<table>
<tr><th colspan="2">Quiz 1</th></tr>
<tr><td>Write a truth table for p ∨ ¬q</td><td>
<table>
<tr><th>p</th><th>q</th><th>$p \lor \neg q$</th></tr>
<tr><td>F</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>F</td></tr>
<tr><td>T</td><td>F</td><td>T</td></tr>
<tr><td>T</td><td>T</td><td>T</td></tr>
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<tr><td>On the island of Smullyan, knights always tell the truth and knaves always like. A says “B is a knave” and B says “We are both knaves”. What are A and B?</td><td>If we look at B’s statement, B can’t be a knight and say it, so B must be a knave. The statement must also be false, so A must be a knight. This is consistent with A’s words.</td></tr>
<tr><td>Demonstrate that (r → q) ∧ (¬p ∧ r) is satisfiable by giving values of p, q, r.</td><td>p=False, q=True and r=True</td></tr>
<tr><td>Draw the truth table for exclusive or: p ⊕ q</td><td>
<table>
<tr><th>p</th><th>q</th><th>$p \oplus q$</th></tr>
<tr><td>F</td><td>F</td><td>F</td></tr>
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</table>
</td></tr>
<tr><td>Let F(x, y) be the statement ”x can fool y”, where the domain is all people in the world. Use quantifiers to express ”Somebody can fool everybody.”</td><td>∃x∀yF(x, y)</td></tr>
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<table>
<tr><th colspan="2">Quiz 2</th></tr>
<tr><td>Show that the following argument isn’t valid: ”If x is a positive real number, then |x| is a positive real number. Therefore if |a| is a positive real number then a is a positive real number.”</td><td>This is the fallacy of affirming the conclusion. Consider a=-1. (Either answer)</td></tr>
<tr><td>Using proof by contraposition, show that for</td><td>Assume n is odd. Then n 2 is odd and n 2 + 7</td></tr>
</table>

all integers n, if n 2 + 7 is odd then n is even.	is even. The result follows by contraposition
Show there exists an integer, n, such that n 3 = 9n	Consider n=3

Quiz 3	
Provide P({a, b, {c}}) (powerset)	{∅, {a}, {b}, {{c}}, {a, b}, {a, {c}}, {b, {c}}, {a, b, {c}}}
Let A= {2,6,8} and B={0,3,6}. Find (A − B) × B	{(2, 0),(2, 3),(2, 6),(8, 0),(8, 3),(8, 6)}
Prove (by counterexample) that the statement jp dxe k = b √ xc for all positive real numbers x is false	Consider x=8.5 (or any number whose ceiling is a perfect square).

Quiz 4	
Prove f(x) = 2x^3+50 is O(x^3)	Consider, e.g. C = 100 and k =2.
Arrange these functions in order of increasing big-O:	$\log n, \sqrt{n}, n\log n, n^2, 2^n$
What is the big-theta θ for $\frac{47x^4+5log(x)}{39x^2}$?	$\Theta(x^2)$
Describe a greedy algorithm.	An optimization algorithm that makes locally optimal choices. It may or may not yield a globally optimal solution.

Quiz 5	
L and M are lists of N elements def bigOh1(L,M): count = 0 for x in L: # loop over all items in list L for y in M: # loop over all items in list M for z in L: # loop over all items in list L if x==y and z%2==0: # count it! count +=1 return count	O(N^3)
S, T are strings of length N def bigOh2(S,T):	O(N^2)

count = 0
for i in range(0,N):
for j in range(i,N):
count += 1
return count	

N is an integer
def bigOh3(N):
count = 0
for j in range(0,N):
X = N
while(X>=1):
count += 14
X = X//3 # Integer division
return count
	O(N*log N)

