9/26/22

Ex: Show $y(t) = t+1$ is a solution to $y' = y - t$ by substitution

$y' = y - t$ Since $y(t) = t+1$,

$y' = 1$ and $y - t = (t+1) - t = 1$

* process of verifying a solution for a function is important

Ex: Show $y(t) = Ce^{-t^2}$ is a solution to $y' = -2ty$

$y' = \frac{d}{dt}(Ce^{-t^2}) = Ce^{-t^2}(-2t) = -2tCe^{-t^2}$

$-2ty = -2t(Ce^{-t^2})$ Same, so it is solution

Both $y(t)$ and $y'(t)$ is defined on $(-\infty, \infty)$. Since $C \in \mathbb{R}$, $y = Ce^{-t^2}$ is a solution of the equation on $(-\infty, \infty)$

☆ ie. $y(t) = Ce^{-t^2}$ gives a different solution to each value of C. Because of this...

$y(t) = Ce^{-t^2}$ is called the general solution of $y' = -2ty$ and the graphs of solutions are called solution curves.

Ex:

Ex: Is $y(t) = \cos t$ a solution to $y' = 1 + y^2$

LHS: $y' = -\sin t$

RHS: $1 + y^2 = 1 + \cos^2 t$

$-\sin t \neq 1 + \cos^2 t$ for most values of

Not a solution

Initial Value Problem (IVP): When constant C is undetermined, ODE has ∞ solutions. You need more info to specify solution completely. This solution is called a particular solution

Ex: If $y(t) = \frac{-1}{t-c}$ is a general solution of $y' = y^2$, find a particular solution for $y(0) = 1$

$$y(0) = \frac{-1}{0-c} = \frac{1}{c} = 1 \Rightarrow c = 1.$$

IVP [So $\boxed{y(t) = \frac{-1}{t-1}}$ is particular solution of $y = y^2$ satisfying $y(0) = 1$

A first-order DE w/ an initial condition:

$$y' = f(t), \quad \underline{y(t_0) = y_0}$$

initial condition (IC)

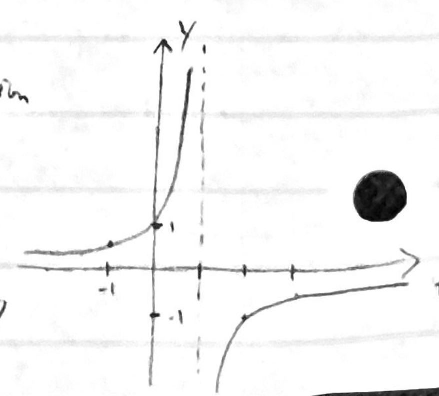
is called an initial value problem

A solution of initial value problem is a diff. function $y(t)$ such that

1. $y'(t) = f(t, y(t))$ for all t in an interval containing t_0 where $y(t)$ is defined
2. $y(t_0) = y_0$

Interval of Existence: the largest interval over which a solution to a DE can be defined and remain a solution. Since solutions to DE's are differentiable, they are CONTINUOUS

Ex: Find interval of existence for solution to IVP. $y' = y^2$ with $y(0) = 1$



prev. example: solution: $y(t) = \frac{-1}{t-1}$ →

Left branch of hyperbola passes $(0,1)$ satisfying IC of $y(0)=1$

$\Rightarrow$ $\boxed{\text{left branch of solution curve needed}}$ ie. left branch is answer b/c it passes thru $(0,1)$

- extends indefinitely to the left, but approaches ∞ as it approaches $t=1$ from left. So

$\boxed{\text{I of E: } (-\infty, 1)}$

<u>Note:</u> Can use variables other than y

<u>Ex:</u> $y' = x + y$ is in form $y' = f(x,y)$ making x ind. variable and requires a solution y that's a function of x. Has general solution $\boxed{y(x) = -1 - x + Ce^{x}}$ which exists on $(-\infty, \infty)$

<u>Ex:</u> verify $x(s) = 2 - Ce^{-s}$ is a solution to $x' = 2 - x$ for any constant C

LHS: $x' = Ce^{-s}$

RHS: $2 - x = 2 - (2 - Ce^{-s}) = Ce^{-s}$

DE solves $s \in (-\infty, \infty)$

$1 = x(0) = 2 - Ce^{-0} = 2 - C$

$1 = 2 - C \Rightarrow C = 1$

$x(s) = 2 - e^{-s}$ solution to IVP, exists for $(-\infty, \infty)$

Both $x(s)$ and $x'(s)$ exist and solve the equation on $(-\infty, \infty)$, therefore the I of E is the whole real line.

Geometric Meaning of DEs & its solutions

$y' = f(t, y)$ $\qquad$ $f(t,y)$ defined for (t, y) in rectangle:

$$R = \{(t, y) \mid a \le t \le b \ \& \ c \le y \le d\}$$

Let $y(t)$ be a solution to $y' = f(t, y)$.

Since $y(t_0) = y_0$, (t_0, y_0) is on solution curve

Notice that DE gives $\underbrace{y'(t_0)}_{\text{slope of solution curve that passes thru } (t_0, y_0)} = f(t_0, y_0)$

At each point (t, y) in a rectangle R, you can make a slope field, just put in $y' =$ whatever you want the slope of