STATS 100A HW6

Problem 1)

1)

$y \backslash x$	1	2
1	0.1	0.1
2	0.1	0.2
3	0.2	0.3

$$P_X(x) = \sum_y p(x,y)$$

$$P_Y(y) = \sum_x p(x,y)$$

x	$P_X(x)$
1	$p(1,1)+p(1,2)+p(1,3)=0.4$
2	$p(2,1)+p(2,2)+p(2,3)=0.6$

y	$P_Y(y)$
1	$p(1,1)+p(2,1)=0.2$
2	$p(1,2)+p(2,2)=0.3$
3	$p(1,3)+p(2,3)=0.5$

2) $p(x|y) = \frac{p(x,y)}{p(y)}$

$p(y|x) = \frac{p(x,y)}{p(x)}$

$p(x|y)$:

$y \backslash x$	1	2
$p(x,1)$	$\frac{p(1,1)}{p(y=1)} = 0.5$	$\frac{p(2,1)}{p(y=1)} = \frac{1}{2}$
$p(x,2)$	$\frac{p(1,2)}{p(y=2)} = \frac{1}{3}$	$\frac{p(2,2)}{p(y=2)} = \frac{2}{3}$
$p(x,3)$	$\frac{p(1,3)}{p(y=3)} = \frac{2}{5}$	$\frac{p(2,3)}{p(y=3)} = \frac{3}{5}$

$p(y|x)$:

$y \backslash x$	$p(y,1)$	$p(y,2)$
1	$\frac{p(1,1)}{p(x=1)} = \frac{1}{4}$	$\frac{p(2,1)}{p(x=2)} = \frac{1}{6}$
2	$\frac{p(1,2)}{p(x=1)} = \frac{1}{4}$	$\frac{p(2,2)}{p(x=2)} = \frac{1}{3}$
3	$\frac{p(1,3)}{p(x=1)} = \frac{1}{2}$	$\frac{p(2,3)}{p(x=2)} = \frac{1}{2}$

3) $E(X) = 1\cdot 0.4 + 2\cdot 0.6 \boxed{= 1.6}$

$E(Y) = 1\cdot 0.2 + 2\cdot 0.3 + 3\cdot 0.5 \boxed{= 2.3}$

$E(XY) = \sum_{x=1}^{2}\sum_{y=1}^{3} (xy)p(x,y) = 1\cdot 0.1 + 2\cdot 0.1 + 2\cdot 0.1 + 4\cdot 0.2 + 3\cdot 0.2 + 6\cdot 0.3 \boxed{= 3.7}$

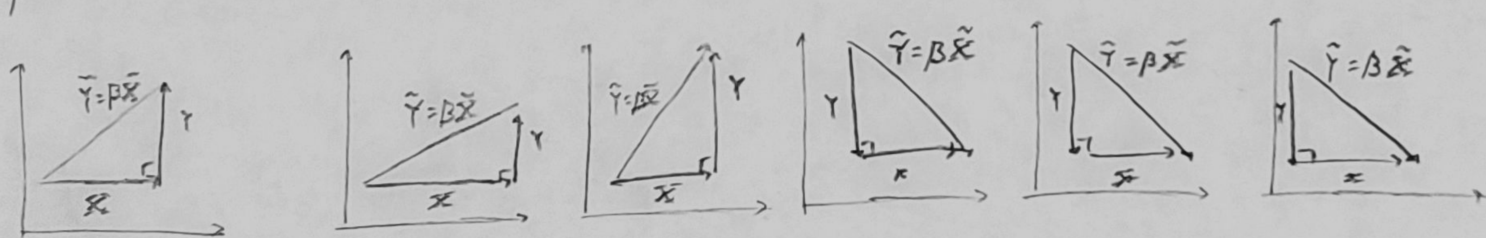
$Cov(X,Y) = E[(X-E(X))(Y-E(Y))] = E[XY - YE(X) - XE(X) + E(X)E(Y)] = E(XY) - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)$

$= 3.7 - 1.6\cdot 2.3 \boxed{= 0.02}$

Problem 2)

1) perfect + correlation (1), high + correlation (0.9), low + correlation (0.5), low - correlation (-0.5), high - correlation (-0.9), perfect - correlation (-1)

2) & 3)



Problem 3)

1) $\boxed{\begin{array}{l} f(x) = \int f(x,y)\,dy \\ f(y|x) = \frac{f(x,y)}{f(x)} \\ f(xy) = f(x)f(y|x) \end{array}}$

2) $E(Y|X=x) = E(\rho x + \epsilon) \boxed{= \rho x}$

$Var(Y|X=x) = Var(\rho x + \epsilon) = Var(\epsilon) \boxed{= 1-\rho^2}$

3) Because $\epsilon \sim N(0, 1-\rho^2)$ ensures that the total variance of Y stays consistent w/ our given values.

4) Because our values can vary by an entire [illegible] w/ variance of ρ.

Problem 4)

1) If X & Y are independent, then $E[XY] = E(X)E(Y)$

so $Cov(X,Y) = E[(X-E(X))(Y-E(Y))]$

$= E(X)E(Y) - E(X)E(Y) = 0.$

Reverse may be false if X & Y are uncorrelated but still have some other form of dependency

2) $Var(X+Y) = E[(X+Y-E[X+Y])^2]$ $\qquad E[X+Y] = E[X]+E[Y]$

$= E[(X+Y-E[X]-E[Y])^2]$

$= Var(X) + 2\,Cov(X,Y)^{0} + Var(Y)$

$= Var(X) + Var(Y)$

3) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$E(\bar{X}) = E(\frac{1}{n}\sum_{i=1}^{n} X_i)$

$= \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\cdot n\mu \boxed{= \mu}$

$Var(\bar{X}) = Var(\frac{1}{n}\sum_{i=1}^{n} X_i)$

$= \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$

$= \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 \boxed{= \frac{\sigma^2}{n}}$