Lecture 5: Nodal & Mesh Analysis

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ECEN 214 – Electrical Circuit Theory (Spring 2020)

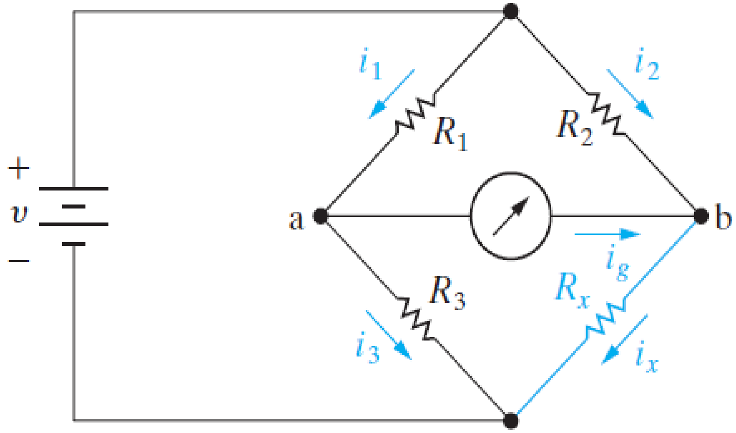
Outline

- Nodal Analysis
- Mesh Analysis

Highlights from Last Lecture

- Resistance Measurement with Wheatstone Bridge

To find the unknown resistance, R_x, for the Wheatstone bridge shown on the right, assuming it is balanced:



$$R_x = \frac{R_2 R_3}{R_1}$$

Highlights from Last Lecture Cont'd

- Δ-to-Y Transformation

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

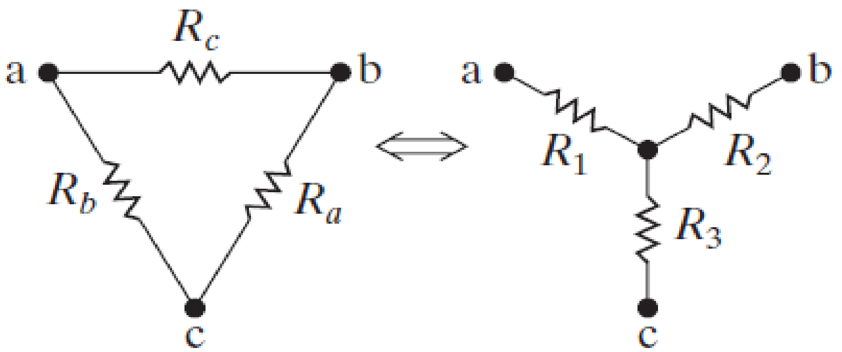
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

- Y-to-Δ Transformation

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$



Nodal Analysis

- Previously we have used voltages and currents of circuit elements as circuit variables for circuit analysis
- Node analysis involves choosing node voltages rather than those of elements as the variables
- Using nodal voltages can reduce the number of variables to solve for and consequently the number of equations to solve
- Nodal Analysis is based on a systematic application of KCL
- Nodal analysis is also called the node-voltage method

Nodal Analysis Procedures

1. Identify all the n essential nodes in the circuit (an essential node is a node where three or more elements are connected)
2. Select one of the essential nodes as the reference node and label it with a ground or reference symbol. The remaining $n-1$ non-reference nodes are assigned the following voltages: v_1, v_2,…, v_{n-1} and these voltages are referenced with respect to the reference node
3. Apply KCL to each of the $n-1$ non-reference nodes and use Ohm's law to express the branch currents in terms of node voltages
4. Solve the resulting system of linear equations to get the nodal voltages

Nodal Analysis Procedures

Identify all the n essential nodes in the circuit

↓

Assign one of the nodes as reference and the others with voltages: $v_1, v_2, \ldots, v_{n-1}$

↓

Apply KCL to $n-1$ non-reference node to get equations in terms of nodal voltages

