THE UNIVERSITY OF CHICAGO
HARRIS SCHOOL OF PUBLIC POLICY

Mini-lecture 4

DAC 2023

Today's **Agenda**

1. Bivariate Regression
2. Goodness of Fit
3. Confidence Intervals
3. T-statistics and P-values

Bivariate Regression

Bivariate regression

- **Regression** is a statistical method to assess:
  - The relationship between two variables and its strentgth
- **When** is regression a good model (a model with explanatory power)?
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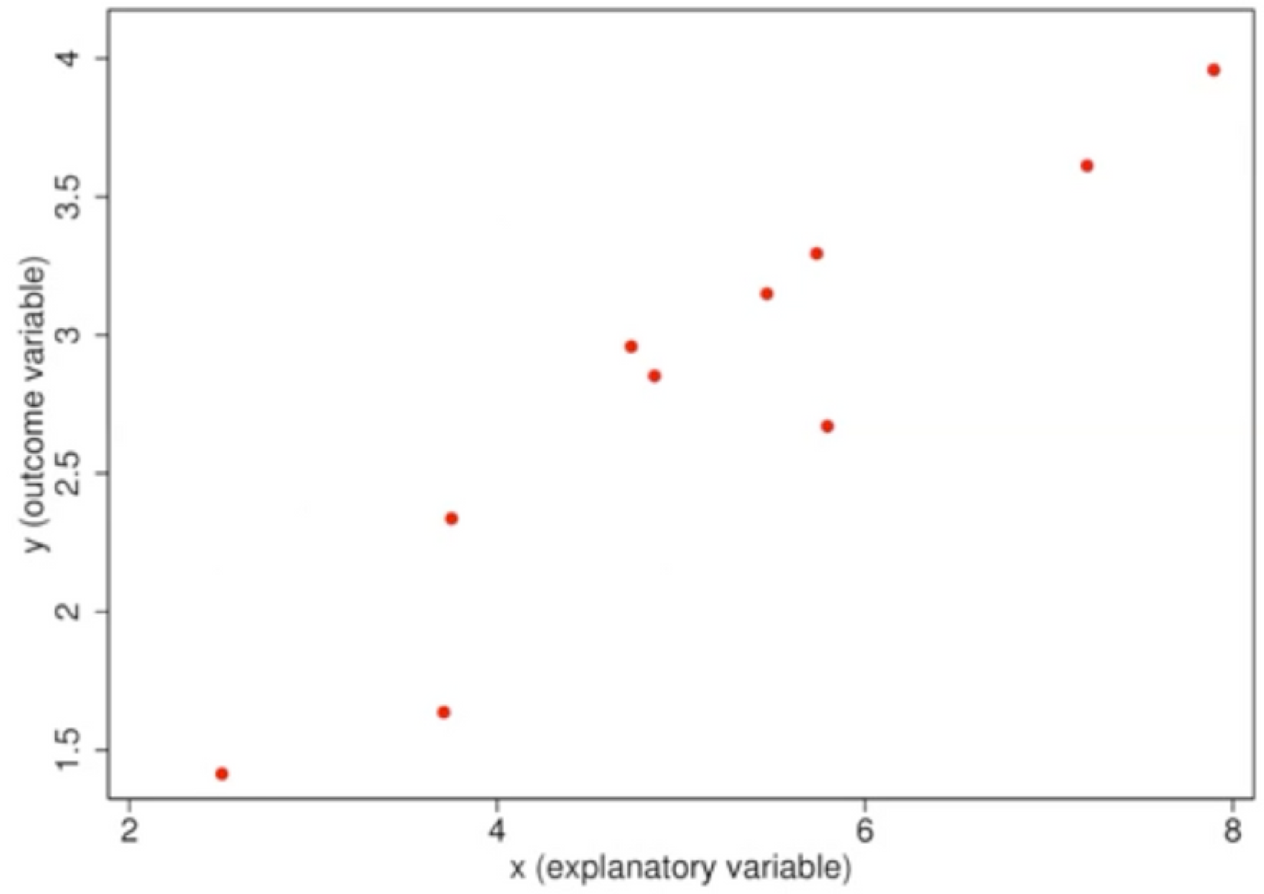

- What does linear mean here?
  - This means that a change in one variable corresponds to a proportional change in the other variable

Bivariate regression

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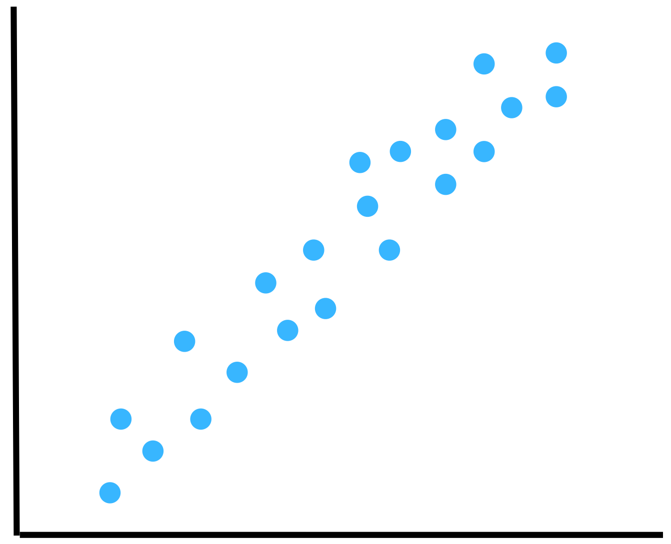

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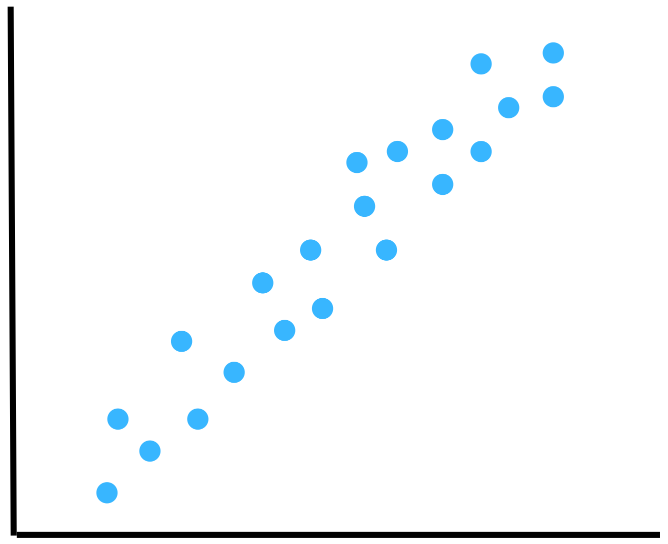
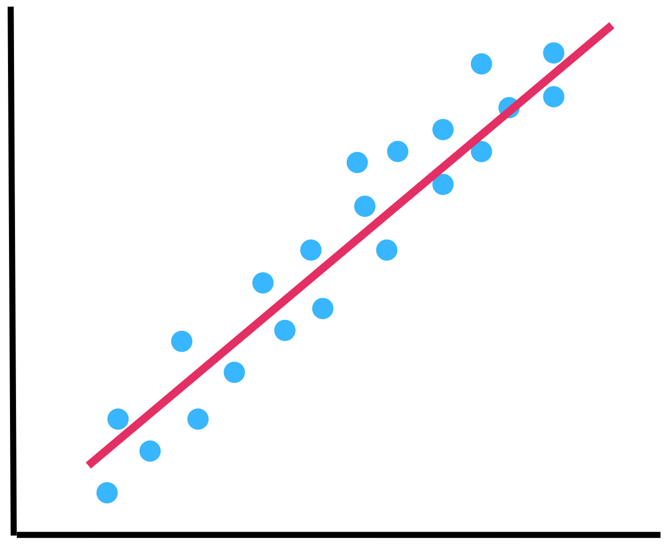


Regression can fit a line...



