STATS 100A HW4

Problem 1)

1) $P(X>4) = P(X=5) + P(X=6) = 0.2+0.3 = \frac{1}{2}$

$P(X=6 \mid X>4) = \frac{0.3}{0.5} = \frac{3}{5}$

2) $E(X) = \sum p(x)x = 1\cdot0.1 + 2\cdot0.1 + 3\cdot0.1 + 4\cdot0.2 + 5\cdot0.2 + 6\cdot0.3 = 4.2$

$Var(X) = \sum (X - E(X))^2 P(X)$

$= \sum (X-4.2)^2 P(X) = 1.024 + 0.484 + 0.144 + 0.008 + 0.128 + 0.972 = 2.76$

$SD(X) = \sqrt{Var(X)} = 1.66$

3) $E(h(X)) = \sum p(x)h(x) = -2 - 1 + 2 + 4 + 30 = \$33$

$Var(h(X)) = \sum (h(x) - E(h(x)))^2 P(X)$

$= \sum (h(x) - 33)^2 P(X) = 280.9 + 184.9 + 106.9 + 105.8 + 33.8 + 1346.7 = 2061 \text{ dollars}^2$

$SD(h(X)) = \sqrt{Var(h(X))} = \$45.40$

Problem 2)

$E(Z) = 0\cdot(1-p) + 1\cdot p = p$

$E(Z^2) = p$

$Var(Z) = (0-p)^2\cdot(1-p) + (1-p)^2\cdot p = p(1-p)$

plug in $p = \frac{1}{2}$:

$E(Z) = \frac{1}{2}$

$E(Z^2) = \frac{1}{2}$

$Var(Z) = \frac{1}{4}$

Replace 0 w/ -1:

$E(Z) = -1\cdot(1-p) + p = 2p - 1$

$E(Z^2) = (-1)^2(1-p) + (1)^2 p = 1$

$Var(Z) = E(Z^3) - E(Z)^2 = 1 - (2p-1)^2 = 1 - 4p^2 + 4p - 1$

$= -4p(p-1)$

Problem 3) $n = 100$, $p = \frac{1}{2}$

$E(X) = np = 50$

$Var(X) = np(1-p) = 100\cdot\frac{1}{2}\cdot\frac{1}{2} = 25$

$SD(X) = \sqrt{Var(X)} = 5$

$E(\frac{X}{100}) = \frac{1}{100}E(X) = \frac{1}{2}$

$Var(\frac{X}{100}) = (\frac{1}{100})^2 Var(X) = 0.0025$

$SD(\frac{X}{100}) = \sqrt{0.0025} = 0.05$

$P(X \in [40,60]) = \sum_{x=40}^{60} {}_{100}C_x (\frac{1}{2})^x (\frac{1}{2})^{100-x}$

Problem 4) $p = 0.2$ $1-p = 0.8$

$E(X) = np = 100\cdot0.2 = 20$

$Var(X) = np(1-p) = 100\cdot0.2\cdot0.8 = 16$

$SD(X) = \sqrt{Var(X)} = 4$

$E(\frac{X}{100}) = \frac{1}{100}E(X) = 0.2$

$Var(\frac{X}{100}) = (\frac{1}{100})^2 Var(X) = 0.0016$

$SD(\frac{X}{100}) = \sqrt{0.0016} = 0.04$

Problem 5) $p = \frac{\pi}{4}$, $n = 10k$

m = # of 10,000 pts that fall into circle

$E(m) = np = 2500\pi$

$Var(m) = np(1-p) = 1685.79$

$E(\hat{\pi}) = \frac{E(m)}{2500} = \boxed{\pi}$

$Var(\hat{\pi}) = \frac{Var(m)}{2500^2} = \boxed{2.70 \cdot 10^{-4}}$

$SD(\hat{\pi}) = \sqrt{Var(\hat{\pi})} = \boxed{0.01642}$

Problem 6)

1) $E(aX) = aE(X)$

$Y = aX$

$E(Y) = E(aX) = \sum x \, a \overbrace{P(X=x)}^{p(x)} = a\sum x p(x) = \boxed{aE(X)}$

2) $E(X+b) = E(X) + b$

$Y = X + b$

$E(Y) = E(X+b) = \sum (x+b)p(x) = \sum x p(x) + \sum b p(x) = \sum x p(x) + b \underbrace{\sum p(x)}_{=1} = \boxed{E(X)+b}$

3) $Var(aX) = a^2 Var(X)$, $Y = aX$

$Var(Y) = E(Y^2) - E(Y)^2 = a^2 E(X^2) - a^2 E(X)^2 = a^2(E(X^2) - E(X)^2) = \boxed{a^2 Var(X)}$

4) $Var(X+b) = Var(X)$, $Y = X + b$

$Var(Y) = E(Y^2) - E(Y)^2 = E(X^2 + 2bX + b^2) - [E(X) + E(b)]^2 = E(X^2) + 2bE(X) + b^2 - E(X)^2 - 2bE(X) - b^2$

$= E(X^2) - E(X)^2 = \boxed{Var(X)}$

5) $Var(X) = E(X^2) - E(X)^2$

$Var(X) = E((X-\mu)^2)$

$= E[(X - E(X))^2]$

$= E[(X - E(X))(X - E(X))] = E[X^2 - 2XE(X) + E(X)^2] = E(X^2) - 2E(XE(X)) + E(E(X)^2)$

$= E(X^2) - 2E(X)E(X) + E(X)^2 = \boxed{E(X^2) - E(X)^2}$

6) $E(Z) = p$

$Var(Z) = p(1-p)$