CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 11

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
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Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, April 14, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".
  NONE

Exercises for Section 9.1:

4(a-d): (2 points).

a .

not reflexive, no one is taller than it self

not symmetric, not possible for two person taller than one another at the same time

anti-symmetric, not possible for two person taller than one another at the same time

transitive, if A is taller than B and B is taller than C, A is taller than C.

b .

reflexive, everyone is born as the same date as itself

symmetric, if A was born the same date as B did, B is also born the same date as A did

not anti-symmetric, A's birthday can be same as B and B's birthday can be same as A. A and B can be different person

transitive, if A's birthday = B's birthday, and B's birthday = C's birthday, then A's birthday = C's birthday

c.

reflexive, everyone has the same first name as itself.

symmetric, if A's first name = B's first name, B's first name = A's first name

not anti-symmetric, two individual people can have same first name but not the same person

transitive, if A's first name = B's first name, and B's first name = C's first name, A's first name = C's first name.

d .

reflexive, everyone has the same grandpa as itself

symmetric, if A and B have the same grandpa, B and A have the same grandpa.

not anti-symmetric, two different person can have the same grandpa

transitive, if A's grandpa = B's grandpa, and B's grandpa = C's grandpa, A's grandpa = C's grandpa

6(a,c,e,g): (2 points).

a.

not reflexive, when x+x = 0, x can only be 0, not for all real numbers

symmetric, if x+y=0, y+x=0

not anti-symmetric, it's possible that x+y=0, y+x=0 and x does not equal to y

transitive, for example, 10+(-10)=0, -10+10=0, but 10+10 is not 0

c.

reflexive, since x-x=0 and 0 is a rational number.

symmetric, let x-y be a rational number, since the negative of a rational number is still rational, y-x = -(x-y) is a rational number

not anti-symmetric, for example, 6-9 and 9-6 are both rational, 6 does not equal to 9.

transitive, if x-y is rational, and y-z is rational, x-z=(x-y)-(y-z) is also rational. the difference

between two rational numbers is also rational.
e.
reflexive, $x * x >= 0$ is always true for real numbers
symmetric, if $xy >= 0, yx = xy >= 0$
not anti-symmetric, it's possible that $xy >= 0, yx >= 0$ and x does not equal to y
not transitive, for example, let $x = -10, y = 0, z = 10, xy >= 0, yz >= 0$ but $xz < 0$
g.
not reflexive, not real numbers is 1
not symmetric,
anti-symmetric, if $(x, y) \in R$ and $(y, x) \in R$, $x = y = 1$
transitive, let $(x, y) \in R$ and $(y, z) \in R$, so x=1, therefore, $(x, z) \in R$ is true.

34(a,c,e,g): (2 points).
a.
$\{(a, b)|(a, b) \in R_1$ or $(a, b) \in R_3\}$
$\{(a, b)|a > b$ or $a < b\}$
$\{(a, b)|a \neq b\}$
R_6

c.
$\{(a, b)|(a, b) \in R_2$ and $(a, b) \in R_4\}$
$\{(a, b)|(a \geq b) or (a \leq b)\}$
$\{(a, b)|a = b\}$
R_5

e.
$\{(a, b)|((a, b) \in R_1) and ((a, b) \notin R_2)\}$
$\{(a, b)|(a > b) and (a \ngeq b)\}$
ϕ

g.
$\{(a, b)|(a, b) \in R_1 - R_3 or (a, b) \in R_3 - R_1\}$
$\{(a, b)|((a, b) \in R_1, (a, b) \notin R_3) or ((a, b) \in R_3, (a, b) \notin R_1)\}$
$\{(a, b)|(a > b, a \nless b) or (a < b, a \ngtr b)\}$
$\{(a, b)|(a > b, a \geq b) or (a < b, a \leq b)\}$
$\{(a, b)|(a > b) or (a < b)\}$
$\{(a, b)|a \neq b\}$
R_6

Exercises for Section 9.5:

2(a,c,e): (2 points).
a. Equivalence relation, satisfy all 3 requirements
c. Not transitive, let x and y have the same father and yz have the same mother, but xz might not have the same father nor mother
e. Not transitive, let xy speak Chinese, yz speak Japanese, it doesn't mean xz share any common language

12: (2 points).
Let A = set of all bit strings that have a length of at least 3
and R = $\{(x, y)|$All bits of x and y agree, except perhaps the first three bits$\}$
now proving R satisfy all three requirements.
Reflexive: Let x $\in$ A
Since all bits of x are the same as itself (as the two strings are same):
Symmetric: let(x,y) $\in$ R
since all bits of x is same as y, it can be another way around.
so (y,x) $\in$ R is true
transitive: let(x,y) $\in$ R, and (y,z) $\in$ R
so x and y have same bits, y and z have the same bits, so z and x have the same bits, thus it's transitive
In conclusion, R is an equivalence relation.

16: (2 points).
Reflexive Let((a,b),(c,d)) $\in$R
so that ad=bc,thus bc=ad, then cb=da
so ((c,d),(a,b))$\in$ R
Thus R is symmetric.
Reflexive Let(a,b)
Since ab = ba
((a,b),(c,d))$\in$ R
Thus R is reflexive.
44(b,c,d): (2 points).
44b)Not a partition since 0 isn't positive nor negative
44c)Partition since all divided classes cover all integers
44d)Partition since classes are nonempty, disjoint, and exhaustive thus are subsets of set of integers

Exercises for Section 9.6:

8(a-c): (2 points).
a.
R is reflexive because the matrix's main diagonal is all 1's.
R is anti-symmetric because $m_{ij} = 1$ and $m_{ji} = 1$ is only true when $i = j$.
R is not transitive because $m_{21} = 1$ and $m_{13} = 1$ and $m_{23} = 0$ because $(2, 1) \in R$ and $(1, 3) \in R$ but $(2, 3) \notin R$.
R is not a partial ordering because R is not transitive.
b.
R is reflexive because the matrix contains only 1's on the main diagonal.
R is anti-symmetric because $m_{ij} = 1$ and $m_{ji} = 1$ is only true when $i = j$.
R is transitive because $m_{21} = 0$ and $m_{13} = 0$ and $m_{23} = 0$ because $(2, 1) \notin R$ and $(1, 3) \notin R$ and $(2, 3) \notin R$.
so R is partial ordering.
c.
R is reflexive because the matrix contains only 1's on the main diagonal.
R is anti-symmetric because $m_{ij} = 1$ and $m_{ji} = 1$ is only true when $i = j$.
R is not transitive because $m_{41} = 1$ and $m_{13} = 1$ and $m_{43} = 0$ because $(3, 1) \in R$ and $(1, 3) \in R$ while $(3, 3) \notin R$.
R is not a partial ordering because R is not transitive.

34(a,c,f,h): (2 points).
a) Maximal element = 60, 48, 72, 27
c) Greatest element = does not exist
f) Least upper bound = 18
h) Greatest lower bound = 12

52b: (1 point).
Let the infinite lattice be on the set of natural number with the less than or equal to relation.
$(\mathbb{N}, \leq)$
It has no greatest element because $m + 1 \in \mathbb{B}$ is a greater element for every $m \in \mathbb{B}$. It has a least element because 0 is smaller than all other elements in $\mathbb{N}$.
66: (1 point).
There are more than one possible order of task, after a topological sort, one of the compatible total order is shown below:
foundation, framing roof, exterior siding, wiring, plumbing, flooring, wallboard, exterior painting, interior painting, carpeting, interior fixtures, exterior fixtures, completion.