CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 9

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
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Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, Mar 31, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".
  NONE

Exercises for Section 6.1:

8: (1 point).
by product rule:
since there are 26 letters and cannot be repeated,
the total number of possible cases is 26*25*24 = 15600

16: (2 points).
to calculate the number of string with at least one x, we can calculate the number of string without x, and let it be subtracted by the total number of all cases.
Note that there are 26 letters
the total number of any string is $26^4 = 456976$
the number of string without x is $25^4 = 390625$
so the number of string with at least one x is $456976 - 390625 = 66351$

28: (1 point).
Note that we have 26 letters and 10 digits
by product rule:
for first three digits are letters and last three are numbers:
$26^3 * 10^3 = 17576000$
similarly, for first three digits are numbers and last three are letters:
$10^3 * 26^3 = 17576000$
and the answer is sum of them: $17576000 * 2 = 63273600$
48c: (1 point).
to calculate the number of exactly one groom or bride, we have to first calculate part a and b
part a: since at least one person is bride, so one person is fixed and the rest can be any
$1 * 9 * 8 * 7 * 6 * 5 = 15120$
and bride can be at any position so $6 * 15120 = 90720$
similarly, the number of at least one groom is 90720 as well.
part b: similarly, first two person are fixed and the rest can be any:
$1 * 1 * 8 * 7 * 6 * 5 = 1860$
the bride can be at any position in rest five people, and groom and be at any position among rest of four people, so
$6 * 5 * 1680 = 50400$
the number of exactly one bride is the number of at least one minus the number of number of them appears.
$90720 - 50400 = 40320$
similarly, exactly one groom is 40320
by sum rule, the answer is the sum of them, $40320 + 40320 = 80640$

Exercises for Section 6.2:

18: (2 points).
pigeons are 1,3,5,7,9,11,13,15
pigeonholes are 1,15,3,13,5,11,7,9
we have four holes. Therefore, we need at least 5 to get all numbers in any one of the sets.
which is calculated from $\lceil x/4 \rceil = 2$, and $x = 5$

20b: (2 points).
in this question, male and female are two holes.
assume students are either male or female
if we have less than 3 males, since we have 9 students, we will have more than 6 female, which means at least 7.
if we have less than 7 females, since we have 9 students, we will have more than 2 male, which means at least 3.

40: (2 points).
since we need guarantee the access of four of eight computer to any of the printers, this means the any of other four computers only needs to connect to one of the printers while being chosen.
to connect four of computers to all four printers, we need 4*4=16 cables. Then, to connect rest four computers to either of the printers we need 4*1=4 cables.
so we need 16+4=20 in total

Exercises for Section 6.3:

30: (1 point).

since the order is not important, we are using the definition of combination:

$$\binom{40}{17} = \frac{40!}{17!(40-17)!} = 88,732,378,800$$

32b: (1 point).

to let the committees has at least one of both sex, we have to calculate the number of any, only female, and only male.

without any restriction, C(9+7,5) = C(16,5) =

$$\binom{16}{5} = \frac{16!}{5!(16-5)!} = 4368$$

only male, C(9,5) =

$$\binom{9}{5} = \frac{9!}{5!(9-5)!} = 126$$

only female, C(7,5) =

$$\binom{7}{5} = \frac{7!}{5!(7-5)!} = 21$$

so the possible committee with at least one male and one female will be the total number minus the number of all male and the number of all female:

$4368 - 126 - 21 = 4221$

38: (2 points).

since the order does not matter, we will apply combination definition.

based on the question, we can think 011 as one block. Therefore, we have five blocks and 4 remaining ones, and those ones can be any position between the blocks.

so now we can treat it as nine digits (5+4=9)

since there are nine digits and four ones can be anywhere among nine digits, we are choosing 4 positions of ones within 9 positions.

$$\binom{9}{4} = \frac{9!}{4!(9-4)!} = 126$$

so there are 126 ways.

Exercises for Section 6.4:

8: (1 point).
based on the binomial theorem, n=17, and j=9
the corresponding term is:

$$\binom{n}{j}(3x)^{n-j}(2y)^j = \binom{17}{9}(3x)^{17-9}(2y)^9 = 81,662,929,920x^8y^9$$

26b: (2 points).

$$\binom{n}{k}\binom{n-k}{r-k} = \frac{n!}{k!(n-k)!}*\frac{(n-k)!}{(r-k)!(n-r)!} = \frac{n!}{k!(r-k)!(n-r)!} = \frac{n!}{r!(n-r)!}*\frac{r!}{k!(r-k)!} = \binom{n}{r}\binom{r}{k}$$

32b: (2 points).

$$\binom{2n}{2} = \frac{(2n)!}{2!*(2n-2)!} = \frac{2n*(2n-1)}{2} = 2n^2 - n$$

$$2\binom{n}{2} + n^2 = \frac{2*n!}{2*(n-2)!} + n^2 = n(n-1) + n^2 = n^2 - n + n^2 = 2n^2 - n$$

they are the same