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By numerically, we should understand it gives us an **intercept** (where does the line intercept the Y-axis) and a **slope** (by how much Y is increased by a one-unit increase on the X (independent variable)

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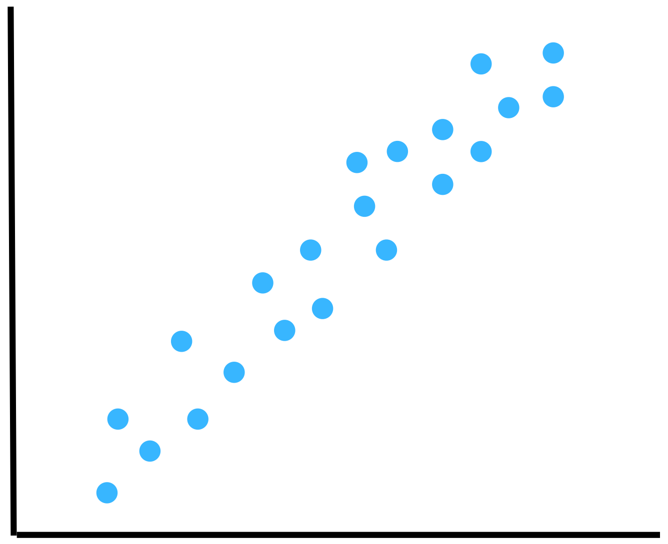

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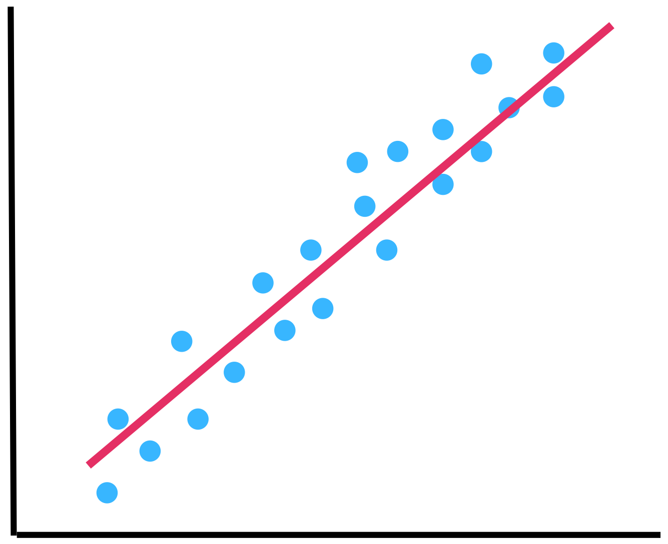

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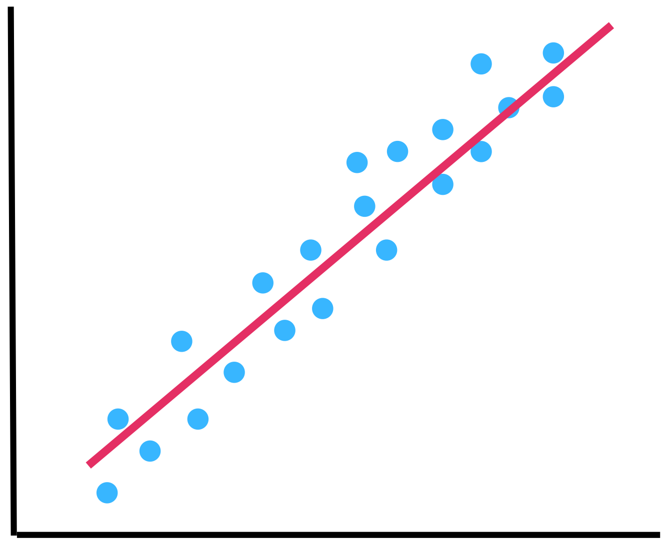
*Y = Bo + B1*X1 +ui*



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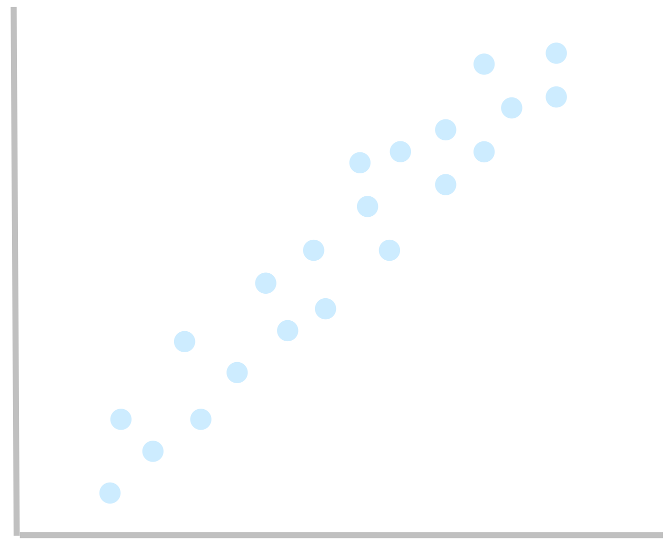

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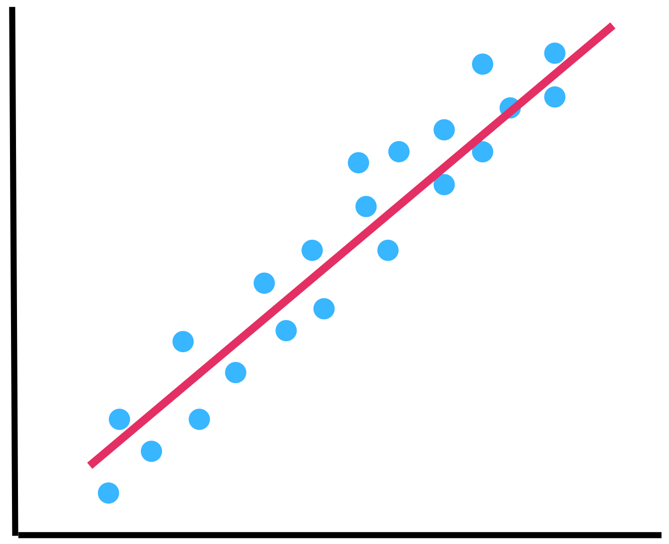

Step 2!