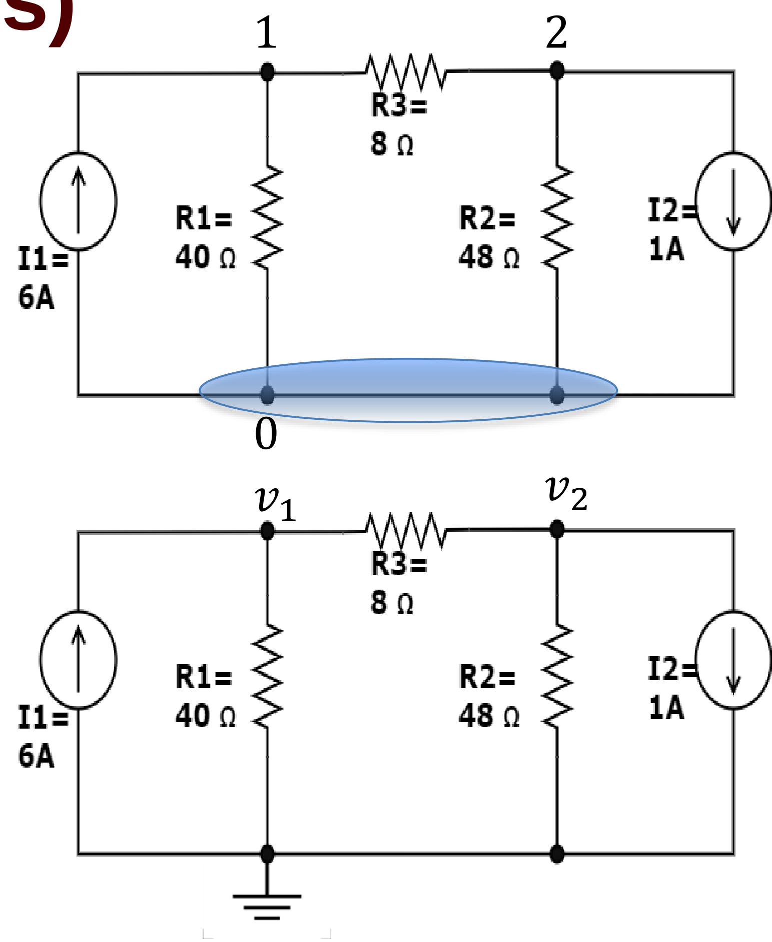
↓

Solve the system of equations simultaneously

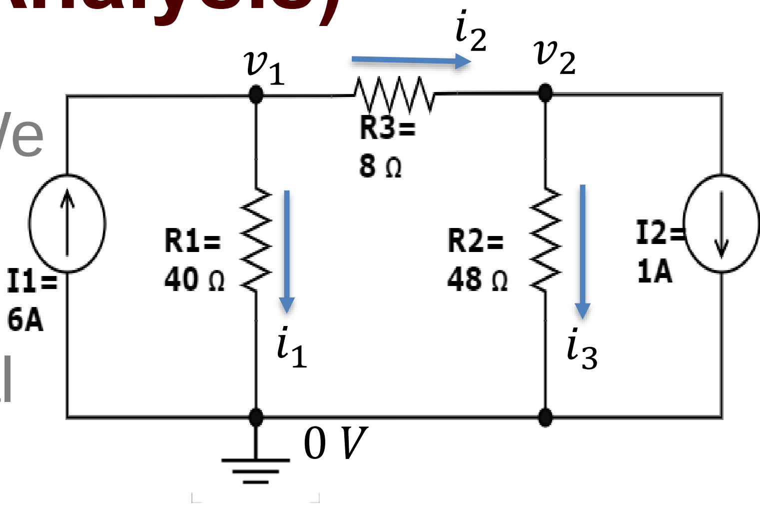
Example 1 (Nodal Analysis)

Consider the circuit on the top right. We apply nodal analysis as follows.

- Step 1: The essential nodes are identified and labeled as 0, 1, 2
- Step 2: Assign node 0 as reference node and nodes 1 & 2 are assigned v_1 and v_2 voltages



Example 1 cont'd (Nodal Analysis)



