Quiz 8 Arthur Chen 3270033~~?~~68

Aggie Code of Honor:

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Arthur Chen

1(a) $x'' - x' - 6x = \delta(t-3)\cos(\pi t),\ x(0)=1,\ x'(0)=3$

$\Rightarrow \mathcal{L}\{x''-x'-6x\} = \mathcal{L}\{\delta(t-3)\cos(\pi t)\}$

$\Rightarrow [s^2\mathcal{L}(x(t)) - s\,x(0) - x'(0)] - [s\mathcal{L}(x(t)) - x(0)] - 6\mathcal{L}(x(t)) = e^{-3s}\frac{s}{s^2+\pi^2}$

$\Rightarrow (s^2-s-6)\mathcal{L}(x(t)) - s - 3 + 1 = e^{-3s}\frac{s}{s^2+\pi^2}$

$\Rightarrow \boxed{\mathcal{L}(x(t)) = \dfrac{e^{-3s}\frac{s}{s^2+\pi^2} + s + 2}{s^2-s-6}}$

1(b) $\frac{1}{s^2-s-6} = \frac{1}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2} \Rightarrow 1 = A(s+2) + B(s-3) \Rightarrow \begin{cases} A+B=0 \\ 2A-3B=1 \end{cases}$

$\Rightarrow \begin{cases} A = \frac{1}{5} \\ B = -\frac{1}{5} \end{cases} \Rightarrow \mathcal{L}(x(t)) = \left(\frac{1}{5(s-3)} - \frac{1}{5(s+2)}\right)\left(e^{-3s}\frac{s}{s^2+\pi^2} + s + 2\right)$

$= \left(\frac{s}{5(s-3)(s^2+\pi^2)} - \frac{s}{5(s+2)(s^2+\pi^2)}\right)e^{-3s} + \frac{(s+2)}{5(s-3)} - \frac{1}{5}$

$\boxed{F(t) = \frac{s}{s^2+\pi^2}\left(\frac{1}{5(s-3)} - \frac{1}{5(s+2)}\right)}$

$\mathcal{L}^{-1}(F(t)) = \mathcal{L}^{-1}\left(\frac{s}{(s-3)(s+2)(s^2+\pi^2)}\right)$

$= \beta$ (assumtion, will continou by this)

$\frac{s}{(s-3)(s+2)(s^2+\pi^2)} = \frac{A}{(s-3)} + \frac{B}{s+2} + \frac{Cs+D}{s^2+\pi^2}$

$= A(s^3+2s^2+\pi^2 s+2\pi^2) + B(s^3-3s^2+\pi^2 s-3\pi^2)$
$C(s^3-s^2-6s) + D(s^2-s-6)$

$\Rightarrow \begin{cases} (A+B+C)=0 \\ 2A-3B-C+D=0 \\ \pi^2 A+\pi^2 B-6C-D=1 \\ 2\pi^2 A-3\pi^2 B-6D=0 \end{cases} \Rightarrow$ can't solve

$x(t) = \beta h(t-3) + \mathcal{L}^{-1}\left\{\frac{1}{5} - \frac{1}{5s+15}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{5}\right\}$

$= \beta h(t-3) + \frac{1}{5}\delta(t) - \frac{1}{5}e^{-3t} - \frac{1}{5}\delta(t)$

$\boxed{\begin{aligned} &= \beta h(t-3) - \tfrac{1}{5}e^{-3t} \\ &\text{where } \beta = \mathcal{L}^{-1}(F(t)) \end{aligned}}$

2. $H(s) = \frac{6e^{-9s}}{2s^6+4s^5+20s^4}$, $\varphi(t) = (f * g)(t)$

$f(t) = (t-9)^3 h(t-9)$

$L\{f(t)\} = L\{t^3 h(t-9)\} = \boxed{\frac{6}{s^4} e^{-9s}}$

$G(t) = H(s)/F(t) = \frac{6e^{-9s}}{2s^6+4s^5+20s^4} \Big/ \left(\frac{6}{s^4} e^{-9s}\right)$

$= \frac{s^4}{(2s^6+4s^5+20s^4)} = \frac{1}{2s^2+4s+20} = \frac{1}{2} \cdot \frac{1}{(s+1)^2+9}$

$g(t) = L^{-1}\{G(t\} = \frac{1}{2} L^{-1}\{\frac{1}{(s+1)^2+9}\} = \frac{1}{2}\left(\frac{1}{3} e^{-t} \sin(3t)\right) = \boxed{\frac{1}{6} e^{-t} \sin(3t)}$

$L^{-1}\{H(s)\}(t) = \int_0^t f(w) g(t-w)\,dw$

$= \int_0^t (w-9)^3 h(t-9) \frac{1}{6} e^{-(t-w)} \sin(3(t-w))\,dw$

$= \int_0^t (w-9)^3 \left(\frac{1}{6} e^{w-t}\right)(\sin(3t-3w))\,dw \quad t \geq 9$