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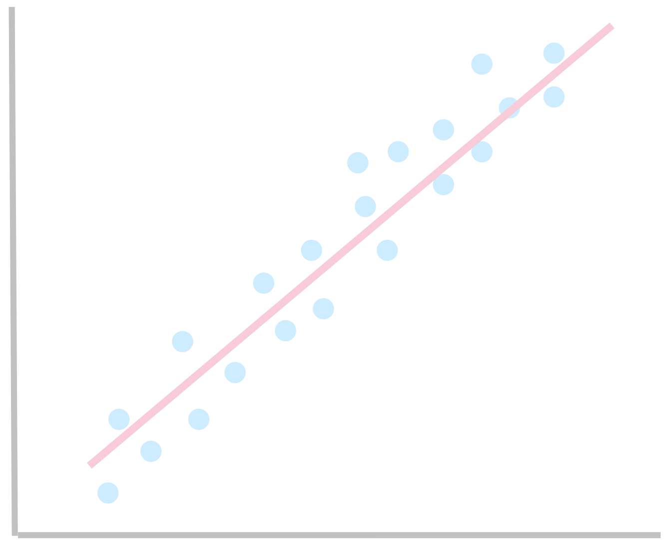

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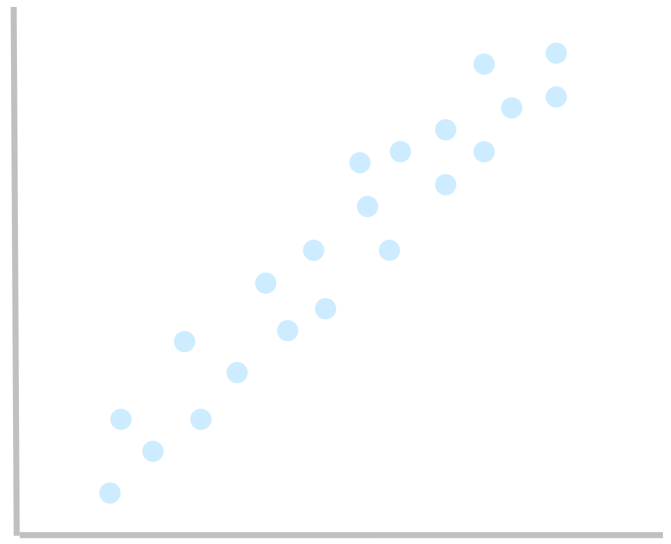

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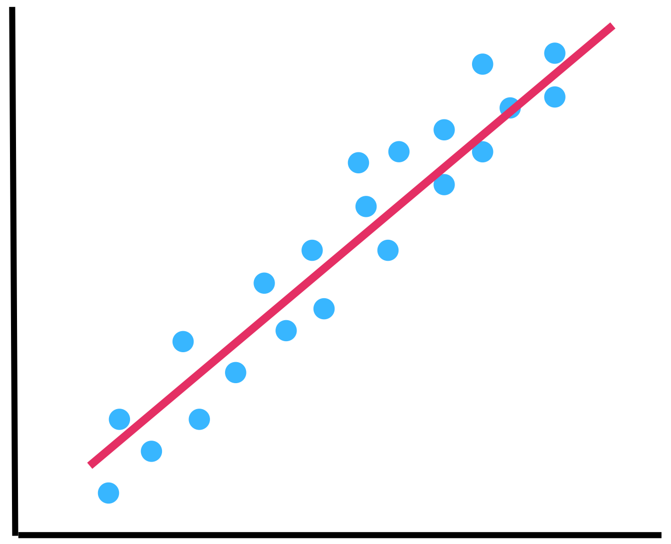

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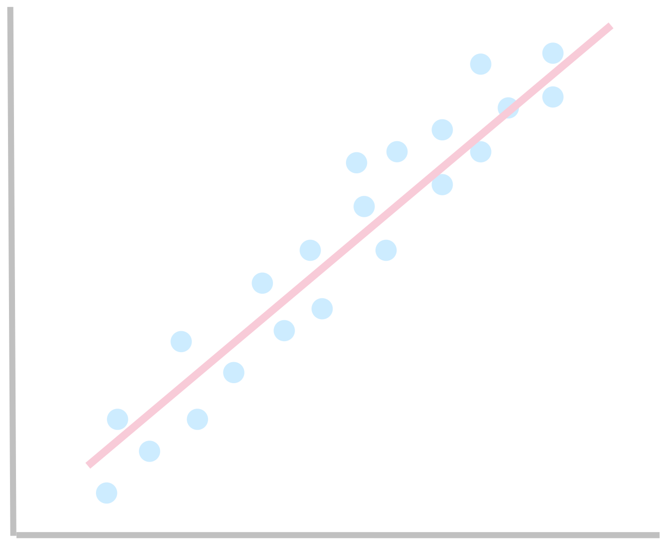
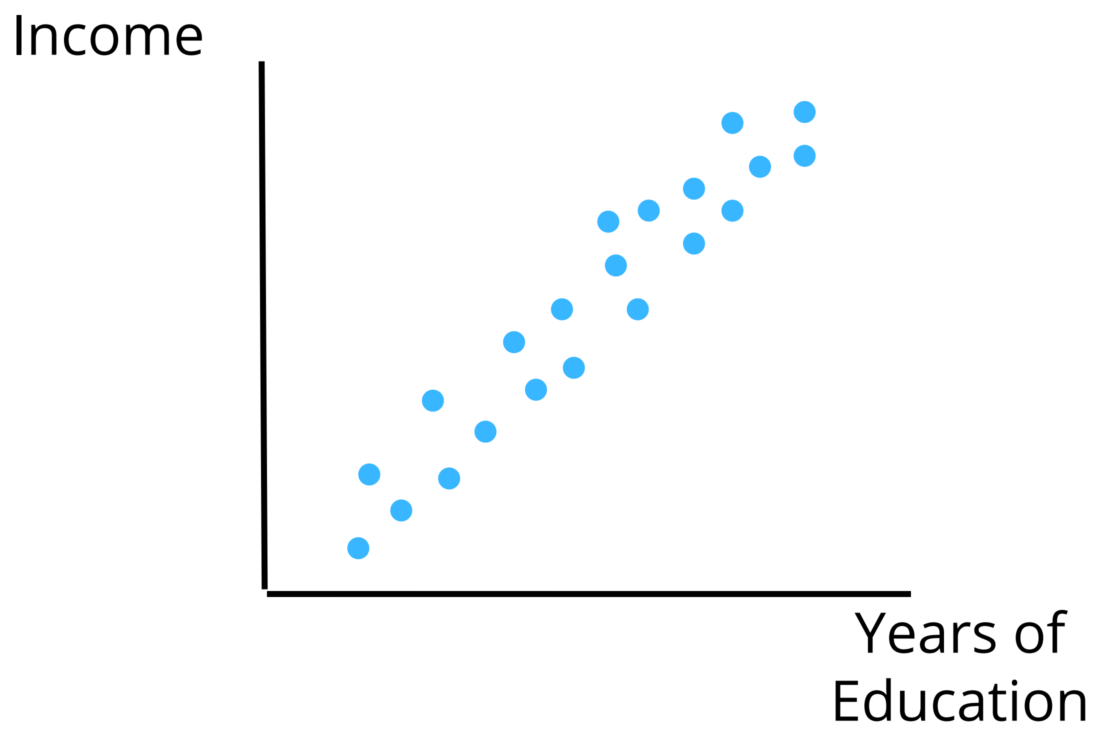


Step 2!

