CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 10

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
ARTHUR CHEN

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, April 7, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".
  NONE

Exercises for Section 8.1:

8(a-c): (2 points).
a. Since it is 3 consecutive 0's, there are four cases.
first, ending in 1 , the length is $n-1$ and number of possible strings is a_{n-1}
second, ending in 10 , the length is $n-2$ and number of possible strings is a_{n-2}
third, ending in 100 , the length is $n-3$ and number of possible strings is a_{n-3}
fourth, ending in 000 ,number of possible strings is 2^{n-3}, where length is $n-3$
conclusion: $a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}$
b.
recall that n is length
when length is 0, $a_0 = 0$, since three continuous 0's needs at least 3 bits
when length is 1, $a_1 = 0$, for same reason
when length is 2, $a_2 = 0$, for same reason
when length is 3, $a_3 = a_2 + a_1 + a_0 + 2^{3-3} = 0 + 0 + 0 + 1 = 1$, only one case will satisfy given question, which is 000.
the recurrence relation holds for n=3 so n=3 does not need to be initial condition
c. Using the recurrence relation from above,
$a_0 = 0$
$a_1 = 0$
$a_2 = 0$
$a_3 = a_2 + a_1 + a_0 + 2^{3-3} = 1$
$a_4 = a_3 + a_2 + a_1 + 2^{4-3} = 3$
$a_5 = a_4 + a_3 + a_2 + 2^{5-3} = 8$
$a_6 = a_5 + a_4 + a_3 + 2^{6-3} = 20$
$a_7 = a_6 + a_5 + a_4 + 2^{7-3} = 47$
47 is the answer
20(a-b): (2 points).
a.
rewriting n by 5n since both nickel and dime worth the multiplies of 5cents.
first case: first coin is nickel, there are a_{n-1} ways to pay $5n - 5 = 5(n-1)$cents
second case: first coin is dime, there are a_{n-2} ways to pay $5n - 10 = 5(n-2)$cents
since the order of which coin is used matters, $a_n = a_{n-1} + a_{n-2}$
initial conditions:
when n=0, only 1 way to pay 0 cents, using 0 nickel and 0 dime.
when n=1, only 1 way to pay 5 cents, using 1 nickel and 0 dime.
b.
45 = 5n, so n=9.
$a_0 = 1$

$a_1 = 1$
$a_2 = a_1 + a_0 = 2$
$a_3 = a_2 + a_1 = 3$
$a_4 = a_3 + a_2 = 5$
$a_5 = a_4 + a_3 = 8$
$a_6 = a_5 + a_4 = 13$
$a_7 = a_6 + a_5 = 21$
$a_8 = a_7 + a_6 = 34$
$a_9 = a_8 + a_7 = 55$

56(b-e): (3 points).

b.

when finding the maximum sum of consecutive terms, we initialize the first term to the sum at first. the maximum sum should be updated only if the sum increases as it adds the next number, which means negative numbers, or else continue adding,and see if the new sum is greater than the recorded max sum.

So in the recurrence relation, M(k-1) indicates the maximum sum before a_{k-1}. by adding a_k, new maximum sum will be created. we need a max() in the recurrence relation because: If M(k-1) is negative, a_k itself will be the new max sum. If a_k is negative, the maximum sum will not be achieved.

c.

```
int maxSum(int[] array) {
    M(1) = array[0];
    for (int k = 2, k <= n, k++){
        M(k)=max(M(k-1)+array[k],array[k];
    {
    return M(k);
}
```

d.

first M(k) is assigned with 2,
then M(k) = $-3 + 2 = -1 > -3$
then M(k) = $4 > -1 + 4 = 3$
then M(k) = $4 + 1 = 5 > 1$
then M(k) = $5 - 2 = 3 > -2$
then M(k) = $3 + 3 = 6 > 3$
so the max sum is 6 from 4,1,-2,3

e. the addition and comparisons only perform in the for loop, the number of iteration is n-2+1=n-1 therefore, we will have n-1 times of additions and comparisons.
since this is linear relationship between n and the number of operation, the complexity is O(n).

