CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 1

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Arthur Chen

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Each exercise is worth 1 point.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, January 21, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".

$NONE$

LaTeX hints: Read this .tex file for some explanations that are in the comments.

Math formulas are enclosed in $ signs, e.g., `$x + y = z$` becomes $x + y = z$.

Logical operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$.

Here is a truth table using the “tabular” environment:

p	$\neg p$
T	F
F	T

Exercises for Section 1.1:

8(e):

let’s say the statement p is ”A has more RAM than B” and q is ”B has more RAM than A”. ”if and only if” indicates that $p \iff q$, this statement is true only if p and q are the same, but apparently they are not.
test: p is true so A is more than B which means B is less than A and makes q false.
So this statement is false.
12(h):
the votes have not been counted or the election is not decided and the votes have been counted

34(f):

p	q	$(p \iff q) \oplus (p \iff \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

46(a,c,e): (1 pt each)
a. since the statement is true, so x plus one. now is two.
c. since the statement is true, so x plus one. now is two.
e. since the statement is true, so x plus one. now is two.

Exercises for Section 1.2:

10:
set up three statements:
p: ”software is being upgraded”
q: ”user can access the file system”
r: ”users can save the files”
then we have:
$p \implies \neg q$
$q \implies r$
$\neg r \implies \neg p$
”system specifications consistent” means that, from all the possible states, there is at least one state that makes all the propositions true. So we need a full truth table.

p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \implies \neg q$	$q \implies r$	$\neg r \implies \neg p$
T	T	T	F	F	F	F	T	T
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T	T
T	F	F	T	T	T	T	T	F
F	T	T	F	F	T	T	T	T
F	T	F	F	T	T	T	F	T
F	F	T	T	F	T	T	T	T
F	F	F	T	T	T	T	T	T

from the truth table above we can see that, in some of the cases, all the propositions are true. Therefore, the system specification is consistent. **18(c):**
it is possible that exactly 2 of them are true.
consider trunk 1 and 2 has treasure, in this case, inscriptions of trunk 2 and 3 are true which matches the statement.

38:
with the given question, we can set up five statements:
p: Either Kevin or Heather, or both, are chatting
q: Either Randy or Vijay, but not both, are chatting
r: If Abby is chatting, so is Randy
s: Vijay and Kevin are either both chatting or neither is
t: If Heather is chatting, then so are Abby and Kevin
time for truth table again!
note that only the cases that satisfy p and q are listed for the clearness of truth table

$Heather$	$Kevin$	$Randy$	$Vijay$	$Abby$	p	q	r	s	t
T	T	T	F	T	T	T	T	F	T
T	F	T	F	T	T	T	T	T	F
F	T	T	F	T	T	T	T	F	T
T	T	F	T	T	T	T	F	T	T
T	F	F	T	T	T	T	F	F	F
F	T	F	T	T	T	T	F	T	T
T	T	T	F	F	T	T	T	F	F
T	F	T	F	F	T	T	T	T	F
F	T	T	F	F	T	T	T	F	T
T	T	F	T	F	T	T	T	T	F
T	F	F	T	F	T	T	T	F	F
F	T	F	T	F	T	T	T	T	T

there is one case that all the propositions are true when Kevin and Vijay are chatting.
44(a): $\neg q \vee \neg q$

Exercises for Section 1.3:

8(c)
p: ”James is young”
q: ”James is string”
”James is young and strong”: $p \wedge q$
$\neg p$: ”James is not young”
$\neg q$: ”James is not string”
De Morgan’s law: $\neg(p \wedge q) = \neg p \vee \neg q$
Therefore, the negative of the statement will be ”James is not young or James is not strong”

10(c)
Q: $(p \implies \neg q) \implies (\neg p \implies q)$
A: $(\neg p \vee \neg q) \implies (p \vee q)$
= $\neg(\neg p \vee \neg q) \vee (p \vee q)$
= $(p \wedge q) \vee (p \vee q)$
= $((p \wedge q) \vee) p \vee q$
= $(p \wedge (q \vee True)) \vee q$
= $p \wedge q$

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transfroming $p \iff q$:

$(p \implies q) \wedge (q \implies p)$
$(\neg p \vee q) \wedge (\neg q \wedge p)$
$(\neg p \wedge (\neg q \vee p)) \vee (q \wedge (\neg q \wedge p))$
$(\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p)$
$(\neg p \wedge \neg q) \vee False \vee False \vee (q \wedge p)$
$(\neg p \wedge \neg q) \vee (q \wedge p)$
$(p \wedge q) \vee (\neg p \wedge \neg q)$

Exercises for Section 1.4:

10(e):
"For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet." means that there is at least one cat owner, one dog owner, and one ferret owner therfore, we can state that:
$(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$

12(g):
translate the statement into English would be "For all the x, there is none of them will make Q true. And that is false, consider when $x = 0.5$

42(b):
$A(x)$: "directory x can be opened"
$B(x)$: "File x can be closed"
C: "system errors can be detected"
consider the question statement, then we have $C \implies (\neg A(x) \wedge \neg B(x))$
46:
Assuming they are not equivalent and try to find an example to prove it
P(x): x is a positive number
Q(x): x is a negative number
For $\forall x(P(x) \iff Q(x))$, since no number can be positive and negative at the same time, so this statement is false.
For $\forall x P(x) \iff \forall x Q(x)$, not all numbers are positive and not all numbers are negative so both $\forall x P(x)$ and $\forall x Q(x)$ are both false which means the truth value of both expressions are equivalent.

Exercises for Section 1.5:

16(e):
let P(a,b,c) be such statement: "student a has a class b and major in c"

”There is a major such that there is a student in the class in every year of study with that major.” means $\exists c \forall b \exists a P(a, b, c)$
based on the givens, this statement is false because, for computer science, not all of students existed.

32(d):
$\neg(\forall y \exists xz(T(x, y, z) \vee Q(x, y))$
$\implies \exists y \forall x \forall(\neg z(T(x, y, z) \wedge \neg Q(x, y))$

44:
$\forall a \forall b \forall c \exists x \exists y((ax^2+bx+c=0) \wedge (ay^2+by+c=0) \wedge [\forall((x \neq z) \wedge (y \neq z)) \implies (az^2+bz+c \neq 0)])$
In English, this means ”for all a, for all b, for all c, there exists one x and there exists one y that x is the root of polynomial and y is the root of polynomial and for all the number those are not x nor y, those are not the root of the polynomial