TEXAS A&M UNIVERSITY

Quiz 6

MATH 308, Texas A&M University
Spring 2020, Lynch

NAME (PRINT LAST, FIRST): CHEN, Arthur

The work on this quiz should be your own. Speaking about it with others is NOT allowed. Please sign the Aggie Code of Honor with this understanding.

If using scrap paper, write this statement down on your own and sign:

AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

SIGNATURE: Arthur Chen

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b. A mass of 16 kg stretches a spring 4.9 cm. The mass moves in a medium that imparts a viscous force of 4 N when the speed of the mass is 1 cm/s. The mass is set in motion from its equilibrium position with an initial upward velocity of 5 cm/s. No external force is applied. Write the IVP so that u would be in meters if solved.

Given : mass = 16kg , $L = 0.049m$, $U'(0) = -0.05\,m/s$

Solution : $mg = kL \Rightarrow k = \frac{mg}{L} = \frac{16\times 9.8}{0.049} = 3200\,N/m$, $\delta = 400\,N.s/m$

$mu'' + \delta u' + ku = 0$

$\Rightarrow 16u'' + 400u' + 3200u = 0$

$\Rightarrow u'' + 25u' + 200u = 0$

And $u(0) = 0$, $u'(0) = -0.05\,m/s$

1. (12 pts) Find the corresponding initial value problem for each of the following spring-mass systems, that if solved, would give the position u of the mass at any time t. Use 32 ft/s^2 or 9.8 m/s^2 for the acceleration due to gravity. **DO NOT SOLVE THE IVP. Show and explain all of your work. Use only exact numbers – NO DECIMALS.**

   a. A mass weighing 10 lb stretches a spring 5 in. The mass is pushed up, contracting the spring 2 in and then dropped. The mass is acted on by an external force of $3\sin(2t)$ lb and no damping is applied. Write the IVP so that u would be in feet if solved.

Given: weigh $= 10\,lb$, length $= 5\,in = \frac{5}{12}ft$, $u(0) = -2\,in = -\frac{1}{6}ft$, $u'(0) = 0$

Force $= 3\sin(2t)$, $\delta = 0$

Solution: $k = \frac{mg}{L} = \frac{10}{\frac{5}{12}} = 24\ lb/ft$, $m = \frac{10}{32} = \frac{5}{16}$ slugs

$\frac{5}{16}r^2 + 24 = 0 \Rightarrow r^2 + \frac{384}{5} = 0 \Rightarrow r = \pm\sqrt{\frac{384}{5}}\,i = \pm 8\sqrt{\frac{6}{5}}\,i$

$u_c = C_1\cos(8\sqrt{\tfrac{6}{5}}t) + C_2\sin(8\sqrt{\tfrac{6}{5}}t)$

$u_p = At\cos(8\sqrt{\tfrac{6}{5}}t) + Bt\sin(8\sqrt{\tfrac{6}{5}}t)$

$u_p' = A\cos(8\sqrt{\tfrac{6}{5}}t) - 8\sqrt{\tfrac{6}{5}}At\sin(8\sqrt{\tfrac{6}{5}}t) + B\sin(8\sqrt{\tfrac{6}{5}}t) + 8\sqrt{\tfrac{6}{5}}Bt\cos(8\sqrt{\tfrac{6}{5}}t)$

$u_p'' = -16\sqrt{\tfrac{6}{5}}A\sin(8\sqrt{\tfrac{6}{5}}t) - \frac{384}{5}At\cos(8\sqrt{\tfrac{6}{5}}t) + 16\sqrt{\tfrac{6}{5}}B\cos(8\sqrt{\tfrac{6}{5}}t) - \frac{384}{5}Bt\sin(8\sqrt{\tfrac{6}{5}}t)$

Substitution to $\frac{5}{16}u'' + 24u = 3\sin(2t)$

by comparasion:

$$\begin{cases} -5\sqrt{\tfrac{6}{5}}A = 3 \\ 5\sqrt{\tfrac{6}{5}}B = 0 \end{cases} \Rightarrow \begin{cases} A = -\frac{3}{5\sqrt{\frac{6}{5}}} \\ B = 0 \end{cases}$$

So, $U_p = -\frac{3}{5\sqrt{\frac{6}{5}}}t\cos(8\sqrt{\tfrac{6}{5}}t)$

$\because U = U_c + U_p$

$\therefore U = C_1\cos(8\sqrt{\tfrac{6}{5}}t) + C_2\sin(8\sqrt{\tfrac{6}{5}}t) - \frac{3}{5\sqrt{\frac{6}{5}}}t\cos(8\sqrt{\tfrac{6}{5}}t)$

$U' = -8\sqrt{\tfrac{6}{5}}C_1\sin(8\sqrt{\tfrac{6}{5}}t) + 8\sqrt{\tfrac{6}{5}}C_2\cos(8\sqrt{\tfrac{6}{5}}t) - \frac{3}{5\sqrt{\frac{6}{5}}}\cos(8\sqrt{\tfrac{6}{5}}t) + \frac{24}{5}t\sin(8\sqrt{\tfrac{6}{5}}t)$

Apply I.C.:

$$\begin{cases} -2 = C_1 - \frac{3}{5\sqrt{\frac{6}{5}}} \times 0 \Rightarrow C_1 = -2 \\ 0 = 8\sqrt{\tfrac{6}{5}}C_2 - \frac{3}{5\sqrt{\frac{6}{5}}} \Rightarrow C_2 = \frac{3}{40} \times \frac{5}{6} = \frac{5}{80} = \frac{1}{16} \end{cases}$$

So $U = -2\cos(8\sqrt{\tfrac{6}{5}}t) + \frac{1}{16}\sin(8\sqrt{\tfrac{6}{5}}t) - \frac{3}{5\sqrt{\frac{6}{5}}}t\cos(8\sqrt{\tfrac{6}{5}}t)$ ft