Lecture 6: Mesh Analysis & Source Transformation

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ECEN 214 – Electrical Circuit Theory (Spring 2020)

Outline

- Mesh Analysis
- Source Transformation

Highlight from last Lecture: Nodal Analysis Procedures

Identify all the n essential nodes in the circuit

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Assign one of the nodes as reference and the others with voltages: $v_1, v_2, \ldots, v_{n-1}$

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Apply KCL to $n-1$ non-reference node to get equations in terms of nodal voltages

↓

Solve the system of equations simultaneously

Highlights from last Lecture: Mesh Analysis Procedures

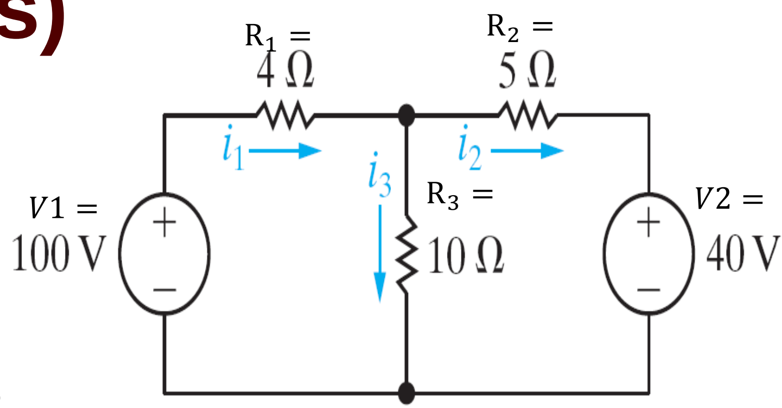
1. Identify the n meshes in the circuit
2. Assign mesh currents to each of the n meshes. A mesh current is the current that exists on the perimeter of a mesh
3. Apply KVL to each of the n meshes and use Ohm's to express the voltages in terms of the mesh currents
4. Solve the resulting system of equations

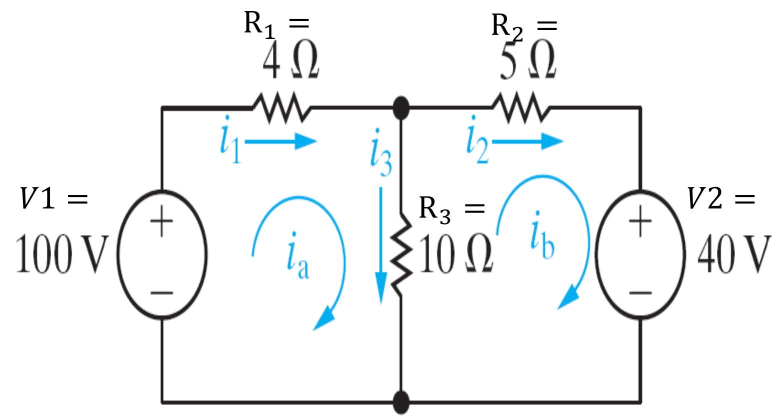
We will demonstrate these steps with an example

Example 1 (Mesh Analysis)

Consider the circuit on the top right. We apply mesh analysis as follows.

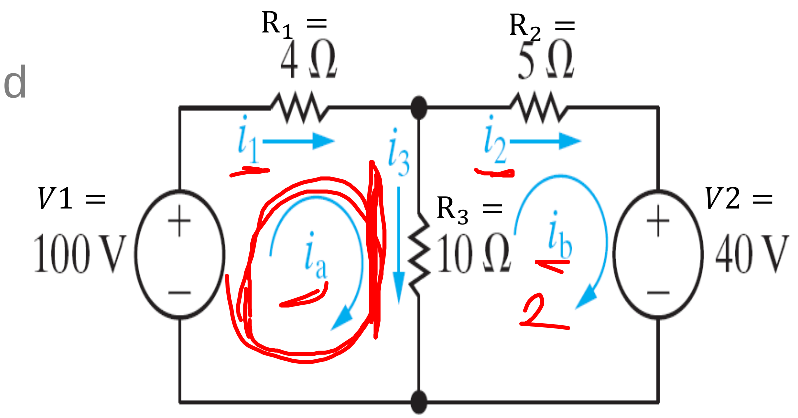
- Step 1: There are two meshes in the circuit with directed curved arrows as shown in the bottom right circuit
- Step 2: Mesh currents i_a and i_b are assigned to each mesh





