CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 2

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Arthur Chen

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday January 28, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".
  NONE

Exercises for Section 1.6:

10(a-f): (4 points)

(a):
r:I play hockey
s:I am sore
t:I use the whirpool.
then we have:
(1)$r \implies s$
(2)$s \implies t$
(3) $\neg t$
combine 2 and 3 we have (4)$\neg s$
from 2 and 1 we have (5) $r \implies t$
since $\neg t$, we can conclude that I did not play hockey

(b):
W(x): I work on x, S(x): it is sunny on x, P(x): it is partly sunny on x
so we have :
(1) $\forall x(W(x) \implies (S(x) \vee P(x)))$
(2)$W(Mon) \vee W(Fri)$
(3)$\neg S(Tue)$
(4)$\neg P(Friday)$
combine 1 and 2 we have:
(5)$W(Mon) \implies (S(Mon) \vee P(Mon))$
(6)$W(Fri) \implies (S(Fri) \vee P(Fri))$
from 4 and 6 we have
(7)$W(Fri) \implies S(Fri)$
from 2,5,6,7 we can conclude that
$S(Mon) \vee P(Mon) \vee S(Fri)$
which means it was either sunny or partly sunny on Mon or sunny on Fri

(c) :
I(x): x is insects
L(x): x has six legs
D(x): x is Dragonflies
S(x): x is spiders
E(x,y): x eats y
from question we can conclude:

$1.\forall[I(x) \implies L(x)]$
$2.\forall[D(x) \implies I(x)]$
$3.\forall[S(x) \implies \neg L(x)]$
$4.\forall((S(x) \wedge D(y)) \implies E(x,y))$
$solution:$
from 1, I(a) $\implies L(a)$
$from 2, D(a) \implies I(a)$
combine two above, we have $D(a) \implies L(a), therefore \forall x(D(x) \implies L(x))$
$5.contrapositive of 1, \neg L(x) \implies \neg I(x)$
$6.instantiation of 3, S(c) \implies \neg L(x)$
with 5 and 6, we have S(c) $\implies \neg I(x)$
$Then\ \forall x(S(x) \implies \neg I(x))$
conclusion: all spiders are not insects

(d):
S(X): X is a student
I(X): X has an internet account
$from question we have:$
$a.\forall X(S(X) \implies I(X))$
$b.\neg I(Homer)$
$c.I(Maggie)$
steps:
1. a premise
2. $S(Homer) \implies I(Homer)$ from 1
3. b premise
4. by 3 and contrapositive of 2, $\neg S(Homer)$
Therefore, Homer is not a student.

(e):
H(X): x is healthy to eat
G(X): x tastes good
E(X): You eat x
we get:
a. $\forall x(H(X) \implies \neg G(X))$
b. $H(tofu)$
c. $\forall x(E(X) \iff G(X))$
d. $\neg E(tofu)$
e. $\neg H(Cheeseburger)$
Solutions:

1. $H(tofu) \implies \neg G(tofu)$ from a
2. $\neg G(tofu)$ from 1 and b
3. from c, $E(c) \iff G(c)$
4. $H(c) \implies \neg G(c)$ from a
5. contrapositive of 3, $\neg E(c) \iff \neg G(c)$
6. from 4 and 5, $H(c) \implies \neg E(C)$
7. from 6, $\forall x(H(x) \implies \neg E(x))$

So, you do not eat any healthy food.

(f):
d: i am dreaming
h: i am hallucinating
e: i see elephants running down the road
then we have:
a. $d \vee h$
b. $\neg d$
c. $h \implies e$
Solutions:
1. from a and b, h
2. from 1 and c, e

So, i see elephants running down the road

16(a-d): (3 points)
a:
E(X): x has enrolled in the university
D(X): x has lived in a dormitory
solution:
1. $\forall x(E(X) \implies D(X))$ (given)
2. from 1, $E(Mia) \implies D(Mia)$
3. $\neg D(Mia)$ (Given)
4. by contrapositive of 2 and 3, $\neg E(Mia)$

so it is true

b:
C(X): x is convertible
D(X): it is fun to drive
solution:
1. $\exists x(C(X) \implies D(X))$ (given)
2. $\neg C(Isac'scar)$ (Given)

However 2 can not conclude that $\neg D(X)$
so it is not valid.

c.
L(x,y): x likes y
1. $\forall actionmovies L(Quincy, actionmovies)$ (Given)
2. $L(Quincy, eightmenout)$ (given)
1 and 2 are not related, so can not conclude further, the statement is not valid.

d.
L(X): x is a lobster man
S(X): x set at least a dozen traps
solution:
1. $\forall x(L(X) \implies S(X))$ (given)
2. L(Hamilton) (Given)
3. S(Hamilton) (from 1 and 2)
so it is true

24: (1 point)
3 has an error: simplification must from a $\wedge$ relationship
5 has an error: simplification must from a $\wedge$ relationship
7 has an error: conjunction yields a $\wedge$ relationship
28: (2 points)
steps:
1. $\forall x(P(X) \vee Q(X))$ premise
2. $\forall x((\neg P(X) \wedge Q(X)) \implies R(X))$ premise
3. $P(C) \vee Q(C)$ from 1
4. $(\neg P(C) \wedge Q(C)) \implies R(C)$ from 2
5. $\neg(\neg P(C) \wedge Q(C)) \vee R(C)$ from 4
6. $(\neg(\neg P(C)) \vee \neg Q(C)) \vee R(C)$ from 5
7. (P(C)) $\vee \neg Q(C)) \vee R(C) from 6$
8.$(\neg Q(c) \vee P(C)) \vee R(C) from 7$
9.$\neg Q(C) \vee (P(C) \vee R(C)) from 8$
10.$P(C) \vee P(C) \vee R(C) from 3 and 9$
11.$R(C) \vee P(C) from 10$
12.$\neg(\neg R(C)) \vee P(C) from 11$
13.$\neg R(C) \implies P(C) from 12$
14.$\forall x(\not R(X) \implies P(X)) from 13$
$so it is true$

Exercises for Section 1.7:

20(a-b): (2 points)
a. contraposition:
since 3n+2 is even, 3n+2 = 2p, and p is an integer
n = 2(p-1)/3
since n is an integer too, (p-1)/3 should be integer
hence 2(p-1)/3 is an even integer

b. contradiction:
assume n is odd, so 3n is odd too.
so 3n + 2 is also odd
but it contradicts to given.

26: (2 points)
in the worst case, for each month there are 2 chosen days.
and that is 12*2 = 24 days
however 25 days is given, this additional day must goes into any month and makes that month has three chosen days in the same month

Exercises for Section 1.8:

8: (2 points)
since x and y are from opposite parity, let's assume x is odd and y is even:
x = 2m+1 and y = 2n for some integer m and n
then 5x+5y will be 10m+5+10n = 10(m+n)+5, which is an odd number.

20: (2 points)
to make the statement invalid, the distance between n and r must be exactly 1/2
that means the value of r will be an integer plus 1/2, which is half of a odd number, can express as a/2 and a is an odd number
this means r is not rational.

32: (2 points)
given that x and y are integers

so x^2 and y^2 are integers too, and not negative.
to satisfy the equation y^2 can not be larger than 2, so only 0,1,2 are possible
similarly, x^2 can only be 0 to 7.
and the possible combinations are:
$x^2 = 2$ and $y^2 = 2$
$x^2 = 7$ and $y^2 = 0$
but none of the combination contains two numbers that is both the squares of an integer, so there is a conflict
therefore, there is no solution.