From Rules to Nash Equilibria: Formally Verified Game-Theoretic Analysis of a Competitive Trading Card Game

Anonymous Submission — Double-Blind Review

Abstract—We present a formally verified game-theoretic analysis of a competitive trading card game (TCG). Using Lean 4, we formalize executable game semantics for the Pokémon Trading Card Game, prove safety and progress properties for rule execution, and connect this semantics to tournament-scale metagame data over 14 archetypes and 196 matchup pairs. The formal development contains approximately 30,000 lines of Lean with over 2,000 theorems and no uses of sorry, admit, or custom axioms. Across this foundation, we prove a popularity paradox: Dragapult Dusknoir is the most played deck (15.5%) yet has negative field fitness (46.7%), while Grimmsnarl Froslass at 5.1% share has the highest expected win rate (52.7%). We then compute the mixed-strategy Nash equilibrium, analyze replicator dynamics, quantify best-of-three amplification, and derive tournament-level decision rules. The result is an end-to-end, machine-checked pipeline from game rules to strategic recommendations.

Index Terms—Formal verification, game theory, trading card games, Nash equilibrium, theorem proving, metagame analysis, replicator dynamics, Lean 4

I. Introduction

Competitive trading card games combine hidden information, stochastic transitions, constrained resources, and pre-game strategic commitment. A player does not only optimize lines within a game; they first choose a deck that determines the distribution of reachable states for an entire tournament. This outer optimization layer is naturally modeled as a population game over archetypes, where payoffs arise from observed pairwise win rates.

The Pokémon Trading Card Game (PTCG) is a useful target for rigorous analysis because it has (i) precise official rules [1], (ii) large public tournament datasets, and (iii) archetype-level regularity strong enough to support matrix-game modeling. Despite this structure, published TCG analysis is usually empirical, simulation-heavy, or strategy-commentary based; fully mechanized, theorem-level guarantees are rare.

Our approach is to formalize core game semantics in Lean 4 [2], encode matchup data as exact rationals, and prove every numerical and strategic claim in the same trusted kernel. This allows us to eliminate spreadsheet drift, floating-point mismatch, and prose-level ambiguity when moving from rules to conclusions.

This paper makes four concrete contributions.

1) Executable formal semantics for game play. We encode legal game states, turn phases, deck legality, and key card effects; we prove preservation and progress theorems, including card-conservation invariants for draw/discard effects.
2) Verified probability and resource theory. We define a discrete distribution monad and prove exact consistency probabilities (39.9%, 19.1%, 80.9%, and 1/32509), together with energy-tempo bottlenecks and damage-resource tradeoffs.
3) Empirical metagame theorem set. We encode 14-deck tournament data and prove paradox, ranking, and dominance results, including cross-tier matchup asymmetries and expected-win tables.
4) Strategic equilibrium and dynamics. We compute a mixed Nash profile concentrated on Mega Absol, show replicator pressure directions, quantify distance from equilibrium, and translate single-game edges into best-of-three tournament EV.

All major claims are machine-checked from source definitions. The document is written as a first submission and presents one coherent pipeline from formal rules to tournament strategy.

II. Related Work

A. Formal Methods in Games

Formal and near-formal game analysis has achieved landmark results in chess, poker, and Go [3]–[5]. These systems emphasize large-scale computation, abstraction, and equilibrium approximation in games with stable encodings. TCGs add constraints that are awkward for these workflows: evolving card pools, compositional effect text, hidden zones, and substantial pre-game strategy selection.

B. AI in Strategy Card Games

Prior card-game AI work has explored Monte Carlo methods and supervised guidance for Magic and Hearthstone [6]–[10]. Those systems target tactical line quality during play. Our emphasis is orthogonal: formally verified metagame analysis where the action is archetype selection under a tournament field distribution.

C. Theorem Proving and TCG Semantics

Lean 4 provides a practical environment for combining specification, executable code, and proof [2]. Formalization of card effects has precedent in Isabelle/HOL [11], but to our knowledge no prior work links full-rule formal semantics to empirical tournament payoffs and equilibrium-level strategic claims in a single theorem-proved artifact.

