STATS 100A HW2

Problem 1 Suppose we flip a fair coin 100 times independently.

(1) What is the probability we get 50 heads?

(2) Let X be the number of heads. What is $P(40 \leq X \leq 60)$?

(3) Let $Z_i = 1$ if the i-th flip is head, and $Z_i = 0$ if the i-th flip is tail. Express X in terms of Z_i.

Note: For (1) and (2), you only need to write down the math formulas. You do not need to calculate the concrete numbers.

Problem 2 Draw a Galton board with 5 layers.

(1) Write down the corresponding Pascal triangle. Explain the meaning of the numbers.

(2) Suppose we label the bins by 0, 1, 2, 3, 4, 5. Calculate the probability that the ball drops into each bin.

(3) Suppose we drop 1 million balls. What are the proportions of balls in these bins?

Problem 3 Consider a random walk on integers. We start from $X_0 = 0$, and at each step, we flip a fair coin. If it is head, we move forward by 1, and if it is tail, we move backward by 1. In math notation, $X_{t+1} = X_t + \epsilon_t$, where $\epsilon_t = 1$ with probability 1/2, and $\epsilon_t = -1$ with probability 1/2.

(1) At time $t = 5$, what are the possible values of X_t?

(2) What is the probability of each possible value in (1)?

(3) What is $P(X_{t+1} = j | X_t = i)$?

(4) Interpret (2) in terms of 1 million people doing the random walk simultaneously and independently, all starting from 0.

Problem 4 Write R code to simulate flipping a fair coin 100 times independently. Let X be the number of heads. Repeat the experiment 1000 times. Plot the histogram of these 1000 X. Also plot the histogram of $X/100$.

Repeat the above for flipping a biased coin where the probability of head is 0.2. (Hint: you can generate a biased coin flipping by first generating a uniform random variable and then comparing it with 0.2.)