Example 1 Cont'd (Mesh Analysis)

- Step 3: Apply KVL to each of the 2 meshes and use Ohm's to express the voltages in terms of the mesh currents



Note that $i_1 = i_a$, $i_2 = i_b$, and $i_3 = i_a - i_b$

KVL around i_a mesh:

$-V1 + R_1 i_a + R_3(i_a - i_b) = 0$

$\rightarrow (R_1 + R_3)i_a - R_3 i_b = V1 \quad (1)$

col 4 row 3

KVL around i_b mesh:

$R_3(i_b - i_a) + R_2 i_b + V2 = 0$

$\rightarrow -R_3 i_a + (R_2 + R_3)i_b = -V2 \quad (2)$

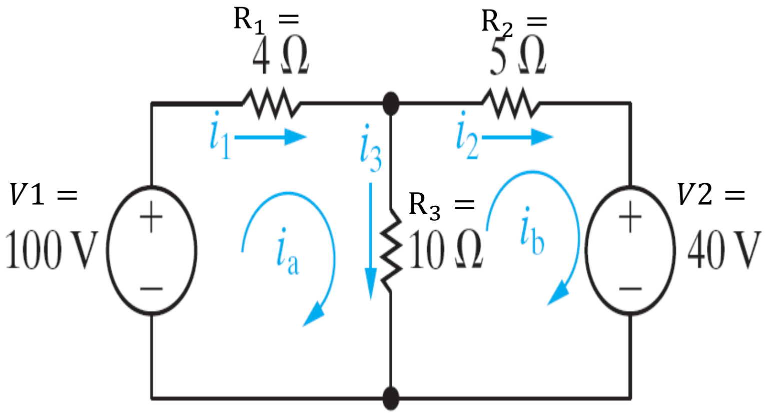
Equation (1) and (2) in matrix form gives:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} V1 \\ -V2 \end{bmatrix}$$

Did you notice any pattern in the matrix form? You can write down the matrix form by inspection if the circuit contains only resistors and independent voltage sources

Example 1 Cont'd (Mesh Analysis)

- Step 4: Solve the resulting system of equations



$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} V1 \\ -V2 \end{bmatrix}$$

$$\begin{bmatrix} 14 & -10 \\ -10 & 15 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 100 \\ -40 \end{bmatrix}$$

$$i_a = 10\ A, i_b = 4\ A$$

$$i_1 = i_a = 10A$$

$$i_2 = i_b = 4A$$

$$i_3 = i_a - i_b = 6A$$

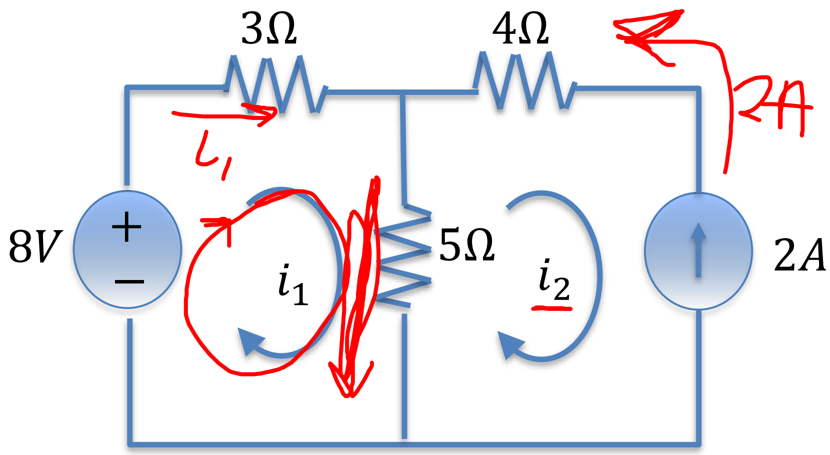
Solving Circuits with Current Sources using Mesh Analysis