- This is when the model will discover what is the numeric relationship
  - The "line of best fit" is precisely a method to plug numbers into the linear model
  - **OLS!**
  - It turns out that minimizing the squared sum of the residuals (=OLS) is a very good guess of what Y (dependent variable) would be given our X (independent variable)

Example!

If we were trying to understand by **how much** increasing the years of education of a citizen impacts his/her income



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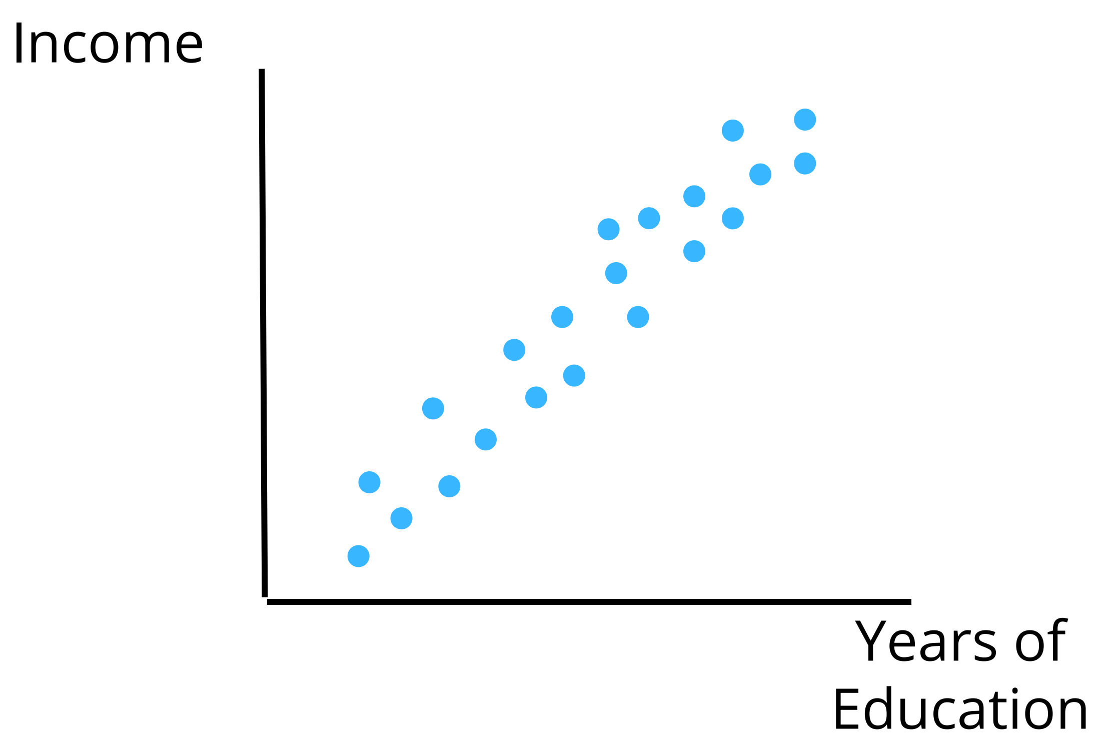
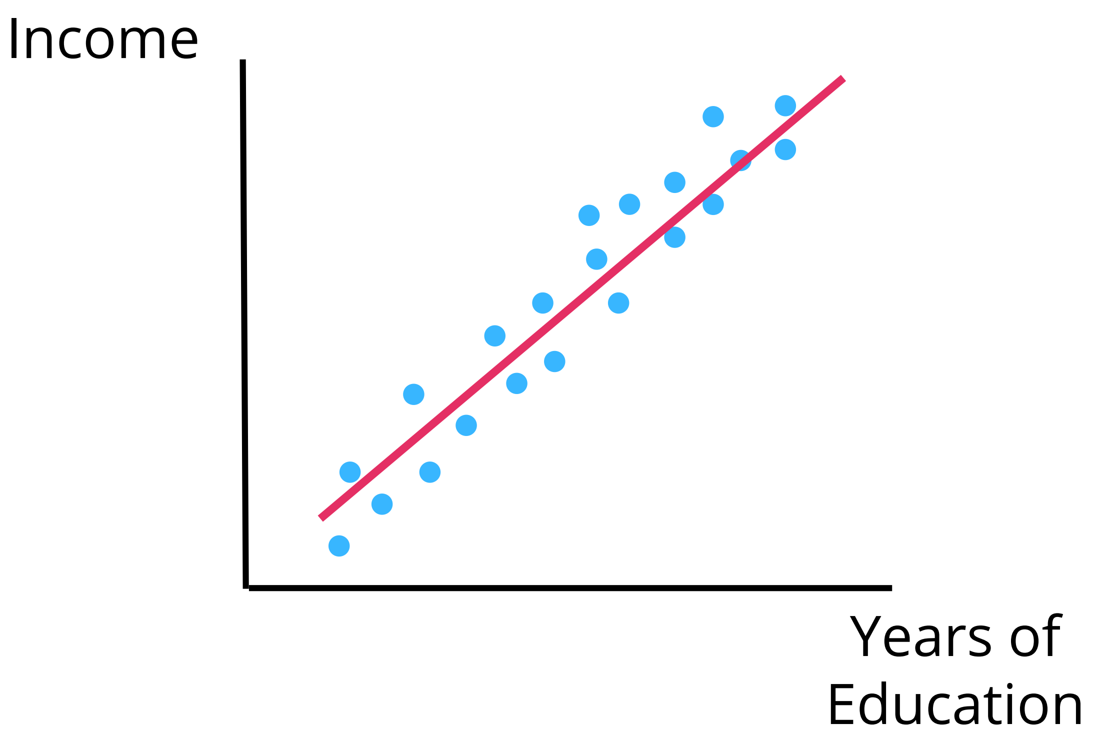