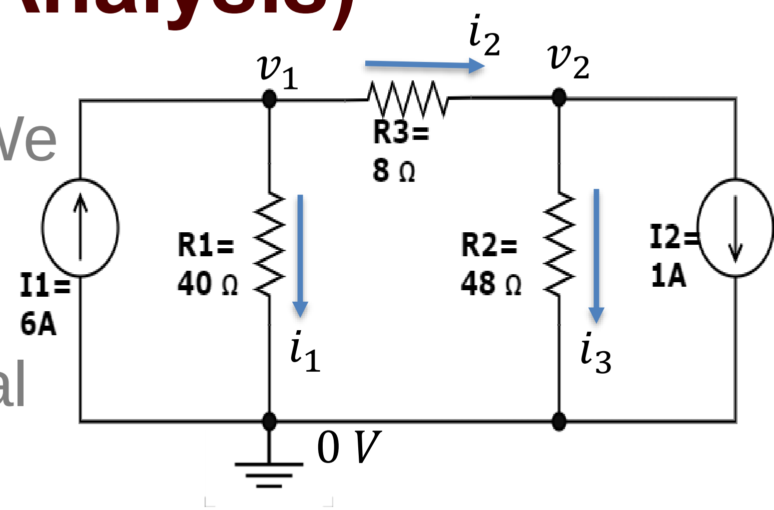
Consider the circuit on the top right. We apply nodal analysis as follows.

- Step 3: Apply KCL and write resulting equation in terms of nodal voltages

Node 1: $I1 = i_1 + i_2 \rightarrow I1 = \frac{v_1 - 0}{R1} + \frac{v_1 - v_2}{R3}$

$$\rightarrow v_1\left(\frac{1}{R1} + \frac{1}{R3}\right) + v_2\left(-\frac{1}{R3}\right) = I1 \quad (1)$$

Example 1 cont'd (Nodal Analysis)



Consider the circuit on the top right. We apply nodal analysis as follows.

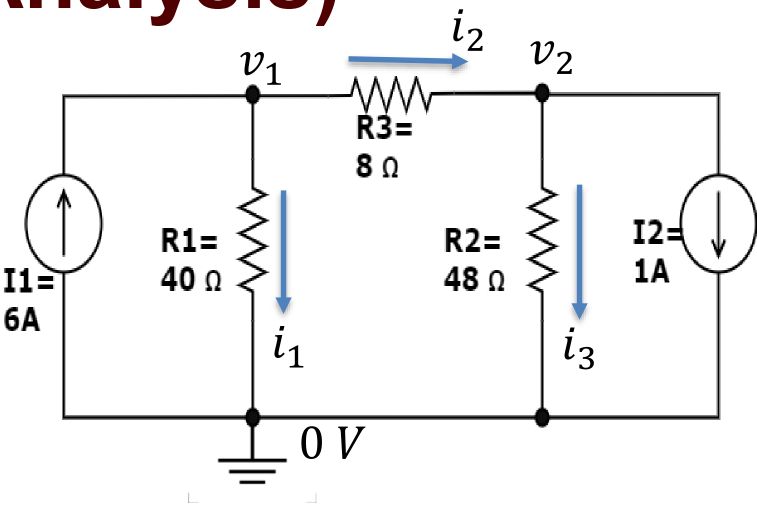
- Step 3: Apply KCL and write resulting equation in terms of nodal voltages

Node 2: $i_2 = i_3 + I2 \rightarrow \frac{v_1 - v_2}{R3} = \frac{v_2 - 0}{R2} + I2$

$\rightarrow v_1\left(-\frac{1}{R3}\right) + v_2\left(\frac{1}{R2} + \frac{1}{R3}\right) = -I2 \quad (2)$

Write equations (1) & (2) in matrix form

Example 1 cont'd (Nodal Analysis)



Consider the circuit on the top right. We apply nodal analysis as follows.

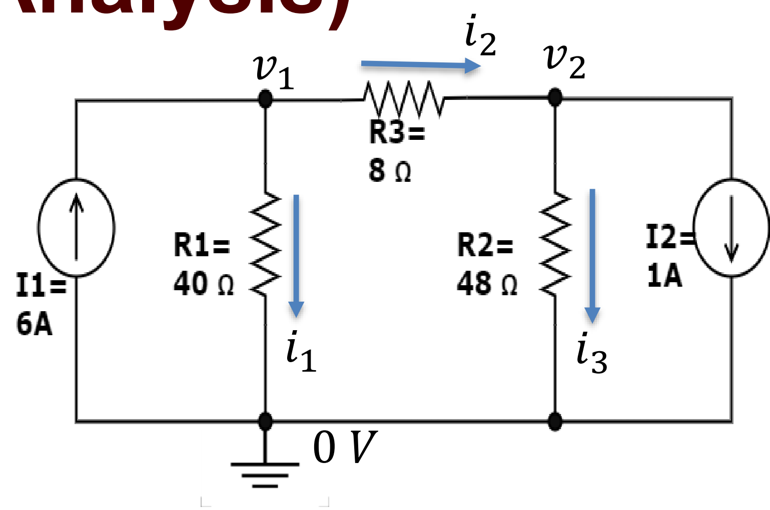
- Step 3: Apply KCL and write resulting equation in terms of nodal voltages

