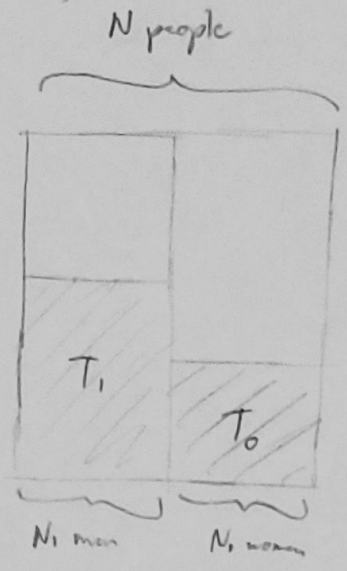
STATS 100A HW1

Problem 1)



A: male

B: over 6ft

1) $P(A) = \frac{N_1}{N}$

$P(B) = \frac{T_1 + T_0}{N}$

$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{T_1}{T_1 + T_0}$

$P(B|A) = \frac{|A \cap B|}{|A|} = \frac{T_1}{N_1}$

$P(A \cap B) = \frac{T_1}{N}$

2) $P(A \cap B) = P(A)P(B|A)$

$= \frac{|A|}{|\Omega|} \cdot \frac{|A \cap B|}{|A|} = \frac{|A \cap B|}{|\Omega|}$

$= P(A \cap B)$ ✓

$P(A \cap B) = P(B)P(A|B)$

$= \frac{|B|}{|\Omega|} \cdot \frac{|A \cap B|}{|B|} = \frac{|A \cap B|}{|\Omega|}$

$= P(A \cap B)$ ✓

3) $P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$ (NOT A, NOT A)

$= \frac{|A|}{|\Omega|} \cdot \frac{|A \cap B|}{|A|} + \frac{|A^c|}{|\Omega|} \cdot \frac{|A^c \cap B|}{|A^c|}$

$= \frac{|A \cap B|}{|\Omega|} + \frac{|A^c \cap B|}{|\Omega|} = \frac{|A \cap B| + |A^c \cap B|}{|\Omega|}$

$= \frac{|B|}{|\Omega|} = P(B)$ ✓

No matter if it is A or isn't A (called A^c) it'll always be B probability

4) We know $P(A|B) = \frac{|A \cap B|}{|B|} = \frac{\frac{|A \cap B|}{|\Omega|}}{\frac{|B|}{|\Omega|}} = \frac{P(A \cap B)}{P(B)}$

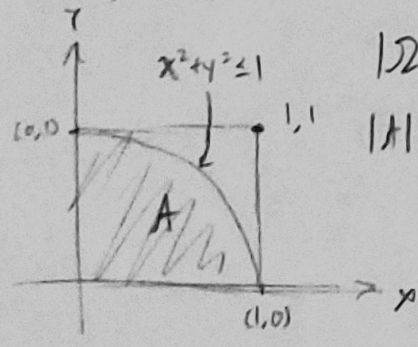
$\frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} = \frac{\frac{|A|}{|\Omega|} \cdot \frac{|A \cap B|}{|A|}}{\frac{|A|}{|\Omega|} \cdot \frac{|A \cap B|}{|A|} + \frac{|A^c|}{|\Omega|} \cdot \frac{|A^c \cap B|}{|A^c|}}$

$= \frac{\frac{|A \cap B|}{|\Omega|}}{\frac{|A \cap B|}{|\Omega|} + \frac{|A^c \cap B|}{|\Omega|}} = \frac{\frac{|A \cap B|}{|\Omega|}}{\frac{|B|}{|\Omega|}} = \frac{P(A \cap B)}{P(B)}$

By transitive property,

$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$

Problem 2)

1) 

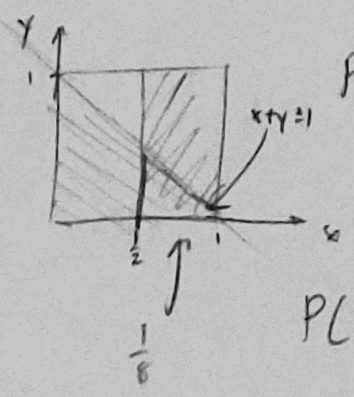
$|\Omega| = 1^2 = 1$

$|A|$ = quarter unit circle ($r=1$)

$= \frac{\pi r^2}{4} = \frac{\pi}{4}$

$\frac{|A|}{|\Omega|} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$

2) $\frac{\pi}{4} = \frac{|A|}{|\Omega|}$

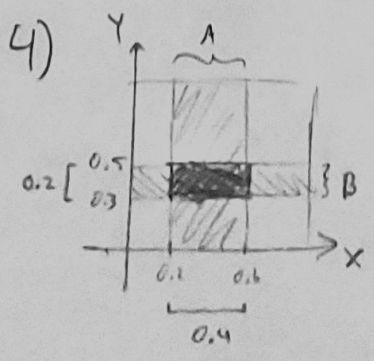
3) 

$P(X \ge \frac{1}{2}) = \frac{0.5}{1} = \frac{1}{2}$

$P(x+y \ge 1) = \frac{0.5}{1} = \frac{1}{2}$

$P(X \ge \frac{1}{2} | x+y \ge 1) = \frac{|x \ge \frac{1}{2} \cap x+y \ge 1|}{|x+y \ge 1|}$

$= \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$

4) 

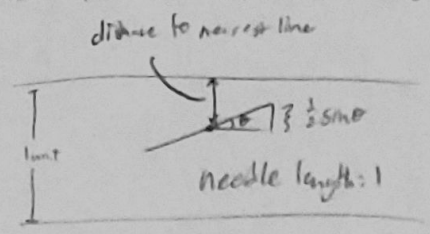
$P(A \cap B) = 0.2 \cdot 0.4 = 0.08$

$P(A)P(B) = \frac{|A|}{|\Omega|} \cdot \frac{|B|}{|\Omega|} = \frac{1(0.5-0.3) \cdot 1(0.6-0.2)}{1^2} = 0.08$

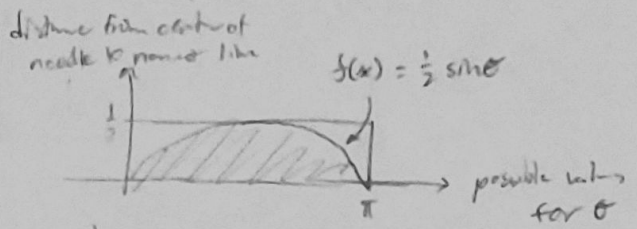
$\therefore P(A)P(B) = P(A \cap B)$

Problem 3)

1) To calculate probability in this case, we drop a needle between 2 lines. We measure the center the distance to the nearest line like this:



How often does the needle touch the line as a measurement of θ?



Shaded portion is $\frac{1}{2}$

Probability is the sum of shaded area over total area: $\frac{1}{\frac{1}{2}\cdot\pi} = \boxed{\frac{\pi}{2}}$

2) To calculate/approximate π, take # of drops, $\times 2$, then divide by # of hits / area within the shaded region. We can approximate π.

↓

The more drops we do the closer we get to π!