.62C_1-1.38C_2=1$

$-2.24C_1=3.76$

$C_1\approx-1.68$ $\quad C_2=3.68$

$\boxed{x(t)=-1.68e^{-3.62t}+3.68e^{-1.38t}}$

What's special abt crit damp?
It's the fastest curve that reaches the x-axis (or t axis).

Why do you want to adjust spring?
To let the door close faster

HW 4.5 #3, 6, 7, 9, 11, 12

3. $y''+2y'+5y=12e^{-t}$

$y=ae^{-t}$

$y'=-ae^{-t}$

$y''=ae^{-t}$

$ae^{-t}-2ae^{-t}+5ae^{-t}=12e^{-t}$

$4a=12$

$a=3$

$\boxed{y=3e^{-t}}$

6. $y''+9y=\sin 2t$

$y(t)=a\cos 2t+b\sin 2t$

$y'(t)=-2a\sin 2t+2b\cos 2t$

$y''(t)=-4a\cos 2t-4b\sin 2t$

$-4a\cos 2t-4b\sin 2t+9a\cos 2t+9b\sin 2t=\sin 2t$

$5a\cos 2t+5b\sin 2t=0\cos 2t+\sin 2t$

$a=0,\ b=\frac{1}{5}$

$\boxed{y(t)=\frac{\sin 2t}{5}}$

7. $y''+7y'+6y=3\sin 2t$

$y(t)=a\cos 2t+b\sin 2t$

$y'(t)=-2a\sin 2t+2b\cos 2t$

$y''(t)=-4a\cos 2t-4b\sin 2t$

$-4a\cos 2t-4b\sin 2t-14a\sin 2t+14b\cos 2t+6a\cos 2t+6b\sin 2t=3\sin 2t$

$\cos 2t(2a+14b)+\sin 2t(-14a+2b)=0\cos 2t+3\sin 2t$

$2a+14b=0$ $\Rightarrow$ $a+7b=0$ $\quad a=-0.21$

$-14a+2b=3$ $\quad -7a+b=1.5$ $\quad b=0.03$

$\boxed{y(t)=-0.21\cos 2t+0.03\sin 2t}$

9. $z''+pz'+qz=Ae^{i\omega t}$

$z(t)=x(t)+iy(t)$

$z'(t)=x'(t)+iy'(t)$

$z''(t)=x''(t)+iy''(t)$

$x''+iy''+px'+ipy'+qx+iqy=Ae^{i\omega t}=A(\cos\omega t+i\sin\omega t)$

$\underline{(x''+px'+qx)}+i\,\underline{(y''+py'+qy)}=A\underline{\cos\omega t}+A\underline{i\sin\omega t}$

they are equivalent

$\therefore$ $Z(t)=x(t)+iy(t)$ is a solution to $z''+pz'+qz=Ae^{i\omega t}$. The x and y DE make up real & imaginary part respectively.

11. $y''+9y=\sin 2t$

$=\mathrm{Im}(e^{2it})$

$z''+9z=e^{2it}$

$-4ae^{2it}+9ae^{2it}=e^{2it}$

$5ae^{2it}=e^{2it}$

$5a=1,\ a=\frac{1}{5}$

$Z(t)=ae^{2it}$

$z'(t)=2aie^{2it}$

$z''(t)=-4ae^{2it}$

$\boxed{z(t)=\frac{1}{5}e^{2it}}$

$\boxed{y(t)=\frac{1}{5}\sin 2t}$

12. $y'' + 7y' + 6y = 3\sin 2t$ $= \mathrm{Im}(3e^{2it})$

$z = ae^{2it}$
$z' = 2aie^{2it}$
$z'' = -4ae^{2it}$

$z'' + 7z' + 6z = 3e^{2it}$

$-4ae^{2it} + 14aie^{2it} + 6ae^{2it} = 3e^{2it}$

$2a + 14ai = 3$

$a = \frac{3}{2+14i} = \frac{3}{100} - \frac{21}{100}i$

$z(t) = (0.03 - 0.21i)e^{2it}$

Euler's $= (0.03 - 0.21i)(\cos 2t + i\sin 2t)$

$= (0.03\cos 2t + 0.21\sin 2t) + (-0.21\cos 2t + 0.03\sin 2t)i$

$y(t) = \mathrm{Im}(z(t)) \boxed{= -0.21\cos 2t + 0.03\sin 2t}$