CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 7

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
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Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, March 3, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".
- NONE

Exercises for Section 5.1:

6: (2 point).
proof by induction
base case n = 1:
$1 * 1! = 1$ and $(1 + 1)! - 1 = 1$, proven true
so P(1) is true
assume P(m) = (m+1)!-1 is true, if P(m+1) is also true, question statement is proven
P(m+1)
= (m+1)!-1+(m+1)*(m+1)!
factoring (m+1)! we have:
= (1+m+1)*(m+1)!-1
= (m+2)*(m+1)!-1
= (m+2)!-1
= ((m+1)+1)!-1
which is the definition of P(m+1), and proven to be true
18(a-f): (3 points).
a. $2! < 2^2$
b. $2! = 2 * 1 = 3 < 4 = 2^2$
c. the hypothesis is P(m) is true: $m! < m^m$
d. we need to prove that P(m+1) is also true: $(m + 1)! < (m + 1)^{m+1}$
e. $(m + 1)! = (m + 1) * m! < (m + 1) * m^m < (m + 1) * (m + 1)^m = (m + 1)^{m+1}$ (note that k¡k+1)
so P(m+1) is also true
f. through the mathematical induction, P(n) is proven to be true for all positive integer n¿1.
32: (3 points).
Proving: P(n): " 3 divides $n^3 + 2n$"
base case: n=1: $1^3 + 2 * 1 = 3$, so P(1) is true
assume P(k) to be true that: 3 divides $k^3 + 2 * k$
now proving P(k+1) is also true
$(k + 1)^3 + 2 * (k + 1)$
$= k^3 + 3 * k^2 + 3 * k + 1 + 2 * k + 2$
$= k^3 + 3 * k^2 + 5 * k + 3$
$= (k^3 + 2 * k) + 3 * (k^2 + k + 1)$
since $k^3 + 2 * k$ is proven to be divisible by 3, and $3 * (k^2 + k + 1)$ as well (with a coefficient of 3), so $(k + 1)^3 + 2 * (k + 1)$ is divisible by 3.
thus P(k+1) is true

Exercises for Section 5.2:

6(a-c): (3 points)

a. Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
b. Prove your answer to (a) using the principle of math-medical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.
c.Prove your answer to (a) using strong induction. How does the inductive hypothesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?

a. listing all the possibilities: 3,6,9,10,12,13,15,16,18,19,20
from 18, there are three consecutive possibilities, so all stamps equal or more than 18 can be made by 3 and 10.

b. to prove part a, set up P(n) to be "n cents can be made up by 3 cents and 10 cents"
base case n = 18: because 3*6=18, base case is true
assuming P(k) is true that k is divisible by 3, now proving P(k+1) is true as well
case one: number containing k+1:
since k is divisible by 3 and including at least three 3 cents, k+1 be represented by replacing those three 3 cents by one 10 cents.
case two: number containing k+2:
if k is formed by less than three 3 cents, then k can be formed by at least two 10 cents(now k is at least 20 and 1819 cents both use at least three 3 cents).
if we have at least two 10 cents, we can replace those two with seven 3 cents to obtain k+1+1, which is the k+2 we want
case three: number including k+3:
just add one more 3 cents and here goes a cycle.
now P(k+1) is proven to be true in all the cases by mathematical induction

c. now let's prove it by strong induction an keep the definition of P(k) the same
the base cases is n=18, n=19, and n=20
P(18) is true, because $3 * 6 = 18$
P(19) is true, because $1 * 10 + 3 * 3 = 19$
P(20) is true, because $2 * 10 = 20$
therefore, we can assume that P(18), P(19), ... P(k) is all true.
to prove P(k+1) is also true, since P(k+1) is interchangeable with P(k-2) (by adding/deleting one 3 cents), and P(k-2) is also true, therefore, P(k+1) is also true.
and P(n) is proven to be turn by strong induction for all positive integer n.

10: (3 points)

base case n = 1, it takes 0 breaks
and n=2 takes 1 breaks, n=3 takes 2 breaks.
so let's assume P(n): it takes n-1 breaks to break n pieces.
with base case is true, assume that P(1), P(2)... P(m) are all true, and it takes x-1 breaks to break a x pieces bar
now proving P(m+1) is also true, m+1 is the number of pieces in the bar and can be represented by a rows and b columns: m+1=a*b
considering breaking the last column of bar first, the bar will be divided into (a-1)*b pieces and 1*b pieces.
since P((a-1)*b) is true, (note that (a-1)*b ¡ m), it needs (a-1)*b-1 breaks to break (a-1)*b pieces
since P(b) is true, it needs b-1 breaks to break b pieces
recall that m+1 pieces is equal to (a-1)*b pieces plus 1*b pieces.
with the "first column break" mentioned above, it needs [(a-1)*b-1]+[b-1]+1 to break m+1 pieces bar
simplifying [(a-1)*b-1]+[b-1]+1:
= a*b -b-1+b-1+1
= a*b-1
= (m+1) - 1
now I have proven that it takes (m+1) - 1 breaks to break k+1 pieces so P(m+1) is true

12: (3 points)

P(n) : a positive integer n can be written as sum of distinct power of 2.
base case: P(1): $1 = 2^0$, P(2): $2 = 2^1$, are true
assuming P(n) is true for all k equal or greater than n, now proving P(k+1) is true
by hint, there are two cases:
case 1: k+1 is odd and k is even
since k is even, k is be written as sum of distinct powers of 2 and none of the powers can be 0.
demo: $k = 2^{e_1} + 2^{e_2} + 2^{e_3} + ... + 2^{e_n}$, where e_i is not 0 and i is within the range of (1,m)
and k+1 can be written as $k = 2^{e_1} + 2^{e_2} + 2^{e_3} + ... + 2^{e_n} + 2^0$

case 2: when k+1 is even
since k+1 is even, (k+1)/2 is an integer, then is can be written is sum of distinct powers of 2.
if we double (k+1)/2, we have 2((k+1)/2), which can be written as $2 * (2^{e_1} + 2^{e_2} + 2^{e_3} + ... + 2^{e_n})$, then $2^{e_1+1} + 2^{e_2+1} + 2^{e_3+1} + ... + 2^{e_n+1}$ where all the e are distinct.
so, regardless k+1 is odd or even, the question statement is true.

32: (3 points)

the proof is invalid because the writer assume that every postage of three cents or more is either including one 3 cents and two 4 cents. For postage value of 5 cents, it doesn't include one 3 cents or two 4 cents, thus this assumption is false and make her proof invalid.