- Since mesh analysis is based on applying KVL at each mesh, unlike the voltage drop through resistors that can be expressed by Ohm’s law, the voltage drop through a current source cannot be easily expressed
- We demonstrate this situation using two examples for two possible cases

Example 2: Solving Circuits with Current Sources using Mesh Analysis (Case 1)

Case 1: When the current exist only in one mesh

- Consider the circuit below, set $i_2 = -2A$
- Write KVL equation for i_1 mesh as usual



$$-8 + 3i_1 + 5(i_1 - i_2) = 0$$

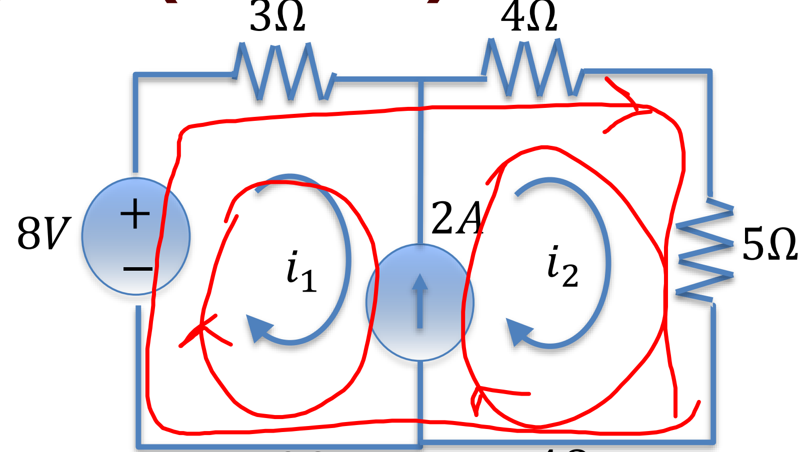
$$-8 + 3i_1 + 5(i_1 + 2) = 0$$

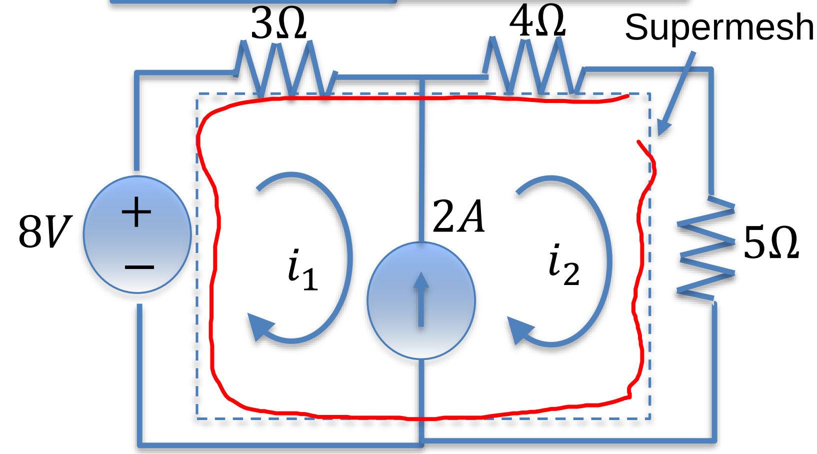
$$i_1 = -\frac{1}{4}A$$

Example 3: Solving Circuits with Current Sources using Mesh Analysis (Case 2)

Case 2: When the current exist between two meshes

- If you want to write the KVL for each mesh shown in the figure below, there is no way to tell the voltage drop across the current source
- To get around this, create what is called a supermesh