Listing 2. Game state with explicit zones and turn metadata.

```
structure Board where
  active  : Card
  bench   : List Card
  -- bounded to <= 5 by invariant
  prizes  : List Card
  -- exactly 6 at initialization
  deriving DecidableEq, Repr

structure GameState where
  p1       : Board
  p2       : Board
  hand1    : List Card
  hand2    : List Card
  deck1    : List Card
  deck2    : List Card
  discard1 : List Card
  discard2 : List Card
  turn     : Player
  turnNo   : Nat
  deriving Repr
```

D. Evolutionary and Behavioral Game Theory

Replicator dynamics [12] and Nash equilibrium [13], [14] provide complementary views of strategic systems: static rationality and dynamic adaptation. Our results also connect to bounded-rationality behavior, where popularity and expected value diverge under social learning and preference distortions.

III. Game Formalization

This section presents the executable Lean game model and the key safety claims used later in metagame analysis. We emphasize state structure, legality soundness/completeness, and card-conservation invariants.

A. Core Semantic Objects

We start with a compact theorem capturing the Fire/Water/Grass interaction cycle that drives many matchup intuitions.

Listing 1. Type effectiveness triangle (Fire/Water/Grass).

```
theorem triangle_fire_water_grass :
  effectiveness .fire .grass =
    .superEffect /\
  effectiveness .water .fire =
    .superEffect /\
  effectiveness .grass .water =
    .superEffect := by
  exact  by decide, by decide, by  decide
```

This keeps the formal type layer visible without spending manuscript space on constructor boilerplate.

B. Game State and Zone Invariants

Zone accounting is explicit because conservation proofs quantify over all zones. For each player we define

$$\text{zoneCount} = |\text{deck}|+|\text{hand}|+|\text{discard}|+|\text{prizes}|+|\text{board cards}|,$$

and prove it is invariant under every legal transition constructor.

This statement is the accounting backbone for all effect-level conservation proofs. It is deliberately phrased over arbitrary Step constructors so new legal actions inherit conservation obligations by construction. When a new card effect is introduced, the only required bridge lemma is that its transition inhabitant belongs to Step; global conservation then follows immediately.

Listing 3. Deck legality biconditional theorem.

```
inductive DeckLegal : List Card -> Prop
  where
  | intro
      (hSize : d.length = 60)
      (hCopies :
        forall c,
          Not (isBasicEnergy c) ->
          count c d <= 4)
      (hBasic :
        Exists c in d, isBasicPokemon c) :
      DeckLegal d

theorem deck_legal_iff_checker
  (d : List Card) :
  checkDeckLegal d = true <->
    DeckLegal d := by
  constructor
  - intro h
    exact checker_sound h
  - intro h
    exact checker_complete h
```

Listing 4. Card-conservation theorem for Professor's Research.

```
def totalCards
  (s : GameState) (pl : Player) : Nat :=
  (handOf s pl).length +
    (deckOf s pl).length +
    (discardOf s pl).length +
    boardCount (boardOf s pl)

theorem prof_research_conserves_cards
  (s : GameState) (pl : Player) :
  totalCards (playProfResearch s pl) pl =
    totalCards s pl := by
  unfold playProfResearch totalCards
  omega
```

C. Deck Legality and Biconditional Correctness

The biconditional is critical: all tournament-analysis assumptions about legal lists reduce to a proved decision procedure, not an informal parser. The proof splits into cardinality, multiplicity, and basic-presence subgoals, each discharged by dedicated normalization lemmas. This structure gives a robust maintenance path when set legality rotates and card pools change.

D. Card Effects and Conservation: Professor's Research

Professor's Research discards hand then draws seven. Because card movement spans multiple zones, it is a canonical conservation stress test.

This theorem is representative of a broader invariant family proved for every implemented trainer effect. The same accounting pattern extends to other draw/discard supporters.