Quiz 6	
Using induction, prove Pn i=0 2 i = 2n+1 − 1	Base case: n=0. $2^0 = 1$. $20 + 1 - 1 = 2 - 1 = 1$. Check. Inductive case. Assume $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$. Need to show $\sum_{i=0}^{n+1} 2^i = 2^{(n+1)+1} - 1$ $\sum_{i=0}^{n+1} 2^i = \sum_{i=0}^{n} 2^i + 2^{n+1}$ Using induction hypothesis, $\sum_{i=0}^{n+1} 2^i = 2^{n+1} - 1 + 2^{n+1}$ Rearranging terms, $\sum_{i=0}^{n+1} 2^i = 2 * 2^{n+1} - 1$ Thus, $\sum_{i=0}^{n+1} 2^i = 2^{(n+1)+1} - 1$. The result follows by induction.
Using induction, prove n! > 2 n for n a positive integer greater than or equal to 4	Base case: n=4. $4! = 24$. $2^4 = 16$. $24 > 16$. Check. Inductive case. Assume $n! > 2^n$. Need to show $(n+1)! > 2^{(n+1)}$. $(n+1)! = (n+1) * n!$. Using induction hypothesis, $(n+1)! > (n+1) * 2^n$. Since $n > 4$, $(n+1) * 2^n > 2^{(n+1)}$. The result follows by induction.

Quiz 7	
Explain why f(0) = 0, f(1) = 1, f(n) = 2f(n − 3) for n ≥ 2 is an invalid function definition.	Because f(2) is undefined (it would equal 2f(n-3)=2f(-1), which is undefined).
Write a recursive function that counts the number of elements in a list.	def countElements(L : List) bool: if isNull(L):

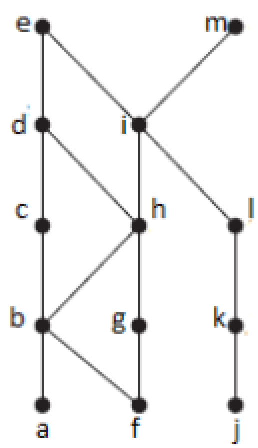
	return 0 else: return 1+countElements(next(L))

Write a recursive function that returns the largest value in the list. You may assume the list has at least 1 element	def largestElement(L : List) bool:
	  if isNull(next(L)):
	    return value(L)
	  else:
	    restLargest = largestElement(next(L))
	    if value(L)>restLargest:
	      return value(L)
	    else:
	      return restLargest

Arthur： quiz 8, quiz 9, quiz 10

Quiz 8	
Give a recurrence relation for the number of ways to pay tolls using nickels, dimes, and quarters, where the order of the coins matters. (Assume all tolls are a multiple of 5 cents)	$P(0)=1, P(5)=1, P(10)=2, P(15)=3, P(20)=5$ $P(n)=P(n-5)+P(n-10)+P(n-25), n \geq 25 \text{ and } 5 \mid n$
Give the general form of the solution of the recurrence relation: A_n = 2a_{n-1} + 3a_{n-2}	r^2-2r-3 has two real roots, 3 and -1 Therefore, a_n = a(3)^n+b*(-1)^n
Give the big-O of the recurrence relation $f(n) = 6f(\frac{n}{3}) + 3n^2$	O(n^2log(n))
(6 points) Give a generating function (but do not solve) you would use to determine how to give 12 identical donuts to 5 students so that each student gets 1-4 donuts. Which coefficient would you want?	$(x+x^2+x^3+x^4)^5$ Want coefficient of x^{12}

Quiz 9	

	a,f,b,j,g,k,c,h,l,d,i,e,m
What are the maximal elements of the Hasse diagram for divisibility on the set {1,2,3,4,5,7,8,10}.	3, 7, 8, 10
Explain why {(a,b)|a and b are the same age} is an equivalence relation.	It is a binary relation that is: reflexive (a is the same age as a) symmetric (if a is the same age as b, b is the same age as a) and transitive (if a is the same age as b, and b is the same age as c, then a is the same age as c)
Name one property that the relation "a weighs more than b" (where a,b taken from the set of all people in the world) is missing which makes it not a partial ordering?	It is not reflexive (a does not weigh more than a)