Example 3 Cont'd: Solving Circuits with Current Sources using Mesh Analysis (Case 2)

Case 2: When the current exist between two meshes

- Apply KVL to the supermesh

$-8 + 3i_1 + 4i_2 + 5i_2 = 0$

$\rightarrow 3i_1 + 9i_2 = 8 \quad (1)$

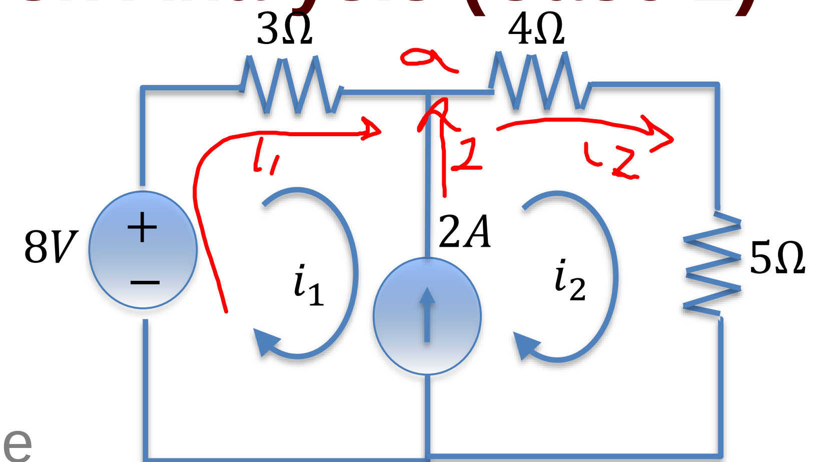
- We need another equation to complete eq. (1). Apply KCL at node a or b of the supermesh