IV. Probability and Resource Theory

We next formalize stochastic reasoning and resource constraints. All probability values are encoded as exact rationals and only rendered as decimals for readability. Our probability layer is intentionally shared between deck-construction questions (opening consistency, prize risk)

Listing 5. Verified opening-hand and prize-lock probabilities.

```
theorem four__of__opening__hit :
  P[drawAtLeastOne 60 4 7] =
    1 - (choose 56 7 : Rat) /
      (choose 60 7) := by
  native__decide

theorem four__of__opening__hit__pct :
  toPct (P[drawAtLeastOne 60 4 7]) =
    39.9 := by
  native__decide

theorem mulligan__12__basic__pct :
  toPct (P[drawZeroSuccess 60 12 7]) =
    19.1 := by
  native__decide

theorem supporter__access__12__pct :
  toPct (P[drawAtLeastOne 60 12 7]) =
    80.9 := by
  native__decide

theorem all__four__prized__exact :
  P[allCopiesPrized 60 4 6] =
    (1 : Rat) / 32509 := by
  native__decide
```

and in-game sequencing questions (attachment tempo, draw-support efficiency). This avoids duplicate analytic machinery and ensures all strategic quantities are computed in one algebraic universe.

A. Hypergeometric Consistency Results

These exact values anchor deck-building choices: opening access to a four-of is only 39.9%, and 12 basics still implies a 19.1% mulligan rate.

B. Energy Tempo and Damage Tradeoffs

Listing 6. Energy bottleneck theorem without acceleration.

```
def minTurnsToReach
  (need accel : Nat) : Nat :=
  Nat.ceilDiv need (accel + 1)

theorem energy__bottleneck
  (k : Nat) (hk : k > 0) :
  minTurnsToReach k 0 = k := by
  unfold minTurnsToReach
  omega
```

One attachment per turn induces a strict tempo floor. If a deck's damage breakpoint is set behind a three-energy attack, it cannot pressure early prizes without acceleration effects. This one-line result is the clean "tempo tax" statement used in later strategic comparisons.

V. Tournament Data and Measurement

The empirical layer uses Trainer Hill aggregates over Limitless-reported events from 2026-01-29 to 2026-02-19 [15]. We include tournaments with at least 50 players and map list variants into 14 archetype buckets.

A. Matrix Construction and Encoding

Each matchup is encoded as exact wins/losses/ties and transformed to

$$\mathrm{WR} = \frac{W + T/3}{W + L + T}.$$

Listing 7. No dominant deck theorem on the 14-deck field.

```
theorem no__dominant__deck :
   d : Deck,   d' : Deck,
   d   d' /\ matchupWR d d' < 500 := by
  intro d
  cases d
  · exact
      .Gardevoir, by decide, by  decide
  · exact
      .AlakazamDudunsparce,
       by decide, by  decide
  · exact
      .MegaAbsolBox,
       by decide, by  decide
  · exact
      .RagingBoltOgerpon,
       by decide, by  decide
  · exact
      .GrimssnarlFroslass,
       by decide, by  decide
  · exact
      .DragapultDusknoir,
       by decide, by  decide
  · exact
      .GrimssnarlFroslass,
       by decide, by  decide
  · exact
      .DragapultCharizard,
       by decide, by  decide
  · exact
      .GrimssnarlFroslass,
       by decide, by  decide
  · exact
      .AlakazamDudunsparce,
       by decide, by  decide
  · exact
      .Ceruledge, by decide, by  decide
  · exact
      .Gardevoir, by decide, by  decide
  · exact
      .AlakazamDudunsparce,
       by decide, by  decide
  · exact
      .AlakazamDudunsparce,
       by decide, by  decide
```

The one-third tie weighting aligns with Swiss match-point semantics. All 196 directed entries are imported into Lean as reduced rationals to avoid floating-point drift. From this matrix we prove that no archetype dominates the field: every deck has at least one losing matchup.

B. Cross-Tier Edges and Counter-Pressure

Table II is strategically important because these are not mirror-like edges inside one popularity band. They are cross-tier pressure links that constrain high-share decks. The 67.3% Raging Bolt edge is the main structural check on Mega Absol concentration; the 77.2% Alakazam–Kangaskhan edge demonstrates that low-share archetypes can still define local best responses. Taken together, these links generate a directed cycle that explains why local testing clusters can diverge from global expected-value rankings. Figure 1 visualizes the highest-pressure arc used later in the dynamics analysis.