In matrix form

$$\begin{bmatrix} \frac{1}{R1}+\frac{1}{R3} & -\frac{1}{R3} \\ -\frac{1}{R3} & \frac{1}{R2}+\frac{1}{R3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I1 \\ -I2 \end{bmatrix}$$

Did you notice any pattern in the matrix form? When the circuit contains only independent current sources and resistors, you can write the matrix equation by inspection

Example 1 cont'd (Nodal Analysis)



Consider the circuit on the top right. We apply nodal analysis as follows.

- Step 4: Solve the system of equations simultaneously

In matrix form

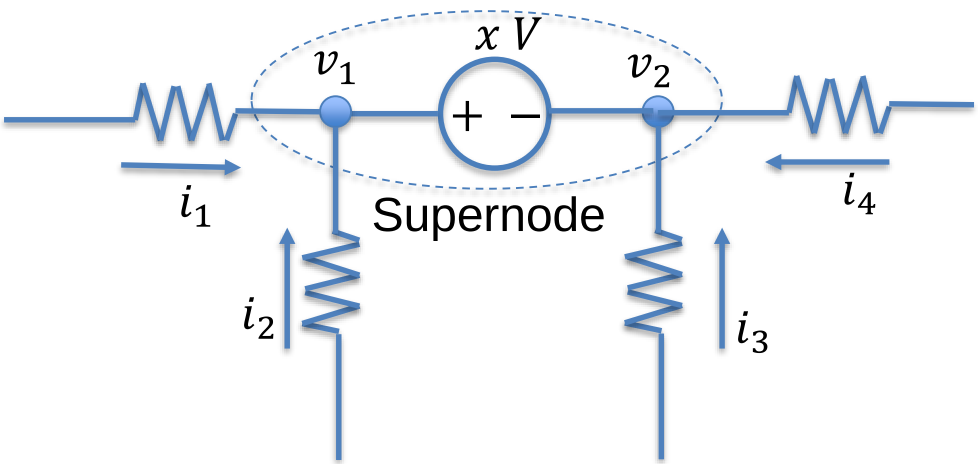
$$\begin{bmatrix} \frac{1}{R1}+\frac{1}{R3} & -\frac{1}{R3} \\ -\frac{1}{R3} & \frac{1}{R2}+\frac{1}{R3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I1 \\ -I2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{40}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{48}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

Which gives $v_1 = 120\ V, v_2 = 96V$. With v_1 and v_2 determined, we can solve for the current through the resistor branches by Ohm's law

Solving Circuits with Voltage Sources using Nodal Analysis

- Since nodal analysis is based on applying KCL at essential nodes, unlike the current through resistors that can be expressed by Ohm's law, the current through a voltage source cannot be easily expressed
- To get around this, we combine two essential node connected by the voltage source into what is called a supernode and apply KCL at this supernode



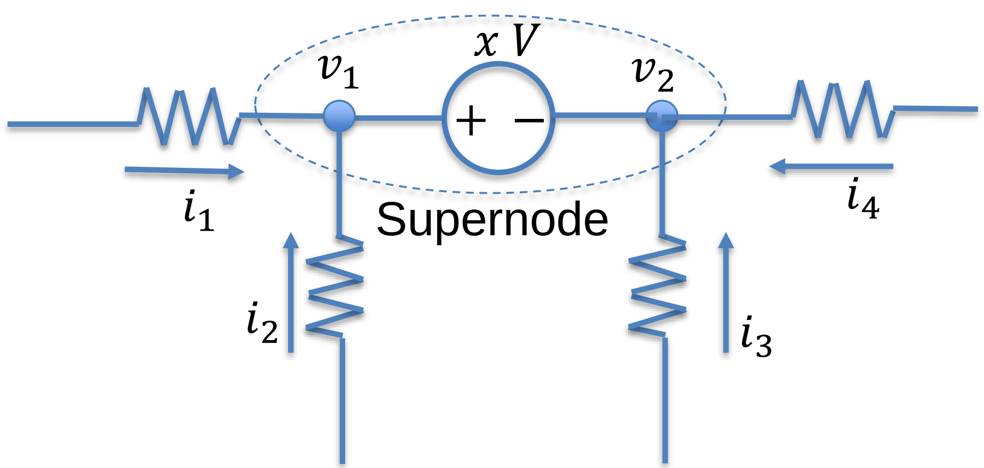
Solving Circuits with Voltage Sources using Nodal Analysis