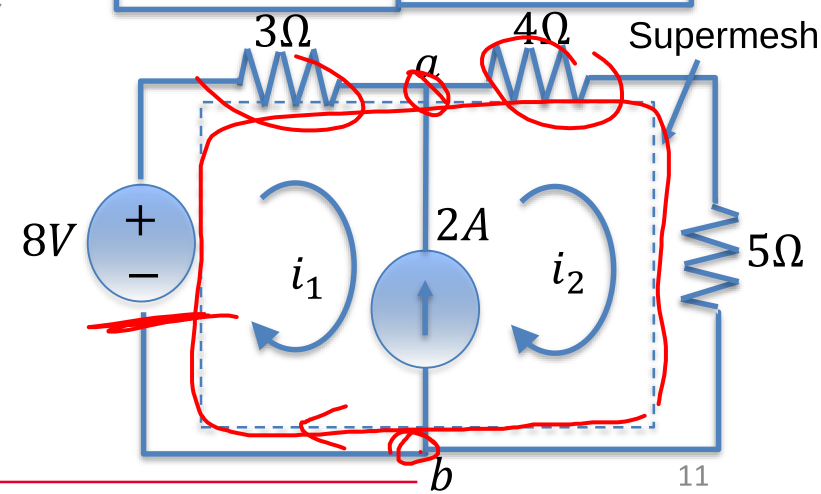
$i_2 - i_1 = 2 \quad (2)$

Eq (2) is called the constraint equation imposed by the supermesh

$i_1 = -\frac{5}{6}, A\ i_2 = \frac{7}{6}A$

$i_1 + 2 = i_2$

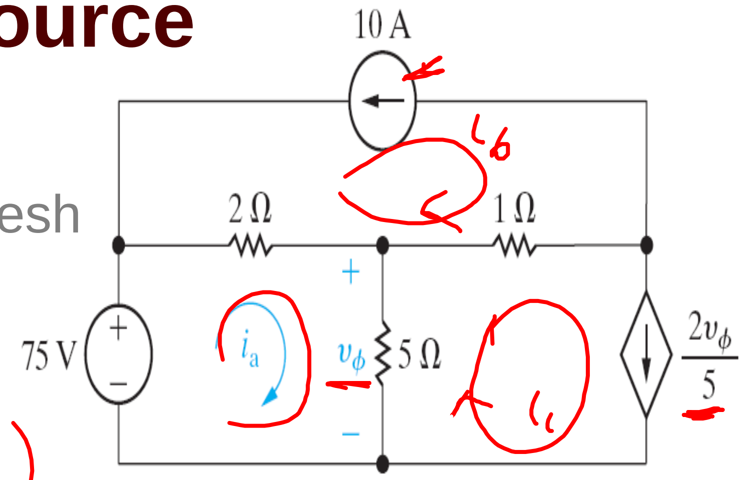




Example 4: With dependent source (Assessment problem 4.11)

- Use the mesh-current method to find the mesh current i_a in the circuit shown on the right.

How many mesh equations do we need?



$i_b = -10A$, $i_c = \frac{2V_\phi}{5}$ —— (1)

KVL for mesh A $\quad -75 + 2(i_a - i_b) + 5(i_a - i_c) = 0$

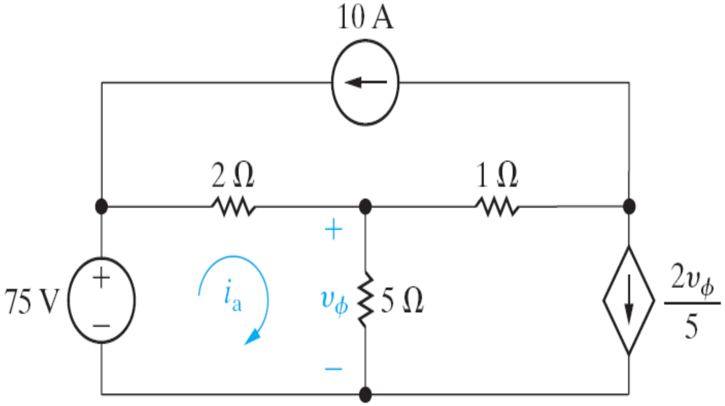
$-75 + 2(i_a + 10) + 5(i_a - i_c) = 0$

$7i_a - 5i_c = 55$ —— (2)

$V_\phi = 5(i_a - i_c)$ —— (3)

$i_a = 15A$, $\quad i_c = 10A$ $\quad V_\phi = 25V$

Example 4 cont'd: With dependent source (Assessment problem 4.11)

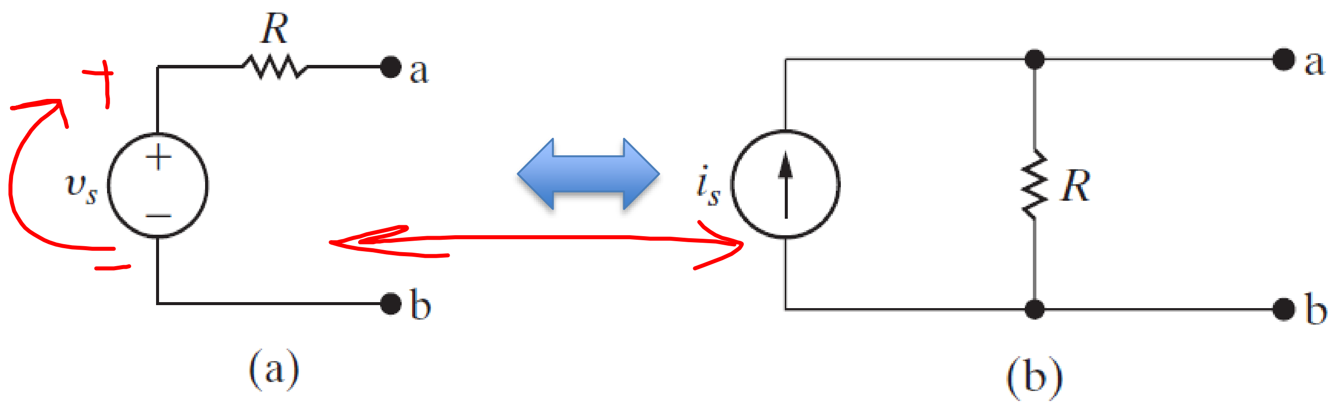


Mesh Analysis vs Nodal Analysis: What is the best approach to choose?

