natural response: no voltage source

Natural response & Step response of Parallel RLC circuit

① voltage is prefered because it's continous all the time

voltage is same for all thee components, and makes us easier to calculate branch current

$$I_C + I_L + I_R = 0$$

$$\Rightarrow C\frac{dV(t)}{dt} + \frac{1}{L}\int_0^t V(x)\,dx + I_0 + \frac{V(t)}{R} = 0$$

$$\Rightarrow \frac{d^2V(t)}{dt^2} + \frac{1}{LC}V(t) + \frac{1}{RC}\frac{dV(t)}{dt} = 0 \text{ (standard form)}$$

Some qualities of this stardard form: ① second order ② Homogenegous ③ Ordinary diff equation ④ has constant coefficients

Which can also be $V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ or $s^2 + (1/RC)s + (1/LC) = 0$

Where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ & $\alpha = \frac{1}{2RC}$ & $\omega_0 = \sqrt{\frac{1}{LC}}$

So the form of S_1 & S_2 depends on $\sqrt{\alpha^2 - \omega_0^2}$:

$\alpha^2 > \omega_0^2$: overdamped, $V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$\alpha^2 < \omega_0^2$: underdamped, $V(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$

And $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$\alpha^2 = \omega_0^2$: Critical damped: $s_1 \& s_2 = -\alpha$, $V(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

For step responce; $\frac{d^2 I_L}{dt^2} + \frac{1}{RC}\frac{dI_L}{dt} + \frac{I_L}{LC} = \frac{I}{LC}$ Char equation: $s^2 + (1/RC)s + (1/LC) = 0$

$I_L(t) = I_F + I_N$ where I_F is particular solution & I_N is homogeneous

Inductor current is preffered because it's only component, final value isn't zero.

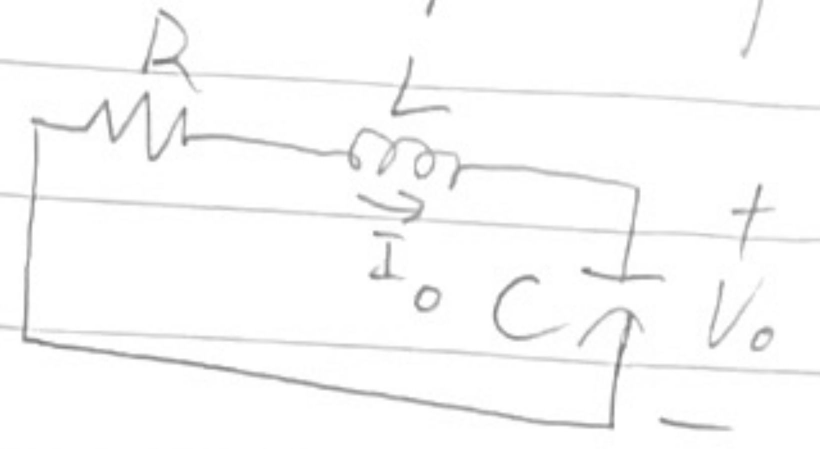
$\alpha = \frac{1}{2RC}$, $\omega_0 = \sqrt{\frac{1}{LC}}$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$\alpha^2 > \omega_0^2$ $I_L(t) = I_f + A'_1 e^{s_1 t} + A'_2 e^{s_2 t}, t \geq 0$

$\alpha^2 < \omega_0^2$ $I_L(t) = I_f + B'_1 e^{-\alpha t}\cos\omega_d t + B'_2 e^{-\alpha t}\sin\omega_d t, t \geq 0$

$\alpha^2 = \omega_0^2$ $I_L(t) = I_f + D'_1 t e^{-\alpha t} + D'_2 e^{-\alpha t}, t \geq 0$

Natural Response for Series RLC circuit



We use current because current is the same through all the circuit

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ $\quad \alpha = \frac{R}{2L}$ $\quad \omega_0 = \sqrt{\frac{1}{LC}}$ $\quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$\alpha^2 > \omega_0^2$: overdamped $I(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$\alpha^2 < \omega_0^2$: underdamped $I(t) = B_1 e^{-\alpha t}\cos \omega_d t + B_2 e^{-\alpha t}\sin \omega_d t$

$\alpha^2 = \omega_0^2$: critically damped $I(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$,

Duality

③ formula for both ~~circle~~ circuit is $\omega_0 = \sqrt{\frac{1}{LC}}$

However, formula for ω_d differs:

parallel : $\omega_d = \sqrt{\alpha^2 - \omega_0^2}$

series : $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

④ Natural respose for parallel:

describing: $\frac{d^2 v(t)}{dt^2} + \frac{1}{LC}v(t) + \frac{1}{RC}\frac{dv(t)}{dt} = 0$

char : $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

Natural respose for series:

describing: $\frac{d^2 I(t)}{dt^2} + \frac{R}{L}\frac{dI(t)}{dt} + \frac{1}{LC}I(t) = 0$

char : $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$