CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2020
Homework 4

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Arthur Chen

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, February 11, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or ”NONE”.
NONE

Exercises for Section 2.4:

4c: (1 point)
$7, 11, 23, 71$

10d: (1 point)
{-1, 0, 1, 3, 13, 74, 613}

14f: (1 points)
$since(n^2) = n + (n-1) + (n-1)^2,$
$n^2 + n = 2n + (n-1) + (n-1)^2$
$then, n^2 + n = a_n = 2n + a_{n-1}$

18(a-c): (2 points)
a. since the annual interest is 9 present, $a_n = 1.09 * a_{n-1}$
b. since the account starts with $1000, the explicit formula will be $a_n = 1000 * 1.09^n$
c. if n = 100, $a_100 = 5529041$

22(a-c): (2 points)
a. based on given, $a_n = 1.05 * a_{n-1} + 1000$

b. $2025 - 2017 = 8$
$a_0 = 50000$
$a_1 = 1.05 * a_0 + 1000 = 53500$
$\ldots$
$a_8 = 1.05 * a_7 + 1000 = \$83421.88$

c. $a_n = 50000 * 1.05^n + 1000 * (1.05^n - 1)/(1.05 - 1)$
$rearrange : a_n = 70000 * 1.05^n - 20000$

24(a-b): (2 points)
a. $B(k) = B(k-1) + I(k) - P$
B(k) is the balance after k months, I(k) is the interest of k_{th} month, and P is the payment.
I(k) = r/12 * B(k-1)
note that r/12 is the rate per month.
from formula above:
$B(k) = B(k-1) + r/12 * B(k-1) - P$
$= (1 + r/12) * B(k-1) - P$

b. after recursively plug in B(k-1) , B(k-2), etc...
we have:

$(1 = r/12)^k B(k-k) - P\sum_{i=1}^{k-1}(1+r/12)^i$
by summation formula:
$= (1+r/12)^n B(0) - P * ((1+r/12)^k - 1)/((1+r/12)-1)$
$= (1+r/12)^n B(0) - (12P/r) * ((1+r/12)^k - 1)$
since the balance is 0 after T months:
$B(T) = 0$
therefore we have:
$= (1+r/12)^T B(0) - (12P/r) * ((1+r/12)^T - 1)$
$afterrearrangement : P = (r(1+r/12)^T B(0))/(12((1+r/12)^T - 1))$
40: (1 points)
$= \sum_{K=1}^{200} K^3 - \sum_{K=1}^{98} K^3$
by table 2:
$((200)^2(200+1)^2)/4 - ((98)^2(98+1)^2)/4$
= 380977799

Exercises for Section 2.5:

4(a-d): (4 points)
a. since all the numbers are one to one, and integer is an infinite set, so it is countably infinite
b. since all the numbers are one to one, and integer is an infinite set, so it is countably infinite
c. since all the numbers are one to one, and integer is an infinite set, so it is countably infinite
d. although the numbers are one to one, however, unless the previous question, we cannot rewrite 9s into fraction form. therefore, this is uncountable.

6: (2 points)
the number of guest in the hotel is countable infinite.
the guest in even room can move to 2(n-1)+1, 2 will move to 3, 3 to 5, 4 to 7. which is odd number so all the guest can still stay inside of the hotel.

8: (2 points)
After moving all the even guest to odd number by previous question, hotel can accept new guest in countable infinite even rooms

10(a-c): (2 points)
a: if A is uncountable even numbers, and B is non zero uncountable even numbers, so the A-B will be a single zero, a finite set.
b: if A is even numbers, and B is odd numbers. A-B is countable infinite.
c: if A is all the prime number and B is all squared numbers, A-B is uncountable.