- KCL at the supernode gives

$i_1 + i_2 + i_3 + i_4 = 0$

- The voltage source at the supernode has a voltage of $x\,V$. This means that there is a voltage rise of x V from node 2 to node 1, that is,

$v_1 - v_2 = x$, This is the additional equation or constraint equation needed to complete the system of equations when there is a supernode in the circuit

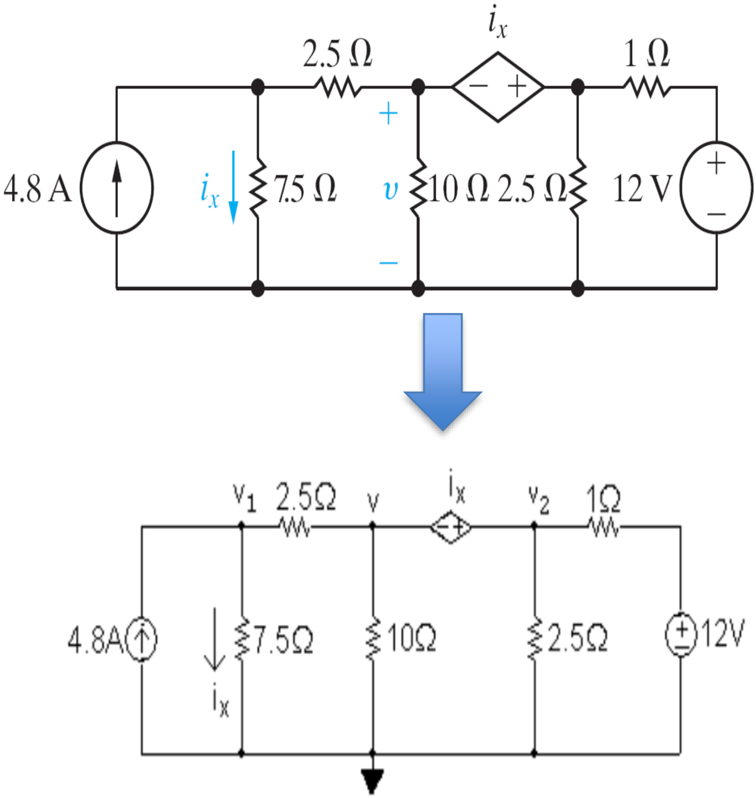


Example 2 (Assessment Problem 4.5)

Use the node-voltage method (that is nodal analysis) to find v in the circuit on the top right

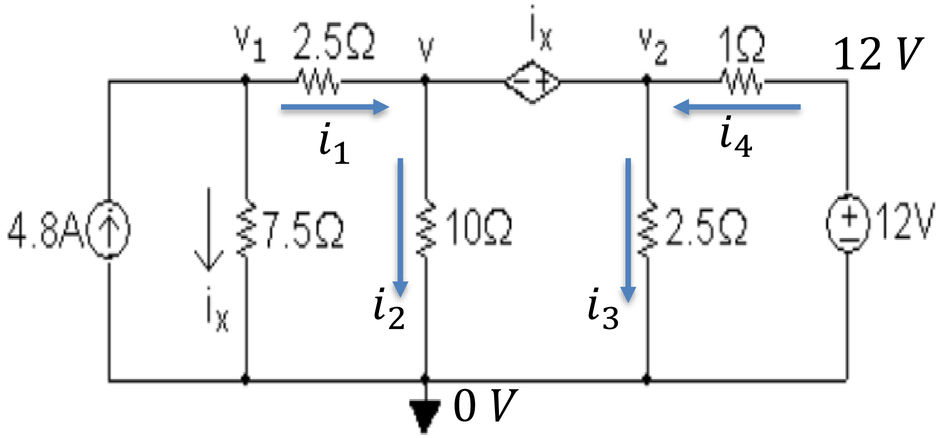
Choose a reference node, specify the nodal voltage variables at the remaining essential nodes

Note that v and v_2 nodes form a super node



Example 2 (Assessment Problem 4.5)

The current directions have been assigned arbitrarily



KCL at v_1 node (eq 1):

$i_1 + i_x = 4.8 \rightarrow$??