We would run our OLS model - that would fit a line between the variables



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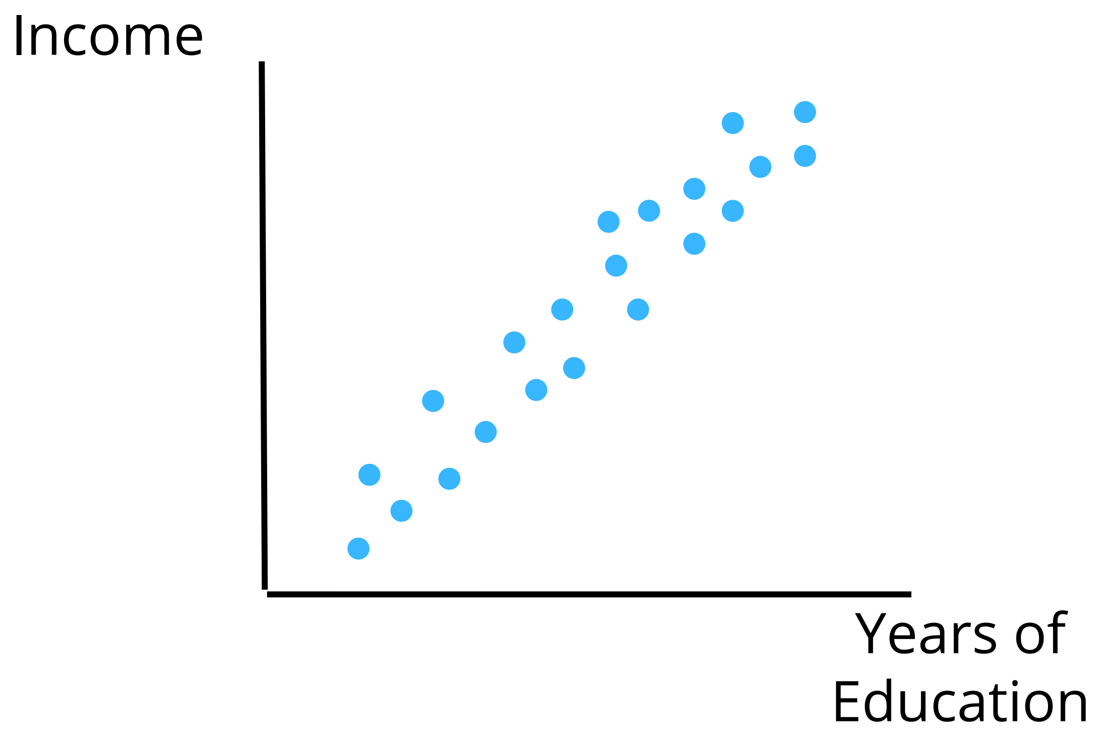

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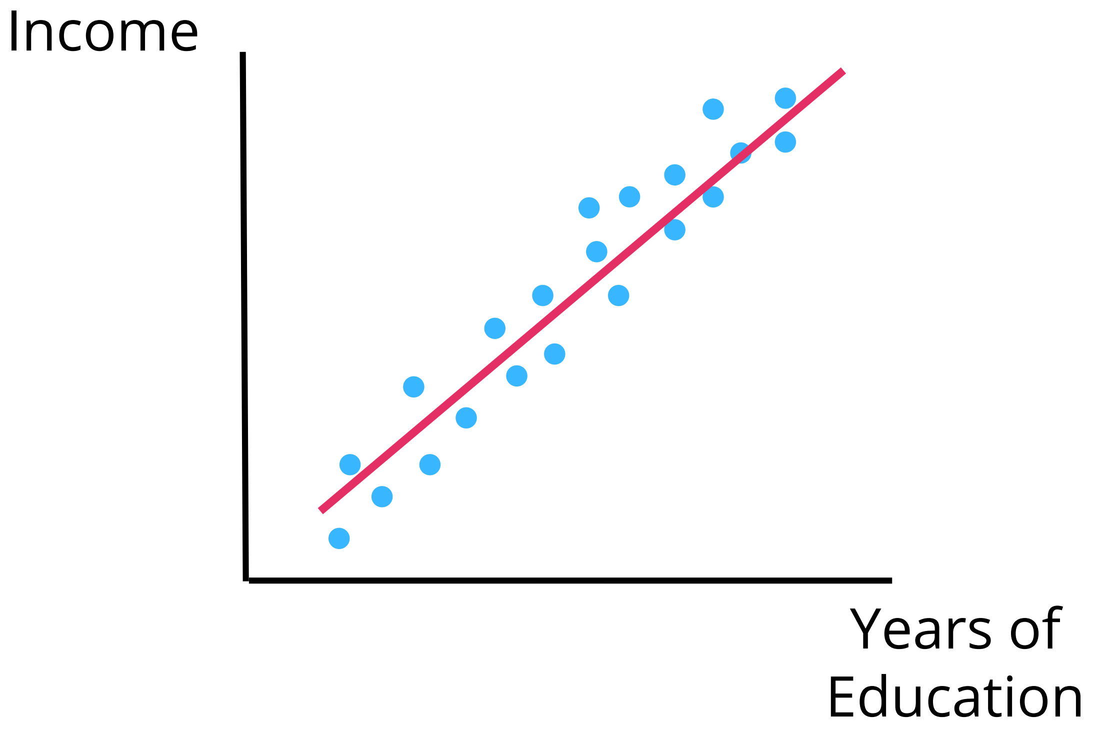

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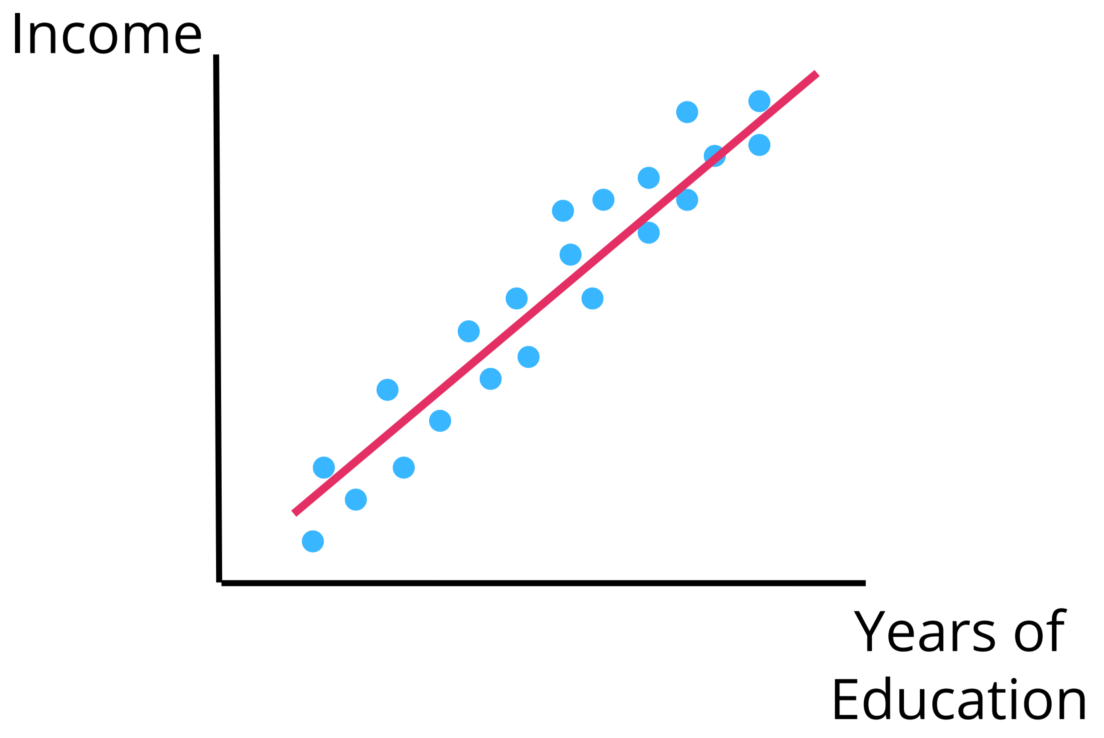
*Income = 10,000 + 15,000 * Years of Education + ui*