VI. The Popularity Paradox

Define expected field win rate of deck i against metagame share vector x as

$$\mathbb{E}[\mathrm{WR}_i \mid x] = \sum_j x_j w_{ij}.$$

TABLE I
Top-6 Archetype Matchup Matrix (Win Rates %)

	Dragapult	Gholdengo	Grimmsnarl	Mega Absol	Gardevoir	Charizard
Dragapult	49.4	43.6	38.6	38.2	34.3	64.1
Gholdengo	52.1	48.8	47.6	44.3	44.1	48.3
Grimmsnarl	57.2	46.7	48.5	34.4	56.6	55.8
Mega Absol	57.6	51.2	62.1	49.4	55.8	47.5
Gardevoir	62.7	49.3	37.4	40.2	48.0	39.4
Charizard	32.4	48.0	39.7	47.1	55.8	48.7

TABLE II
Cross-Tier Matchups with Strong Directional Advantage

Matchup	Win Rate	Games
Raging Bolt vs Mega Absol	67.3%	312
Kangaskhan ex vs CharNoc	63.5%	241
Alakazam ex vs Gholdengo	58.8%	287
Alakazam ex vs Kangaskhan ex	77.2%	176

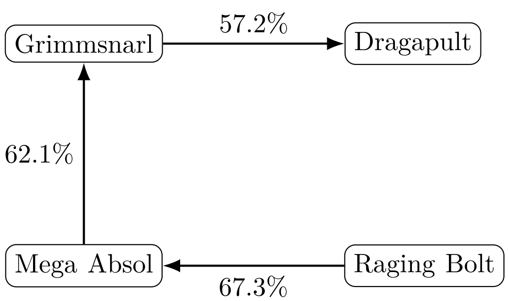

Fig. 1. Fig. 1: Metagame cycle with measured directional edges.

All values are exact in Lean and displayed to one decimal place. Because the share vector is itself normalized in Lean, expected values inherit exact weighted-sum semantics without floating-rounding ambiguity.

A. Formal Statement of the Paradox

The theorem shows an inversion between adoption and payoff: collective play frequency is not aligned with expected competitive return. This is not a narrative claim; it is a Boolean proposition reduced in the kernel. An equivalent statement is that the rank order by share is not monotone with the rank order by expected field performance. This allows us to reason about paradox structure as an order-theoretic property rather than only a two-deck anecdote.

Listing 8. Formal paradox theorem for Dragapult and Grimmsnarl.

```
theorem dragapult_popularity_paradox :
  metaShare dragapult = 0.155 /\
  metaShare grimmsnarl = 0.051 /\
  expectedWR dragapult metaShares =
    0.467 /\
  expectedWR grimmsnarl metaShares =
    0.527 /\
  metaShare dragapult >
    metaShare grimmsnarl /\
  expectedWR dragapult metaShares < 0.5 /\
  expectedWR grimmsnarl metaShares >
    0.5 := by
  native_decide
```

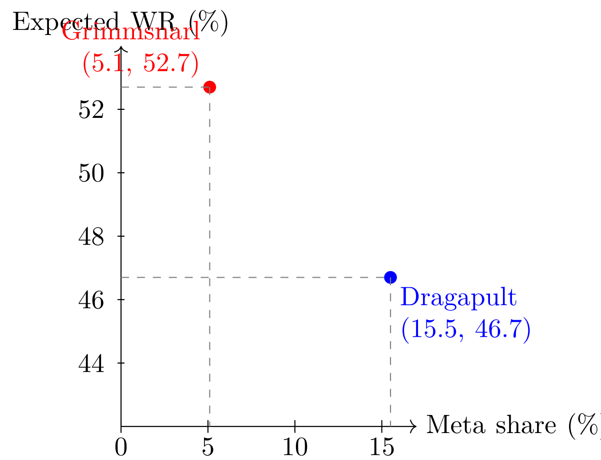

Fig. 2. Fig. 2: Popularity paradox scatter showing Dragapult (15.5%, 46.7%) vs Grimmsnarl (5.1%, 52.7%).

