CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 5

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Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
**Arthur Chen**

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**Instructions:**

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, February 18, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

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**Help Received:**

- List any help received here, or "NONE".

NONE

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## Exercises for Section 3.1:

**8: (2 points). Assume the list of integers is indexed 1,2,3...**
set k to 0, to hold number
for i in range of (1,n) (inclusive)
    if f(i) %2 = 0 (even) and larger than f(k)
        k=i
at the end of this loop, k has the position of the largest even number.

**14: (2 points)**
linear search:
linear search goes through numbers one by one
first compare the 1st number in the sequence, which is 1, but 1 is not 7 so move on to the 2nd one, and so on. Until the algorithm finds 7 in the sequence.
if the end of the sequence is reached, 7 is not found.

binary search
there are 8 numbers in the sequence, and break it into two parts: 1,3,4,5 and 6,8,9,11.
since 7 is larger than the largest number in the 1st group, we break 2nd group into half, 6,8 and 9,11.
similarly, since 7 is smaller than 8, we search within the 6 and 8.
7 is larger than 6, search in 8, but 7 is not 8. so 7 is not found.

**18: (2 points)**
assume the index of the sequence starts from 1, then 2,3...
sequence is noted as $a_1, a_2, a_3...$
set up k = 1, to hold the position of the minimum number, set to the first number initially.
create i to perform loop, and n is the number of elements in the sequence
for i = 2 to n
    if $a_i issmallorequalto a_k$
        k=i
return k
since the if condition includes equal, so the k will be updated tot he last occurrence.

**38: (2 points)**
first pass:
compare 1st and 2nd: d,f,k,m,a,b
compare 2nd and 3rd: d,f,k,m,a,b
compare 3rd and 4th: d,f,k,m,a,b
compare 4th and 5th: d,f,k,a,m,b

compare 5th and 6th: d,f,k,a,b,m
and the last one is sure to be in the right place so no need comparison
second pass:
compare 1st and 2nd: d,f,k,a,b,m
compare 2nd and 3rd: d,f,k,a,b,m
compare 3rd and 4th: d,f,a,k,b,m
compare 4th and 5th: d,f,a,b,k,m
and the last two is sure to be in the right place so no need comparison
third pass:
compare 1st and 2nd: d,f,a,b,k,m
compare 2nd and 3rd: d,a,f,b,k,m
compare 3rd and 4th: d,a,b,f,k,m
and the last three is sure to be in the right place so no need comparison
4th pass:
compare 1st and 2nd: a,d,b,f,k,m
compare 2nd and 3rd: a,b,d,f,k,m
and the last four is sure to be in the right place so no need comparison
5th pass:
compare 1st and 2nd: a,b,d,f,k,m
and the last five is sure to be in the right place so no need comparison, and the bubble sort will stop

**58 (a-d): (2 points)**
based on the greedy algorithm:
a. 3 quarters, 1 dime and 2 pennies
b. 1 quarters, 2 dime and 4 pennies
c. 3 quarters, 2 dime and 4 pennies
d. 1 quarters, 0 dime and 8 pennies
when nickel is possible, 5 pennies can transfer into 1 nickel for fewer coins
since only d satisfy this condition, so d does not have fewest possible number of coins while rest all do.

## Exercises for Section 3.2:

**6: (2 points)**
set k to 2 and x is larger than 2
by long division:
$\frac{x^3+2x}{2x+1} = (\frac{1}{2}x^2 - \frac{1}{4}x + \frac{9}{8}) - \frac{\frac{9}{8}}{2x+1}$
$\leq |\frac{1}{2}x^2 - \frac{1}{4}x + \frac{9}{8}|$

$\leq |\frac{1}{2}x^2 + \frac{9}{8}|$
$= \frac{1}{2}x^2 + \frac{9}{8}$
$< \frac{1}{2}x^2 + x^2$
$= \frac{3}{2}|x^2|$
so $k = 2$, and $C = \frac{3}{2}$

