Lista de Exercícios – IC242 - Cálculo II

Integrais Impróprias

Prof. Renan Teixeira

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1. Calcule as Integrais

   (a) $\displaystyle\int_1^{+\infty} \frac{1}{x^3} dx$

   (b) $\displaystyle\int_0^{2} \frac{1}{t^3} dt$

   (c) $\displaystyle\int_1^{+\infty} \frac{1}{x^k} dx$, para $k \in \mathbb{R}$

   (d) $\displaystyle\int_{-\infty}^{0} e^y dy$

   (e) $\displaystyle\int_{-\infty}^{+\infty} e^{-y} dy$

   (f) $\displaystyle\int_{-4}^{1} \frac{1}{\sqrt[3]{x+2}} dx$

   (g) $\displaystyle\int_0^{1} [\ln x]^2 \, dx$

   (h) $\displaystyle\int_0^{+\infty} \frac{\ln x}{\sqrt{x^5}} dx$

   (i) $\displaystyle\int_0^{+\infty} \frac{1}{1+x^2} dx$

   (j) $\displaystyle\int_1^{+\infty} \frac{1}{\sqrt{x}} dx$

   (k) $\displaystyle\int_0^{+\infty} x e^{-x^2} dx$

2. Sejam α e s, $s > 0$, reais dados. Mostre que:

   (a) $\displaystyle\int_0^{+\infty} e^{-st}(\alpha t) \, dt = \frac{\alpha}{s^2+\alpha^2}$ com $\alpha \neq 0$

   (b) $\displaystyle\int_0^{+\infty} e^{-st} \cos(\alpha t) \, dt = \frac{\alpha}{s^2+\alpha^2}$

   (c) $\displaystyle\int_0^{+\infty} e^{-st} e^{\alpha t} \, dt = \frac{\alpha}{s-\alpha}$ com $s > \alpha$

   (d) $\displaystyle\int_0^{+\infty} e^{-st} \, dt = \frac{1}{s}$

3. Esboce o gráfico de $F(x) = \int_{-\infty}^{x} f(t) \, dt$ onde

   (a) $f(t) = \begin{cases} 2, & \text{se } |t| \leq 1 \\ 0, & \text{se } |t| > 1 \end{cases}$

   (b) $f(t) = \begin{cases} \dfrac{1}{t}, & \text{se } |t| \geq 1 \\ 0, & \text{se } |t| < 1 \end{cases}$

   (c) $f(t) = \begin{cases} 0, & \text{se } |t| > 1 \\ 1-t^2, & \text{se } |t| \leq 1 \end{cases}$

   (d) $f(t) = \begin{cases} 0, & \text{se } t \leq 0 \\ e^{-t}, & \text{se } t > 0 \end{cases}$

4. Calcule

   (a) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

   (b) $\displaystyle\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$

   (c) $\displaystyle\int_{-1}^2 \frac{1}{4-x^2}\,dx$

   (d) $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

5. Determine:

   (a) $\displaystyle\int_{-\infty}^{+\infty} x\,e^{-0,1x^2}\,dx$

   (b) $\displaystyle\int_1^{+\infty} \frac{ln(x)}{\sqrt{x^5}}\,dx$

6. Calcule as integrais impróprias abaixo:

   (a) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{1+x^2}dx.$

   (b) $\displaystyle\int_{-\infty}^{0} xe^x dx.$

7. Determine, justificando sua resposta, se as integrais abaixo são convergentes ou divergentes.

   (a) $\displaystyle\int_1^{+\infty} \frac{3}{x^3+3}dx.$

   (b) $\displaystyle\int_1^{+\infty} \frac{2+e^{-x}}{x}dx.$

   (c) $\displaystyle\int_0^{+\infty} \frac{x}{x^3+1}dx.$

   (d) $\displaystyle\int_0^{+\infty} e^{-x}\sin^3 x\,dx.$

   (e) $\displaystyle\int_1^{+\infty} \frac{\sin x}{x}\,dx.$

8. Mostre que $\displaystyle\lim_{c\to+\infty}\int_{-c}^{c} \sin x\,dx = 0.$

9. Mostre que a $\displaystyle\int_2^{+\infty} \frac{dx}{x(\ln x)^p}$ converge se $p > 1$.