Quiz 7 Arthur Chen Math 308 32700336 8

① $f(t) = te^t$

$\because \mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}} \quad s>a$

$\therefore n=1, a=1$

$\therefore \mathcal{L}\{te^t\} = \frac{1!}{(s-1)^{1+1}} = \frac{1}{(s-1)^2}$

②

$f(t) = t^2 - t^2 h(t-2) + 6h(t-2) - 6h(t-4) + (t+2)h(t-4)$

③ $y'' - 4y = 0, y(0) = 0, y'(0) = 1$

$\because \mathcal{L}\{y''\} = s^2 Y(s) - sY(0) - Y'(0)$

$\mathcal{L}\{4y\} = 4Y(s)$

$\therefore \mathcal{L}\{y'' - 4y = 0\} \Rightarrow s^2 Y(s) - sY(0) - Y'(0) - 4Y(s) = 0$

$\Rightarrow s^2 Y(s) - 0 \cdot s - 1 - 4Y(s) = 0$

$\Rightarrow (s^2 - 4) Y(s) = 1$

$\Rightarrow Y(s) = \frac{1}{s^2-4}$

$Y(s) = \frac{1}{(s+2)(s-2)} = \frac{A}{s+2} + \frac{B}{s-2}$

$\Rightarrow 1 = (s-2)A + (s+2)B$

$\Rightarrow 1 = As - 2A + Bs + 2B$

$\Rightarrow \begin{cases} A+B=0 \\ -2A+2B=1 \end{cases}$

$\Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{1}{4} \end{cases}$

$\Rightarrow Y(s) = -\frac{1}{4(s+2)} + \frac{1}{4(s-2)}$