STATS 100A HW2

Problem 1)

$p = \frac{1}{2}$, $n = 100$

1) $P(X=50) = {}_{100}C_{50} \cdot (\frac{1}{2})^{50} \cdot (\frac{1}{2})^{50}$

$\boxed{= 0.0796}$

I just saw the note
n = # of total trials
m = # of successful trials
p = probability of a successful trial

$P(X=m) = {}_nC_m \cdot p^n \cdot (1-p)^{(n-m)}$

2) Do the same as the problem above but sum the probabilities for $91 \le m \le 60$: $\boxed{\sum_{m=40}^{m=60} {}_nC_m \cdot p^n \cdot (1-p)^{(n-m)}}$

3) $\boxed{X = \sum_{i=1}^{n} Z_i}$

Problem 2)

1)
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        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
1   5  10  10   5   1
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5-layer pascal triangle. The next number in the list is the sum of the left & right numbers above it.

Additionally, the sum of the #'s for each row doubles as it goes down.

row 1: 1
row 2: 2
row 3: 4
row 4: 8
⋮

2) Probability: $\frac{{}_nC_r}{2^n}$ where n is the total # of layers and r is the bin number from left to right.

bin #	0	1	2	3	4	5
probability	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

3) We will see that approx. 31,250, 156,250, 312500, 312500, 156,250, 31,250 would land in bins 0, 1, 2, 3, 4, 5 respectively

Problem 3)

1) Outcomes:

		X_t
5H	0T	5
4H	1T	3
3H	2T	1
2H	3T	-1
1H	4T	-3
0H	5T	-5

2)

5H 0T:	$\frac{1}{32}$
4H 1T:	$\frac{5}{32}$
3H 2T:	$\frac{10}{32}$
2H 3T:	$\frac{10}{32}$
1H 4T:	$\frac{5}{32}$
0H 5T:	$\frac{1}{32}$

3) What's the probability $X_{t+1} = j$ given at time t $X_t = i$?

X_{t+1} can either be i+1 or i-1

$\boxed{P(X_{t+1} = i+1 \mid X_t = i) = P(X_{t-1} = i-1 \mid X_t = i) = \frac{1}{2}}$

4) It would simply result in $\frac{1}{32}, \frac{5}{32}, \frac{10}{32}, \frac{10}{32}, \frac{5}{32}, \frac{1}{32}$ of the people taking 5, 3, 1, -1, -3, -5 steps respectively, approximately to:

X_t	approx.
5	31250
3	156250
1	312500
-1	312500
-3	156250
-5	31250