KCL at the supernode (eq 2):

??

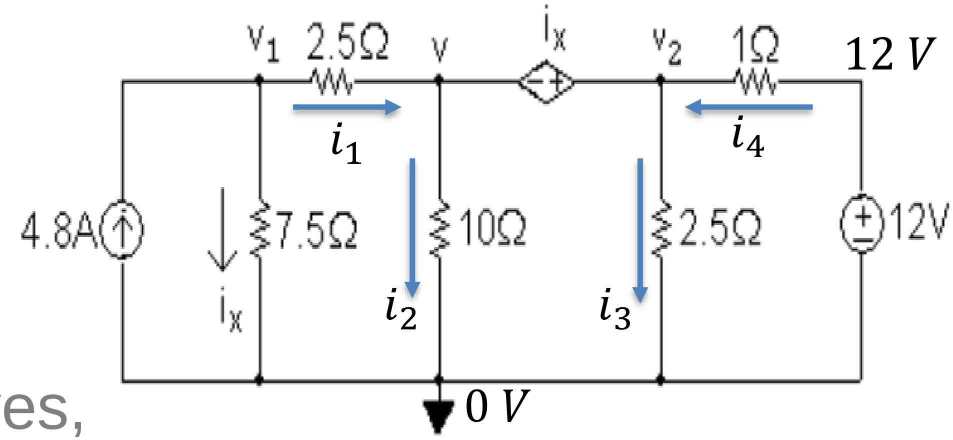
Constraint equation due to supernode (eq 3):

??

Example 2 (Assessment Problem 4.5)

In matrix form

$$\begin{bmatrix} 4 & 0 & -3 \\ -0.4 & 1.4 & 0.5 \\ 1 & -7.5 & 7.5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v \end{bmatrix} = \begin{bmatrix} 36 \\ 12 \\ 0 \end{bmatrix}$$



Solving the matrix equation gives,

??

Mesh Analysis

- Unlike nodal analysis, mesh analysis involves choosing mesh currents rather than node voltages (A mesh is a loop that does not contain any other loops within it)
- Mesh Analysis is based on a systematic application of KVL
- Mesh analysis is also called the mesh-current method

Mesh Analysis Procedures

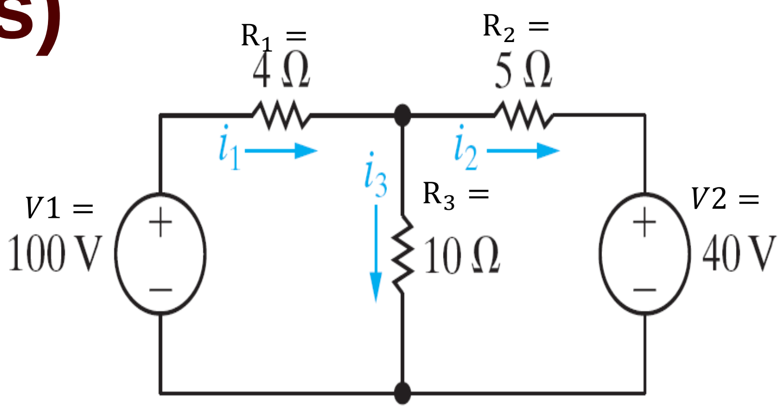
1. Identify the n meshes in the circuit
2. Assign mesh currents to each of the n meshes. A mesh current is the current that exists on the perimeter of a mesh
3. Apply KVL to each of the n meshes and use Ohm's to express the voltages in terms of the mesh currents
4. Solve the resulting system of equations

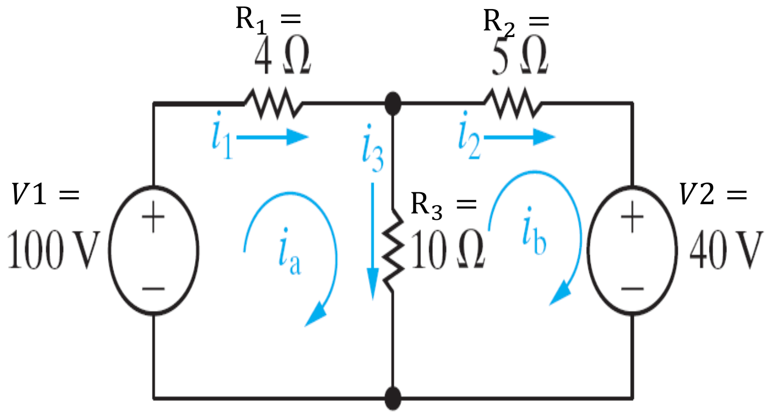
We will demonstrate these steps with an example