Quiz 10	
Fermat's little theorem states that if a is not divisible by p, then a p−1 ≡ 1 (mod p). Use this fact to compute 4 20 mod 7.	4^18 ≡ 1 mod 7, so 4^20 mod 7 equals 4^2 mod 7, which is 2.
Alice has an RSA a public key of (7, 55) and a private key of (23, 55). Bob has an RSA public key of (5, 133) and a private key of (65, 133). Alice wants to encrypt and send the message 13 to Bob. What does she compute? (Do not sign the message). Write the answer as (e.g.2 19 mod 37)	13^5 mod 133.
What is the GCD of 28 and 21?	7

	http://www.alcula.com/calculators/math/gcd/
Encrypt the message “I WANT AN A” using a shift cipher with key k = 4.	“M AERX ER E” https://planetcalc.com/1434/

Jack: 考试们

Let p and q be the propositions p: I own a computer q: I am a comp sci major Write the proposition ”I am a comp sci major, but I do not own a computer” using logical connectives (including negations).	$q \land \neg p$
Using DeMorgan’s law, express the negation of the statement, ”Batman is wealthy and has a butler.”	Batman is not wealthy or Batman does not have a butler.
Let F(x,y) be the statement ”x can fool y,” where the domain consists of people. Use quantifiers to express the statement ”Everyone can fool both Jerry and Sally.”	$\forall x, F(x, \text{Jerry}) \land F(x, \text{Sally})$
Construct a truth table for $(p \oplus q) \rightarrow (p \land q)$.	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline p & q & p \land \neg q & p \oplus q & (p \land \neg q) \rightarrow (p \oplus q) \\ \hline \mathrm{F} & \mathrm{F} & \mathrm{F} & \mathrm{F} & \mathrm{T} \\ \hline \mathrm{F} & \mathrm{T} & \mathrm{F} & \mathrm{T} & \mathrm{T} \\ \hline \mathrm{T} & \mathrm{F} & \mathrm{T} & \mathrm{T} & \mathrm{T} \\ \hline \mathrm{T} & \mathrm{T} & \mathrm{F} & \mathrm{F} & \mathrm{T} \\ \hline \end{array}$
If the professor is sick, then class is cancelled. What can we conclude if class in cancelled?	Nothing
Suppose you are planning to prove that ”if n is an integer and n 3 + 8 is even, then n is even” by contradiction. What would be your assumption? (Do NOT complete the proof).	Assume n is an integer, is odd AND n 3 + 8 is even.
Disprove that Disprove that $\lfloor 3x \rfloor = 3 \lfloor x \rfloor$.	Consider x=4/3. Disproven.
Suppose knights always tell the truth and knaves always lie. You meet three people, Aaron, Bohan, and Crystal. If Bohan and Crystal both say “Exactly one of us is a	Aaron is a knave and Bohan and Crystal are both knights.

knave", and Aaron says "All of us are knaves" what are they?	
Provide a counterexample to the following, where the domain for all variables is $\mathbb{R}$: $\forall x \exists y (y = \sqrt{x})$	Consider x = -2
Let A = {1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7, 8}. Find A ∪ B and A ∩ B	A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8} and A ∩ B = {3, 4, 5}
Let A = {a, {b}, c}. What is the power set of A?	P(A) = {∅, {a}, {{b}}, {c}, {a, {b}}, {{b}, c}, {a, c}, {a, {b}, c}}
$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ Using the fact above, give an expression containing only numbers for $\sum_{k=56}^{100} k^3$. You do not need to simplify.	$\frac{100^2(101)^2}{4} - \frac{55^2(56)^2}{4}$
What is the value of $\sum_{i=0}^{2} \sum_{j=0}^{2} ij$	1*1+1*2+2*1+2*2=9
There are about 600 pounds of sand in my back yard. In 2020, I plan to add 300 more pounds. In 2021, I will add 150. Each year I will add half amount of sand as the previous year. Give an upper bound for the amount of sand in my yard, or infinity if there isn't one.	1200 pounds
Prove that {1, 2} × Z + is countably infininte.	f((a, b)) = b is an onto mapping from {1,2} to Z + f(x) = ((x + 1)mod2 + 1, d x 2 e) is an onto mapping from Z + to {1, 2} × Z +

