

STATS 100A HW3

Problem 1 Consider a random walk over 3 webpages, 1, 2, 3. At any step, if the person is at webpage 1, then with probability $1/6$, she will go to webpage 2, and with probability $1/6$, she will go to webpage 3. If the person is at webpage 2, then with probability $1/2$, she will go to webpage 1, and with probability $1/2$, she will go to webpage 3. If the person is at webpage 3, then with probability $1/2$, she will go to webpage 1, and with probability $1/2$, she will go to webpage 2.

Let X_t be the webpage the person is browsing at time t , and let us assume she starts from webpage 1 at time 0, i.e., $X_0 = 1$.

(1) Let $K_{ij} = P(X_{t+1} = j | X_t = i)$. Let $K = (K_{ij})$ be the 3×3 transition matrix. Write down K .

(2) Let $p_i^{(t)} = P(X_t = i)$. Let $p^{(t)} = (p_i^{(t)}, i = 1, 2, 3)$ be the row vector. Calculate $p^{(t)}$ for $t = 1, 2, 3$ using vector matrix multiplication.

(3) Let π_i be the stationary distribution at webpage i , so that $\pi_j = \sum_{i=1}^3 \pi_i K_{ij}$. Let $\pi = (\pi_i, i = 1, 2, 3)$ be the row vector. Then $\pi = \pi K$. Given K , solve π from this equation. Is $p^{(3)}$ close to π ?

(4) Based on the above calculations, answer the following questions. Suppose there are 1 million people doing the above random walk independently, and suppose they all start from webpage 1 at time $t = 0$. Then on average, what is the distribution of these 1 million people for $t = 1, 2, 3$? What is the stationary distribution of these 1 million people? Which page is the most popular?

Problem 2 Suppose at any moment, the probability of fire in a classroom is α . Suppose the conditional probability of alarm given fire is β , and the conditional probability of alarm given no fire is γ .

(1) Calculate the conditional probability of fire given alarm.

(2) Please explain your calculation by counting hypothetical repetitions. For instance, suppose $\alpha = 1/1000$, and $\beta = 99/100$, and $\gamma = 2/100$. Imagine we repeat the experiments 100,000 times. Then on average, 100 times there is fire. Out of these 100 times, 99 times there is alarm, and so on.

Problem 3 (Optional) Read about Google pagerank

<https://en.wikipedia.org/wiki/PageRank>

write a brief summary, focusing on Markov chain and random walk conceptualization, and population migration interpretation.

Problem 4 (Optional) Read about Bayes net

https://www.norsys.com/tutorials/netica/secA/tut_A1.htm

write a brief summary. See if you can explain the probability calculations by counting a population or counting repetitions.

Problem 5 (Optional) Read about Monte Hall problem

https://en.wikipedia.org/wiki/Monty_Hall_problem

write a brief summary. See if you can explain the probability calculations by counting the repetitions, e.g., we repeat the game 6000 times.