Goodness of Fit

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- **R-squared** measures the proportion of the total variation in the dependent variable (TSS) that is explained by our independent variable (ESS)

$$R^2 = 1 - \frac{RSS}{TSS}$$

R^2 = coefficient of determination

RSS = sum of squares of residuals

TSS = total sum of squares

(RSS): measures the unexplained variability in the dependent variable

(TSS): measures the total variability in the dependent variable

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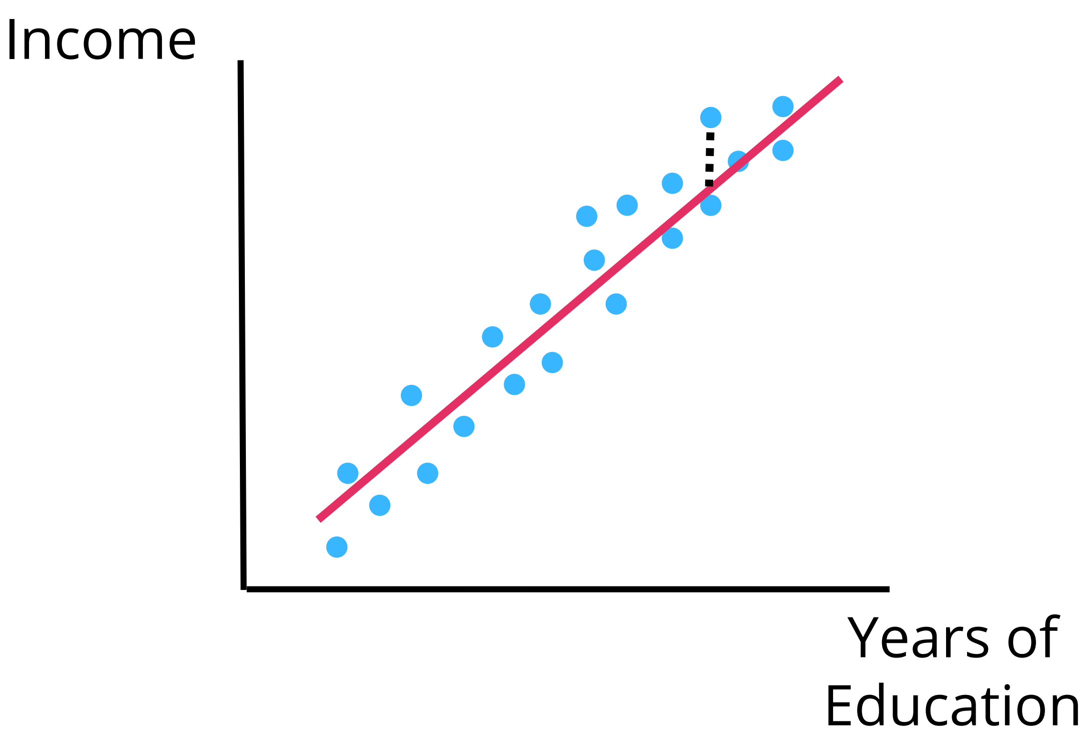