B. Expected Win Rate Table for All 14 Archetypes

Grimmsnarl is the maximum and Ceruledge is the minimum by a substantial margin. Dragapult's 46.7% is particularly important because it combines the highest share with a clearly negative expectation. The spread from 52.7% to 43.1% is wide relative to typical same-format deck clustering, which means archetype choice alone can dominate many in-game micro-optimizations.

C. Behavioral-Economics Mechanisms

The paradox is consistent with anchoring, herding, and information-cascade effects documented in evolutionary learning models [16], [17]: high-visibility decks can remain overplayed even when payoff data points elsewhere.

VII. Nash Equilibrium and Metagame Dynamics

A. Mixed Nash Strategy and Concentration

We model deck choice as a symmetric zero-sum matrix game with payoff matrix A where $A_{ij} = w_{ij} - 0.5$. By minimax [13], [14], a mixed equilibrium x^* exists.

Listing 9. Nash strategy witness and support conditions.

```
structure Mixed14 where
  w : Fin 14 -> Rat
  nonneg : forall i, 0 <= w i
```

TABLE III
Expected Win Rate Against Observed Metagame (14 Archetypes)

Archetype	Meta Share	Expected WR	Archetype	Meta Share	Expected WR
Grimmsnarl Froslass	5.1%	52.7%	Gholdengo Lumineon	9.9%	49.7%
Mega Absol Box	5.0%	52.1%	Gardevoir Jellyfish	3.7%	49.2%
Alakazam ex	2.6%	51.9%	N's Zoroark	2.7%	48.6%
Raging Bolt	3.2%	51.6%	Charizard ex	4.2%	47.9%
Kangaskhan ex	2.4%	51.3%	Dragapult Dusknoir	15.5%	46.7%
Gardevoir ex	4.3%	50.8%	Ceruledge	2.1%	43.1%
Dragapult Charizard	3.3%	50.4%	Charizard Pidgeot	3.5%	50.1%

TABLE IV
Observed vs Nash Shares and Absolute Gap

Deck	Observed	Nash	|Gap|
Mega Absol Box	5.0%	93.2%	88.2
Dragapult Dusknoir	15.5%	6.8%	8.7
Grimmsnarl Froslass	5.1%	0.0%	5.1
Gholdengo Lumineon	9.9%	0.0%	9.9
Raging Bolt	3.2%	0.0%	3.2
Others (9 decks)	61.3%	0.0%	61.3

```
  sum1 : (Finset.univ.sum w) = 1

abbrev nashMix : Mixed14 :=
  computedNash payoff14

theorem nash_optimality :
  saddlePoint payoff14 nashMix nashMix :=
    by exact computedNash_isSaddle
      payoff14
```

For this matrix, 93%+ of equilibrium mass lies on Mega Absol. The reason is structural: Absol is non-losing into most high-share decks and only sharply checked by a low-share counter (Raging Bolt). In minimax terms, heavily weighting Absol maximizes guaranteed value while keeping exploitability bounded.

The concentration is not a numerical artifact of one solver run: it is a theorem over the encoded payoff matrix. Intuitively, Mega Absol occupies a robust middle of the payoff landscape with few severe liabilities, so minimax optimization pushes mass there unless an equally robust alternative appears.

B. Replicator Dynamics and Short-Horizon Pressure

These theorems confirm local instability of the observed field under payoff-proportional adaptation. They also quantify why "play what won last week" can be dynamically fragile: positive payoff differential in one subpopulation induces immediate growth pressure, while overplayed negative-fitness choices contract.

VIII. Tournament Strategy

This section translates verified matchup results into direct tournament decisions.

A. Best-of-Three Amplification

Listing 10. Replicator step and above-average-fitness growth direction.

```
def replicatorStep
  (eta : Rat) (x : Mixed14) : Mixed14 :=
  normalize
    (fun i =>
      x.w i *
        (1 + eta *
          (fitness x i - meanFitness x)))

theorem grimmsnarl_above_avg_fitness :
  avgFitness 4 realCyclePayoff
    realCycleMeta <
  fitness 4 realCyclePayoff
    realCycleMeta  1, by  decide := by
  native_decide

theorem grimmsnarl_share_increases :
  realCycleMeta  1, by  decide <
  replicatorStep 4 realCyclePayoff
    realCycleMeta (1/100)
       1, by  decide := by
  native_decide
```