Exercises for Section 8.2:

4(a,c,e,g): (2 points).
a.
roots characteristic equation:
assume $a_n = r^2, a_{n-1} = r, a_{n-2} = 1$
transferring the given condition:
$r^2 = r + 6$
then solve for r: $r = 3$ or $r = -2$
since the solution of the recurrence relation is in the form of $a_n = Ar_1^n + Br_2^n$
plug in the roots from above:
$a_n = A * 3^n + B * (-2)^n$
applying the initial condition:
$3 = a_0 = A + B$
$6 = a_1 = 3A - 2B$
now solving for A and B:
$A = 2.4$ and $B = 0.6$
so the solution of the recurrence relation is $a_n = 2.4 * 3^n + 0.6 * (-2)^n$

c.
roots characteristic equation:
assume $a_n = r^2, a_{n-1} = r, a_{n-2} = 1$
transferring the given condition:
$r^2 = 6r - 8$
then solve for r: $r = 4$ or $r = 2$
since the solution of the recurrence relation is in the form of $a_n = Ar_1^n + Br_2^n$
plug in the roots from above:
$a_n = A * 2^n + B * 4^n$
applying the initial condition:
$4 = a_0 = A + B$
$10 = a_1 = 2A + 4B$
now solving for A and B:
$A = 3$ and $B = 1$
so the solution of the recurrence relation is $a_n = 3 * 2^n + 1 * 4^n$

e.
roots characteristic equation:
assume $a_n = r^2, a_{n-1} = r, a_{n-2} = 1$
transferring the given condition:

$r^2 = 1$
then solve for r: $r = 1$ or $r = -1$
since the solution of the recurrence relation is in the form of $a_n = Ar_1^n + Br_2^n$
plug in the roots from above:
$a_n = A * 1^n + B * (-1)^n$
applying the initial condition:
$5 = a_0 = A + B$
$-1 = a_1 = A - B$
now solving for A and B:
$A = 2$ and $B = 3$
so the solution of the recurrence relation is $a_n = 2 * 1^n + 3 * (-1)^n = 2 + 3 * (-1)^n$

g.
roots characteristic equation:
assume $a_n = r^2, a_{n-1} = r, a_{n-2} = 1$
transferring the given condition:
$r^2 = -4r + 5$
then solve for r: $r = -5$ or $r = 1$
since the solution of the recurrence relation is in the form of $a_n = Ar_1^n + Br_2^n$
plug in the roots from above:
$a_n = A * 1^n + B * (-5)^n$
applying the initial condition:
$2 = a_0 = A + B$
$8 = a_1 = A - 5B$
now solving for A and B:
$A = 3$ and $B = -1$
so the solution of the recurrence relation is $a_n = 3 * 1^n + (-1) * (-5)^n$
$=> a_n = 3 - (-5)^n$
28(a-b): (2 points).
a.
roots characteristic equation:
assume $a_n = r^2, a_{n-1} = r, a_{n-2} = 1$
transferring the given condition:
$r = 2$, since other function of n is considered as 0.
with one single root: the solution is in the form of:
$a_n = Ar_1^n + Bnr_1^n + ... + Zn^{k-1}r_1^n$
since k is 1 in this case, we have solution of homogeneous recurrence relation:
$a_n^{(h)} = A * 2^n$
b.

the particular solution:
Since $2n^2$ is in the form of r^n, i am guessing $A2^n$
plug back to equation we obtain:
$A2^n * n - 2 * A2^{n-1} * (n-1) = 2^n$
after solving for A and plug back to equation, the particular solution is $-2n^2 - 8n - 12$
applying the initial condition:
we obtain A=13
$=> a_n^{(h)} = 13 * 2^n$
the general solution is the sum of homogeneous and particular solution:
$a_n = 13 * 2^n - 2n^2 - 8n - 12$

Exercises for Section 8.3:

18(a-b): (2 points).

a.

n voters create 2n names, dividing those names into two sub list till the sub lists contain at most 3 names. so that one of the names will at least be repeated twice.

when merging back each sub lists, 12n of comparisons are required.

so the recurrence relation is:

f(n) = 2f(n/2) + 12n

b. By Master theorem,

a = 2, b = 2, c = 12, d = 1

so $a = b^d$ for f(n) = af(n/b) + c n^d, then

$O(n^d logn) = O(nlogn)$

which is the worst case complexity

22: (2 points).