- There is no hard and fast rule on what approach to choose. Typically, we desire to choose an approach that results in a fewer system of equations
- Some questions to consider when making a choice:
  - Which method will result in fewer number of equations?
  - Is there a voltage source between two essential nodes? If yes, then making one of these essential nodes the reference node may reduce the number of equations
  - Is there a current source that a member of one mesh? If yes, then using mesh analysis will reduce the number of mesh equations to be solved
  - Will solving a portion of the circuit give the requested solution? If yes, what method is best for solving that portion of the circuit?

Source Transformation

- Just like series-parallel combination and Y-Δ transformation helps to simplify circuit, so does source transformation
- A source transformation allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa

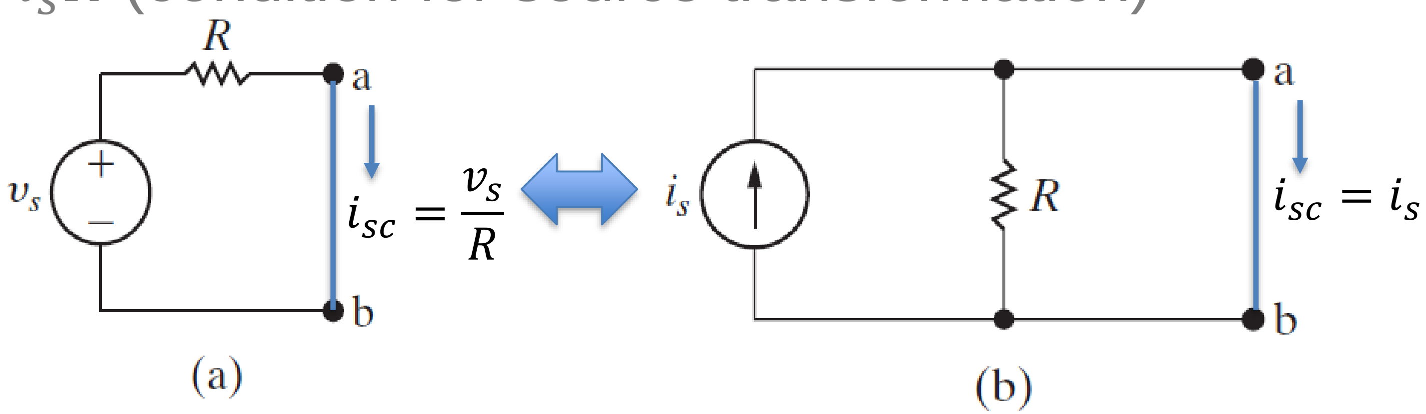


Source Transformation

- The two circuits below are equivalent if they have the same voltage-current relation at terminals a and b.
- When terminals a and b are shorted, then their short circuit current, i_{sc}, must be equal if they are equivalent. Therefore,

$$i_{sc} = \frac{v_s}{R} = i_s$$

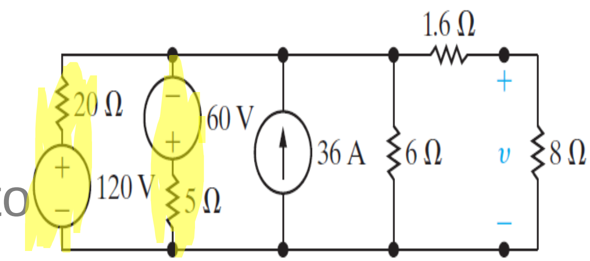
Or $v_s = i_s R$ (condition for source transformation)