Explain why this is not a good algorithm for finding the median of a set of numbers in a sorted array. the parameter a is a 0-indexed array (e.g. a[0]=1, a[1]=2, a[2]=3)
`def findMedian(a):`
`    if len(a) % 2 ==0:`
`        return a[len(a)/2]`
	It does not return a result if len(a) is even

<table>
<tr><td>What is the best big-O limit of $(x^2 + 300x + x * \log(x))^3$?</td><td>O(x^6)</td></tr>
<tr><td>Prove (provide c and k) that $5x^2 + 14x + 10$ is $O(x^2)$.</td><td>Let c be 30 and k be 1, then $5x^2 + 14x + 10 < 30x^2 \ \forall x \geq 1$</td></tr>
<tr><td>What is the big O of the following code?<pre>
for i in range(n):
    for j in range(n):
        d = d+i+j
</pre></td><td>O(n^2)</td></tr>
<tr><td>What is the big O of the following code?<pre>
for i in range(n):
    k=i
    while k>1:
        k=k/2
</pre></td><td>O(n*log(n))</td></tr>
<tr><td>) Does a greedy algorithm with coins of denominations 1,4,16, and 64 cents always produce change using the fewest coins possible?</td><td>Yes! [This is always true when the denomiations are powers of an integer (in this case 4).]</td></tr>
<tr><td>Prove by induction that ∀n ≥ 2, n! < n^n .</td><td>Base case (n=2). 2!=2. $2^2 = 4$. $2 < 4$. Check. Inductive case. Assume $n! < n^n$. Show $(n + 1)! < (n + 1)^{n+1}$. $(n + 1)! = (n + 1) * n!$. This is less than $(n + 1) * n^n$ by the inductive hypothesis. Since $n < n + 1$ and $n > 1$, $n^n < (n + 1)^n$. Thus $(n + 1)! < (n + 1)(n + 1)^n = (n + 1)^{n+1}$. The result follows by induction.</td></tr>
<tr><td>Prove by strong induction that every dollar amount ≥ $4 can be made with only $2 and $5 bills.</td><td>Base cases (n=4,n=5). Use two $2 bills or one $5 bill. Check. Induction case. Assume $k can be made with $2 and $5 bills for all k, 4 ≤ k ≤ n. Need to show $n+1 can be made with $2 and $5 bills. For any amount, n+1, $6 or greater, we can use induction to make n-1 using $2 and $5 bills, and then add one more $2 bill. Check. The result follows by induction.</td></tr>
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Give a recursive definition of the ceiling of the log base 2 of n. (Hint: use a base case of n=1).	$\lceil log_2(n) \rceil = \begin{cases} 0, & \text{if } n = 1. \\ \lceil log_2(\lceil \frac{n}{2} \rceil) \rceil + 1, & \text{otherwise.} \end{cases}$

Write a recursive method to find a 2 n . (Hint: a^2^ (n+1) = (a^2^n)^2)	<pre>def f(a,n):
	    if n==0:
	        return a
	    else:
	        x=f(a,n-1)
	        return x*x</pre>

How many license plates can be formed using three distinct letters followed by four digits? (Write a product of numbers, but do not simplify).	26*25*24*10*10*10*10
A bowl contains 10 red balls, 10 green balls, and 10 yellow balls. How many must be removed to guarantee that at least 3 balls are the same color?	7 [You can have 2 each of red, green and yellow but then have to repeat]
Three distinguishable penguins (wearing different colored bow ties) are in a line with six indistinguishable puffins. How many arrangements are there such that all the puffins stand together?	There are four ways to put the puffins all together (before the first penguin, after the first, after the second, after the third). There are then 3!=6 ways to arrange the penguins. 6*4=24.
What if the puffins and penguins are distinguishable? How many arrangements are there now?	Ok, so it's the same 24, now times 6! = 720*24=17,280.
What is the coefficient of x 2 y 15 in the expansion of (5x^2 + 2y^3)^6 ?	$\binom{6}{1}\binom{5}{5} * 5 * 2^5 = 960.$
Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ if $1 \le k \le n$ (n and k integers).	$k\binom{n}{k} = \frac{k*n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!} = \frac{n*(n-1)!}{(k-1)!(n-k)!} = n\binom{n-1}{k-1}$