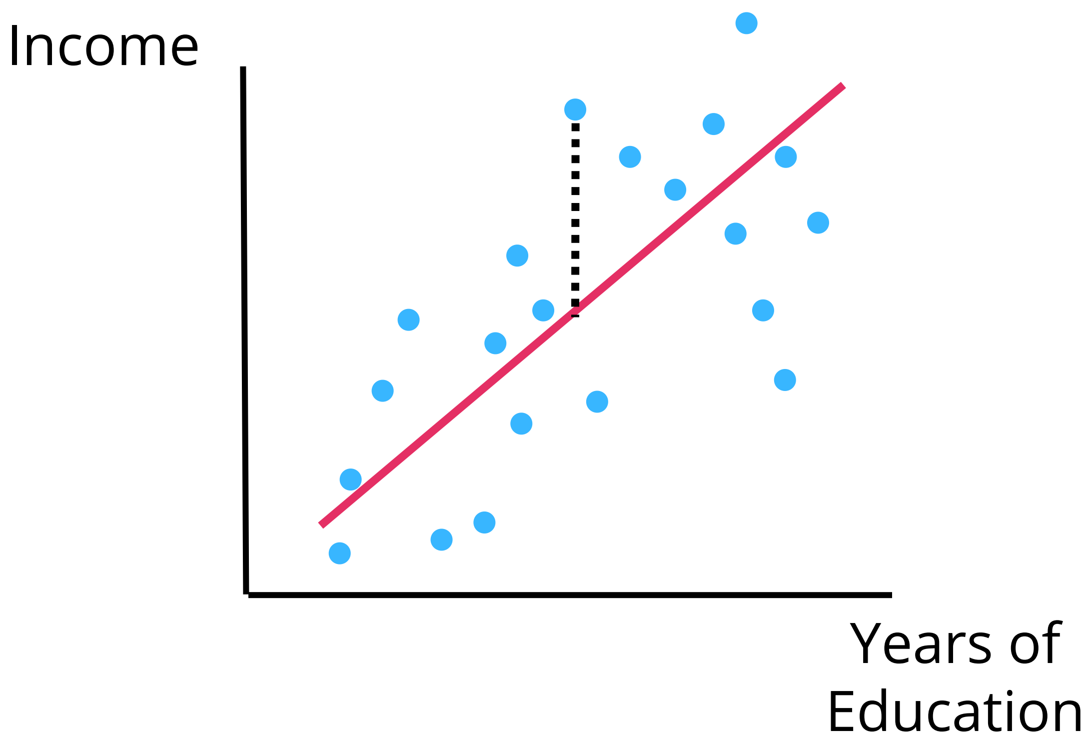

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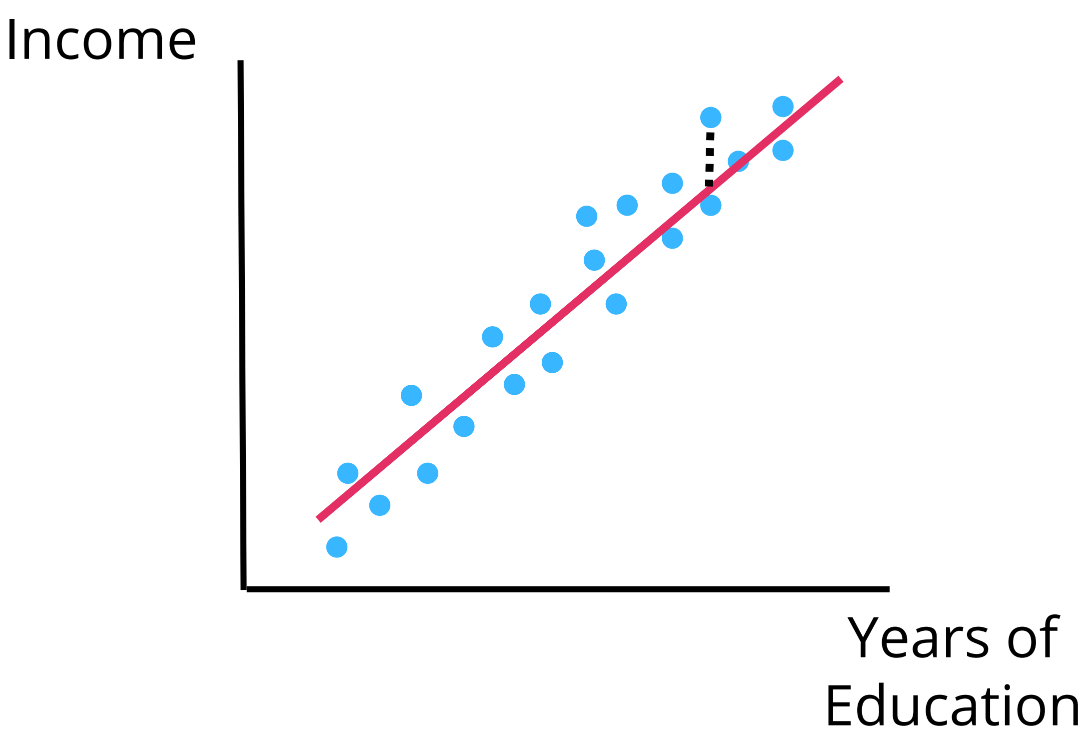
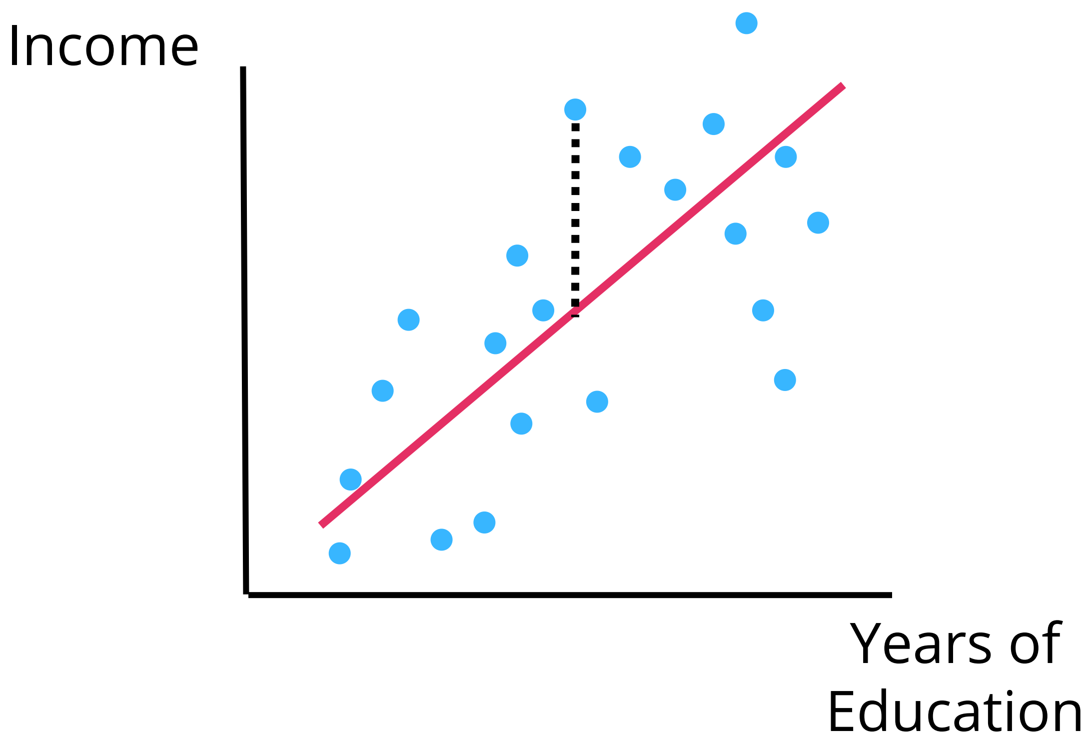


High R-squared



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Without calculating
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Point estimate

24.5

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Calculating
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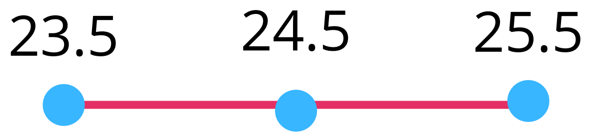
Interval



"Giving the mean of my sample, I'm 99% confident that the average age of this class (population) is between 23.5 and 25.5 years old"

The questions then is: **how to calculate the CI?**

Formula

$$CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$$

CI = confidence interval

$\bar{x}$ = sample mean

z = confidence level value

s = sample standard deviation

n = sample size

But we use the t-test (and not the z)! Why?

When the standard deviation (s) is estimated, then normalizing yields a t-distribution (similar shape to the bell curve)

t-stats and p-values

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- ***t*-stats:** a statistical measure we will need when constructing our confidence intervals (or hypothesis testing) when estimating our population parameters.
  - When running **regressions** our t-stats or t-value is calculated by:
    - t = B / SE
      - B = our estimate
      - SE = the standard error of the coefficient

- ***p-value***: is a statistical measure used in hypothesis testing to determine the strength of evidence against a null hypothesis. If the null hypothesis were true, then how likely (probability) would it be for me to obtain a value like this coefficient I've estimated?

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Thus, if our sample show this:

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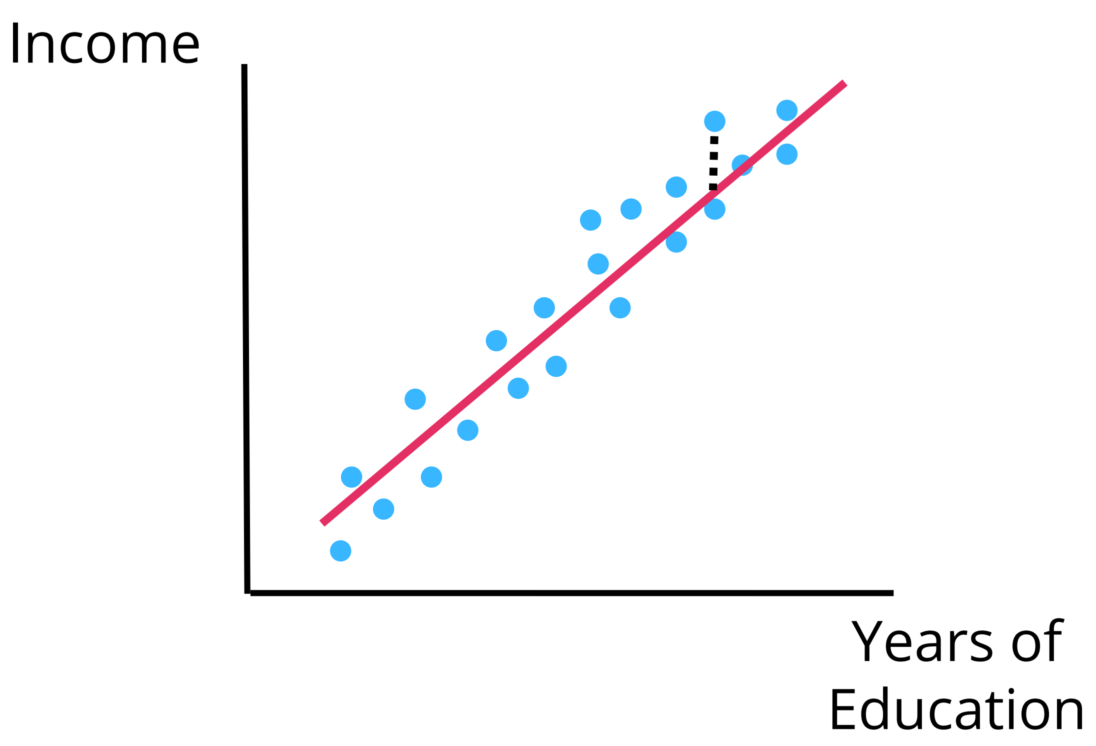

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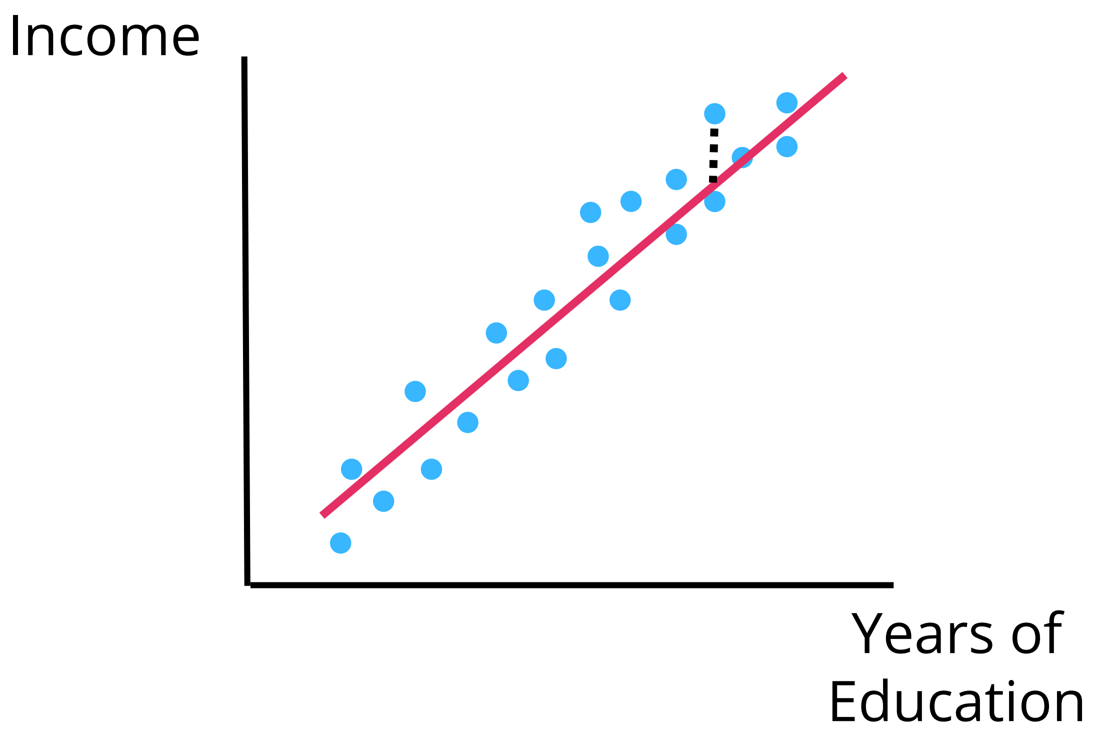
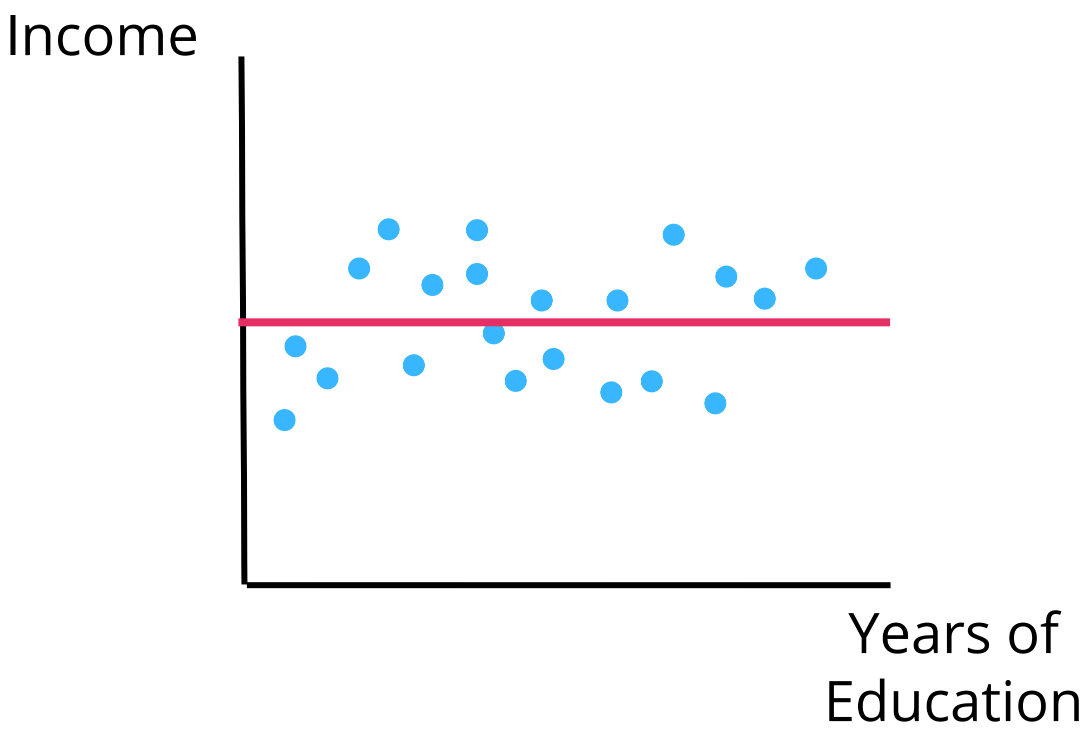


... but the null hypothesis says this

Coefficient estimate (slope) = 0.00



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- **Interpreting the p-value**
  - When our p-value is lower than our rule of thumb (=0.05) or .0.01, we say that the probability is so low that we reject the null hypothesis (which is normally formulated in terms of the estimation being actually equal to zero.

Thank you!

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