Example 3 (Mesh Analysis)

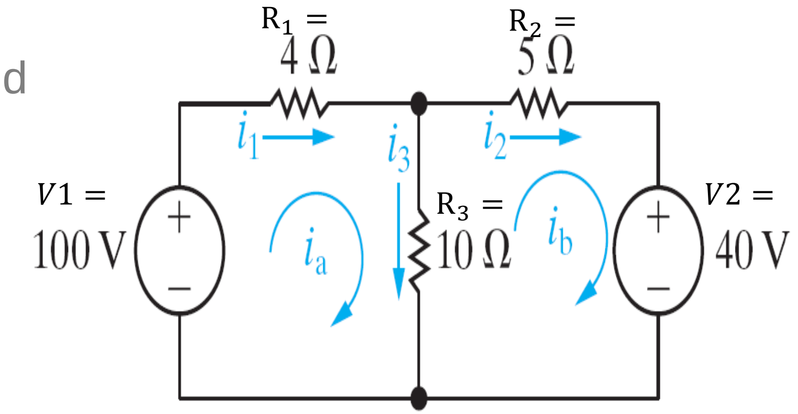
Consider the circuit on the top right. We apply mesh analysis as follows.

- Step 1: There are two meshes in the circuit with directed curved arrows as shown in the bottom right circuit
- Step 2: Mesh currents i_a and i_b are assigned to each mesh





Example 3 Cont'd (Mesh Analysis)



- Step 3: Apply KVL to each of the 2 meshes and use Ohm's to express the voltages in terms of the mesh currents

Note that $i_1 = i_a$, $i_2 = i_b$, and $i_3 = i_a - i_b$

KVL around i_a mesh:

$-V1 + R_1 i_a + R_3(i_a - i_b) = 0$

$\rightarrow (R_1 + R_3)i_a - R_3 i_b = V1 \quad (1)$

KVL around i_b mesh:

$R_3(i_b - i_a) + R_2 i_b + V2 = 0$

$\rightarrow -R_3 i_a + (R_2 + R_3)i_b = -V2 \quad (2)$

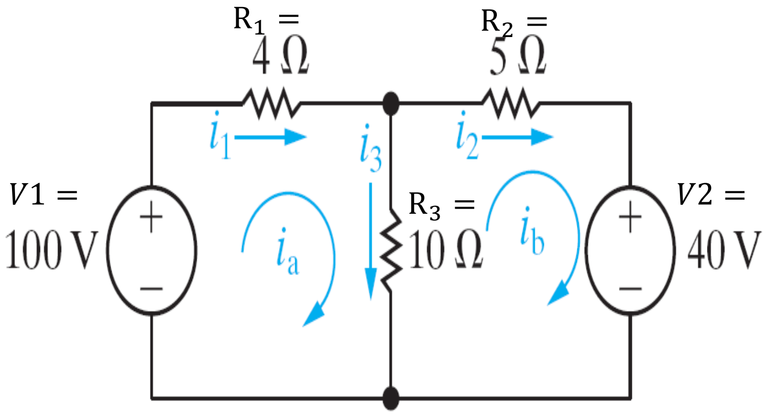
Equation (1) and (2) in matrix form gives:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} V1 \\ -V2 \end{bmatrix}$$

Did you notice any pattern in the matrix form? You can write down the matrix form by inspection if the circuit contains only resistors and independent voltage sources

Example 3 Cont'd (Mesh Analysis)

- Step 4: Solve the resulting system of equations



$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} V1 \\ -V2 \end{bmatrix}$$

$$\begin{bmatrix} 14 & -10 \\ -10 & 15 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 100 \\ -40 \end{bmatrix}$$

$$i_a = 10\ A, i_b = 4\ A$$

$$i_1 = i_a = 10A$$

$$i_2 = i_b = 4A$$

$$i_3 = i_a - i_b = 6A$$