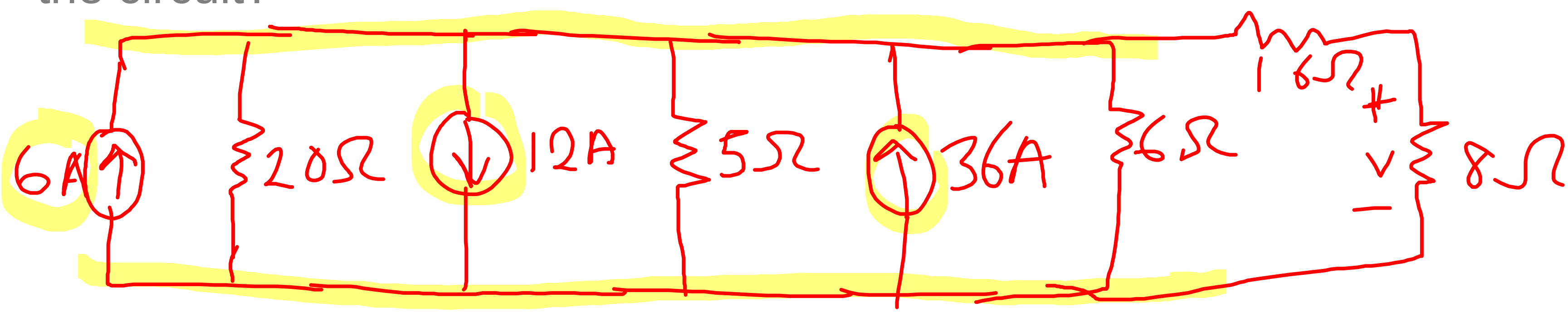


Example 5: (Assessment problem 4.15)

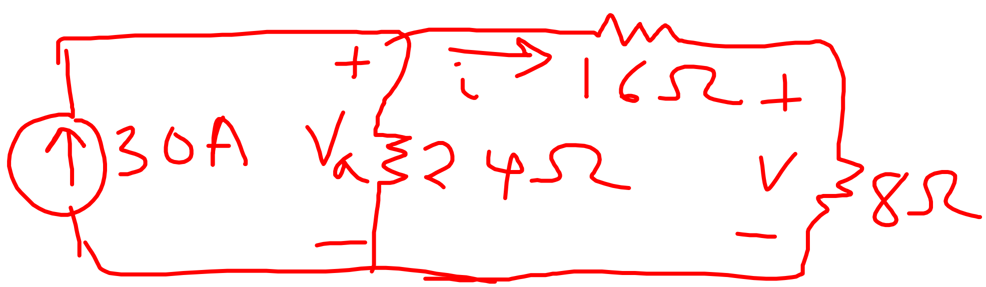
a) Use a series of source transformation to find the voltage v in the circuit

b) How much power does the 120 V source deliver to the circuit?





$20 \| 5 \| 6 = 24\Omega$



$i = \dfrac{24}{1.6+8+24} \times 36A = 6A$

$V = 6A \times 8\Omega = 48V$

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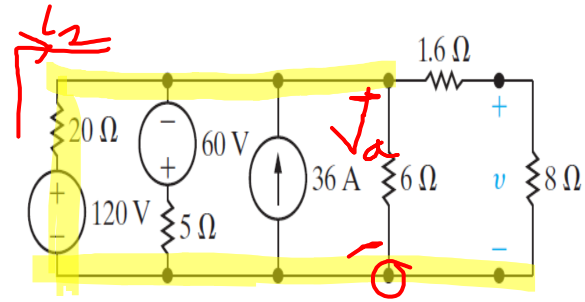
Example 5 cont'd: (Assessment problem 4.15)



b) $V_a = i(1.6+8)$
$= 6 \times 9.6 = 57.6\,V$

$120 - i_2 \times 20 = 57.6\,V$
$i_2 = 3.12\,A$

$P_{120V} = -v i_2 = -120 \times 3.12$
$= -374.4\,W, |P_{120}| = 374.4\,W$