

STATS 100A HW1

Problem 1 Suppose a population has N people. Among them, N_1 are males, and N_0 are females. Among the males, T_1 are taller than 6 feet. Among the females, T_0 are taller than 6 feet. Suppose we randomly sample a person from the population. Let A be the event that the person is male. Let B be the event that the person is taller than 6 feet.

Using the numbers N , N_1 , N_0 , T_1 , T_0 , calculate or verify the following:

- (1) Calculate $P(A)$, $P(B)$, $P(A|B)$, $P(B|A)$, $P(A \cap B)$.
- (2) Verify that $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$. This is called chain rule.
- (3) Verify that $P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$. This is called rule of total probability.
- (4) Verify that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}.$$

This is called the Bayes rule.

Problem 2 Suppose we generate X and Y from the uniform distribution over $[0, 1]$ independently, i.e., (X, Y) is a random point within the unit square $[0, 1]^2$.

- (1) Calculate $P(X^2 + Y^2 \leq 1)$ by measuring the area.
- (2) Suppose we repeat the experiment n times. For large n , how often $X^2 + Y^2 \leq 1$? Suppose it happens m times. Can you approximate π based on m and n ?
- (3) Calculate $P(X \geq 1/2)$ and $P(X \geq 1/2 | X + Y \geq 1)$.
- (4) Let A be the event that $X \in [.2, .6]$, and let B be the event that $Y \in [.3, .5]$. Show that $P(A \cap B) = P(A)P(B)$.

Problem 3 Read the following webpages on Buffon's needle:

<https://mste.illinois.edu/activity/buffon/>

<https://yihui.org/animation/example/buffon-needle/>

Write a brief memo on (1) how to calculate the probability in the simplest case, (2) how to estimate π using Monte Carlo.