**8(a-d): (2 points)**
a.
since log x grows as x increases, we cannot choose $O(x^3)$ , and log x grows much slower than x so we choose $x^4$
$2x^2 + x^3 logx \leq 2x^4 + x^4 = 3x^4$
so $n = 4, C = 3, k = 1$

b.
since $(logx)^4 > 0$ but is insignificant compared to $x^5$, so $n = 5, c = 4, k = 1$

c.
as x increases, the value of this fraction approaches 1, $(x^0)$, so n = 0.
therefore, for x¿1, $f(x) \leq \frac{3x^4}{x^4} = 3$
so c=3,k=1

d.
as x increases, the value of the fraction approaches $\frac{1}{x}$, so n = -1.
therefore, for x¿1, $f(x) \leq \frac{6x^3}{x^4} = \frac{6}{x}$
so c=6,k=1
**22: (2 points)**
1. $(1.5)^n$ is an exponential function with base 1.5
2. $n^{100}$ is a polynomial function of degree 100
3. $(logn)^3$ is a logarithmic function
4. $\sqrt{n}logn$ is a composed of logarithmic and polynomial function of 1/2 degree, higher order than poly functions of 1/2 degree but lower than poly function of 1 degree.
5. $10^n$ is an exponential function with base 10
6. $(n!)^2$ is a factorial function
7. $n^{99} + n^{98}$is a polynomial function of degree 99

sort in increasing order:
3,4,7,2,1,5,6
**26(a-c): (2 points)**
a.
through distributive property:

$= 18n^3logn + n^2(logn)^2 + 20n^3 + n^2logn + 34logn + 38$
so we can assume $g(n) = n^3logn$
since $logn \leq n$ and we choose $n > 10^{20}$ so that $logn > 20$,
$\leq 18n^3logn + n^3logn + n^3logn + n^3logn + n^3logn + n^3logn$
$= 23|n^3logn|$
so C = 23, and $k = 10^{20}, andg(n) = n^3logn$

b.
through distributive property:
$= n^32^n + n^5 + 6^n + n^23^n$
assume $g(n) = 6^n$
using k = 3 and $n > 3$,
$< 3^n2^n + 2^n3^n + 6^n + 2^n3^n$ (note that $n^3 < 3^n$, when $n > 3$, and $n^2 < 2^n$ when $n > 2$
$= 6^n + 6^n + 6^n + 6^n = 4|6^n|$
so C = 4and k = 3 and $g(n) = 6^n$

c.
through distributive property:
$= n^nn! + n2^nn! + 5^nn! + n^n5^n + n2^nn! + 5^n5^n$
assume $g(n) = n^nn!$
when $n > 5$, $5^n < n^n$ and $n2^n < n^n$
when $n \geq 12, 5^n < n!$
choose k = 12,
$< n^nn! + n^nn! + n^nn! + n^nn! + n^nn! + n^nn!$
$= 6|n^nn!|$
so C = 6, k = 12 and $g(n) = n^nn!$ **74: (2 points)**
$nlogn = logn^n$
so now we are comparing between $log(n!)andlog(n^n)$
thus between $n!andn^n$
since $n! = 1 * 2 * 3 * 4.... * n \leq n * n * n * n * ....n = n^n$
so $log(n!) \leq logn^n$, for $n > 1$
thus log(n!) is of O(n log n)

set up a sequence i that 0,1,2,3... n-1
$n \geq i + 1$
$n - 1 - i \geq 0$
$in - i - i^2 \geq 0$
$in - i - i^2 + n \geq n$
$(n - i)(i + 1) \geq n$

$\sum_{i=0}^{n-1} n \leq \sum_{i=0}^{n-1} (n-i)(i+1)$
$n^n \leq (n*1)*(n-1)*2*(n-2)*3...(1*n)$
$n^n \leq (n*(n-1)*...*1)(1*2*...*n)$
$n^n \leq (n!)^2$
$log(n^n) \leq log(n!)^2$
$nlogn \leq 2log(n!)$
so $nlogn$ is of $O(log(n!))$
statement proved