a. n=16

$f(16) = 2f(4) + \log(16)$

$= 2(2f(2) + \log(4)) + \log(16)$

$= 2(2 + 2) + 4$

$= 12$

b. Let $m = \log n$ and $n = 2^m$,

$f(n) = f(2^m)$

$= 2f(\sqrt{2^m}) + \log(2^m)$

$= 2f(2^{\frac{m}{2}}) + m$

Let $R(m) = f(2^m)$

So $R(m) = 2R(\frac{m}{2}) + m$

a = 2, b= 2, c= 1, d =1

$R(m) = O(m^d \log m) = O(m \log m)$

Therefore, $f(n) = O(\log n \log \log n)$

Exercises for Section 8.4:

12(a,c,e): (3 points).

a.

$\frac{1}{1+3x} = \frac{1}{1-(-3x)}$
$= \sum_{k=0}^{+\infty}(-3x)^k$
$= \sum_{k=0}^{+\infty}(-3)^k x^k$
$a_k = (-3)^k$
$k = 12$
$a_{12} = (-3)^{12} = 531441$

c.

$\frac{1}{(1+x)^8} = \frac{1}{(1-(-x))^8}$
$= \sum_{k=0}^{+\infty}\left(\begin{array}{c} 8+k-1 \\ k \end{array} \right)(-x)^k$
$= \sum_{k=0}^{+\infty}\left(\begin{array}{c} 7+k \\ k \end{array} \right)(-1)^k x^k$
$a_k = \left(\begin{array}{c} 7+k \\ k \end{array} \right)(-1)^k$
and $k = 12$
$a_{12} = \left(\begin{array}{c} 7+12 \\ 12 \end{array} \right)(-1)^{12} = \left(\begin{array}{c} 19 \\ 12 \end{array} \right) = 50388$

e.

$\frac{x^3}{(1+4x)^2} = x^3 \cdot \frac{1}{(1-(-4x))^2}$
$= x^3 \cdot \sum_{k=0}^{+\infty}\left(\begin{array}{c} 2+k-1 \\ k \end{array} \right)(-4x)^k$
$= \sum_{k=0}^{+\infty}\left(\begin{array}{c} 1+k \\ k \end{array} \right)(-4)^k x^{k+3}$
$a_k = \left(\begin{array}{c} 1+k \\ k \end{array} \right)(-4)^k$
$k = 9$
$a_{12} = \left(\begin{array}{c} 1+9 \\ 9 \end{array} \right)(-4)^9 = -10 \cdot 4^9 = -2621440$

14: (2 points).

we have five kids with at most 3 figures
$(1 + x + x^2 + x^3)^5 = (\sum_{k=0}^{3} x^k)^5$
$= (\frac{1-x^4}{1-x})^5$
$= (1 - x^4)^5 \cdot (1 - x)^{-5}$
$= \sum_{m=0}^{+\infty}\left(\begin{array}{c} 5 \\ m \end{array} \right)(-x^4)^m \cdot \sum_{k=0}^{+\infty}\left(\begin{array}{c} 5 \\ k \end{array} \right)(-x)^k$

$= \sum_{m=0}^{+\infty} b_m x^4 m \cdot \sum_{k=0}^{+\infty} c_k x^k$

Let $b_m = (\begin{array}{c} 5 \\ m \end{array})(-1)^m$ and $c_k = (\begin{array}{c} -5 \\ m \end{array})(-1)^k$

since we need to find the coefficient of $x^1 2$, $4m + k = 12$

and summing each possible combination of m and k:

a$_1 2 = b_0 c_1 2 + b_1 c_8 + b_2 c_4 + b_3 c_0$

$= (\begin{array}{c} 5 \\ 0 \end{array})(-1)^0(\begin{array}{c} -5 \\ 12 \end{array})(-1)^{12} + (\begin{array}{c} 5 \\ 1 \end{array})(-1)^1(\begin{array}{c} -5 \\ 8 \end{array})(-1)^8$

$+ (\begin{array}{c} 5 \\ 2 \end{array})(-1)^2(\begin{array}{c} -5 \\ 4 \end{array})(-1)^4 + (\begin{array}{c} 5 \\ 3 \end{array})(-1)^3(\begin{array}{c} -5 \\ 0 \end{array})(-1)^0$

$= 1820 - 2475 + 700 - 10$

$= 35$