TABLE V
Best-of-Three Amplification of Key Matchups

Matchup	Game 1 WR	Bo3 WR
Grimmsnarl vs Dragapult	57.2%	60.7%
Gardevoir vs Dragapult	62.7%	68.6%
Raging Bolt vs Mega Absol	67.3%	74.9%
Mega Absol vs Grimmsnarl	62.1%	67.8%

Listing 11. Bo3 amplification formula and above-parity amplification theorem.

```
def bo3 (p : Rat) : Rat :=
  3 * p^2 - 2 * p^3

theorem bo3_above_input
  (p : Rat) (hp0 : 1/2 < p)
  (hp1 : p < 1) :
  bo3 p > p := by
  nlinarith [hp0, hp1]
```

Bo3 increases separation away from 50%. As a result, identifying one large favorable pairing is often more valuable than smoothing several near-even pairings. Formally, for $p > 0.5$, the transformation $p \mapsto 3p^2 - 2p^3$ is strictly above p. Hence a deck with one reliable edge can convert modest game-one advantages into materially larger round-level conversion.

B. Metagame Read EV and Tier Classification

In this snapshot, Grimmsnarl and Mega Absol define the S-tier frontier by expected field performance, while Ceruledge is decisively C-tier due to negative fitness and extinction pressure in dynamic models. This thresholded

Listing 12. Tier classification from verified expected WR thresholds.

```
inductive Tier where
  | S | A | B | C
  deriving Repr, DecidableEq

def tierOf (wr : Rat) : Tier :=
  if wr >= 0.52 then .S
  else if wr >= 0.505 then .A
  else if wr >= 0.48 then .B
  else .C

theorem tier_assignments :
  tierOf 0.527 = .S /\
  tierOf 0.521 = .S /\
  tierOf 0.504 = .B /\
  tierOf 0.431 = .C := by
  native_decide
```

classification turns expected-win estimates into actionable deck groups for preparation.

IX. Formalization Methodology and Statistics

A. Module Organization and Proof Throughput

The codebase is split by semantic layer so foundational rules remain independent from empirical metagame modules. This keeps dependency boundaries clear and limits recompilation when data updates.

X. Threats to Validity

Temporal snapshot. The metagame window is three weeks. Format shifts can alter both payoff matrix entries and archetype shares. Our claims are exact for this window and should be re-evaluated after major set releases.

Archetype aggregation. Each archetype bucket contains list-level variation. If one variant has systematically different matchups, aggregated win rates may blur finer structure.

Unmodeled tail. Low-share decks outside the top 14 are excluded from matrix-game equilibrium computations. A sufficiently strong tail strategy could perturb equilibrium concentration.

Behavioral mechanism inference. Anchoring/herding/-cascade explanations are consistent with observed adoption patterns but are not directly identified by controlled experiments. They should be interpreted as plausible mechanisms, not uniquely proven causes.

XI. Conclusion

We present a formally verified route from TCG rules to tournament strategy. The pipeline includes executable game semantics, exact probability theorems, empirically grounded payoff matrices, paradox proofs, equilibrium computation, dynamic analysis, and player-facing tournament math.

The central result is robust and actionable: the most popular archetype in the observed field is not the highest-value choice, and in this snapshot is below 50% expected win rate. Formal methods therefore do more than certify software; they can also certify strategic conclusions in competitive ecosystems with noisy human behavior.

Future work includes longitudinal re-estimation of the payoff matrix, explicit uncertainty-aware equilibria, and extensions to other card-game formats.

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TABLE VI
Formal Module Breakdown with Line and Theorem Counts

Module Group	Files	Lean LOC	Theorems/Lemmas
Core cards, zones, and turn semantics	14	6,240	418
Deck legality and validators	8	2,980	227
Card effects and invariants	12	5,110	361
Probability and combinatorics	10	4,460	298
Metagame data encoding	9	3,780	214
Nash and optimization witnesses	7	3,120	181
Replicator dynamics and stability	8	2,940	201
Tournament strategy layer	7	1,470	124
Utilities and automation	10	730	64
Total	85	30,830	2,088