Lecture 7: Thevenin & Norton Equivalent Circuits, Maximum Power Transfer

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ECEN 214 – Electrical Circuit Theory (Spring 2020)

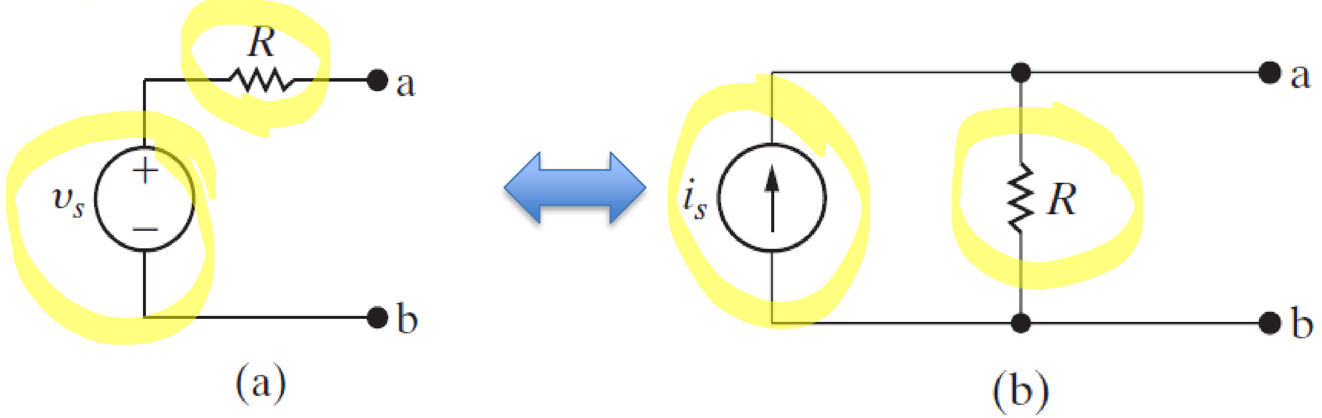
Outline

- Thevenin & Norton Equivalent Circuits
- Maximum Power Transfer

Highlights from Last lecture: Source Transformation

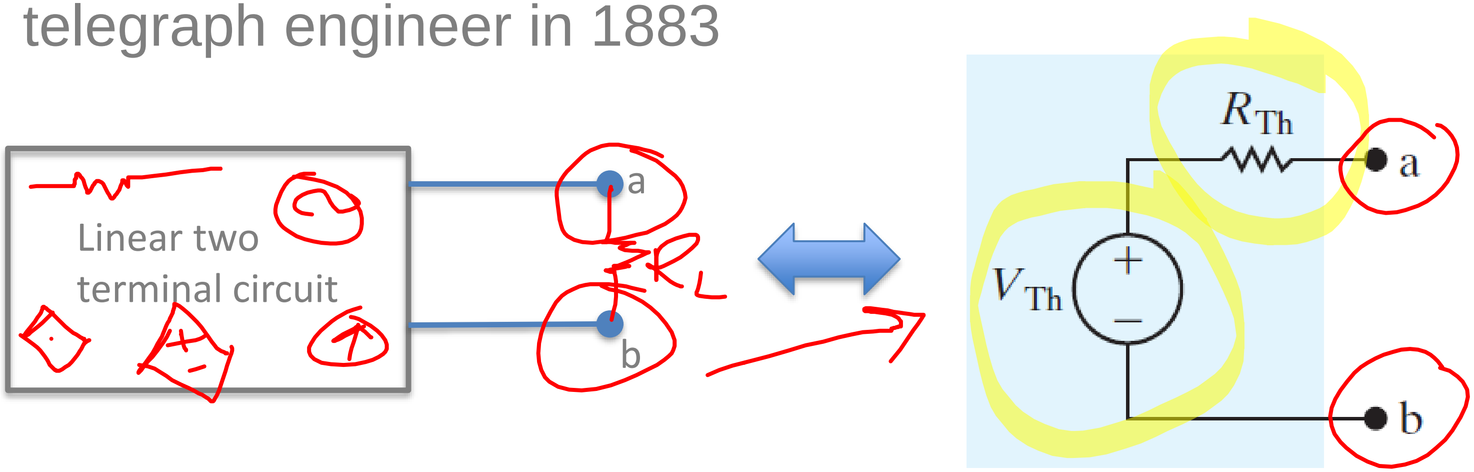
- A source transformation allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa
- The relationship between the two transformation is as follows:

$$\frac{v_s}{R} = i_s \text{ or } v_s = i_s R$$



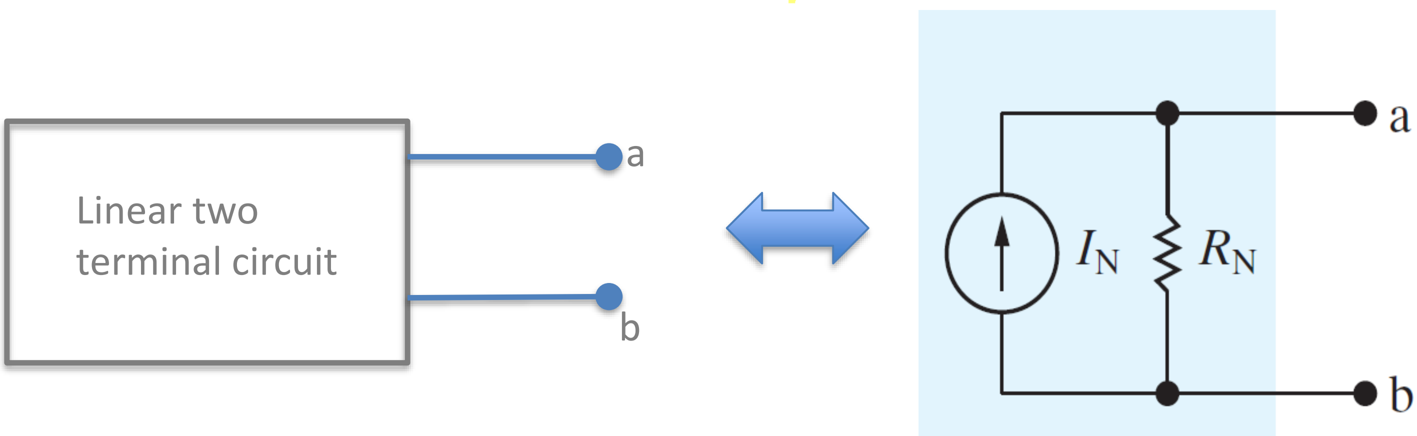
Thevenin's Theorem

- Thevenin's theorem states any linear two terminal circuit can be represented by an equivalent circuit made up of a voltage source, V_{th}, in series with a resistor, R_{th}
- A linear circuit is one whose output is linearly related to its input
- This theorem was derived by M. Leon Thevenin, a French telegraph engineer in 1883



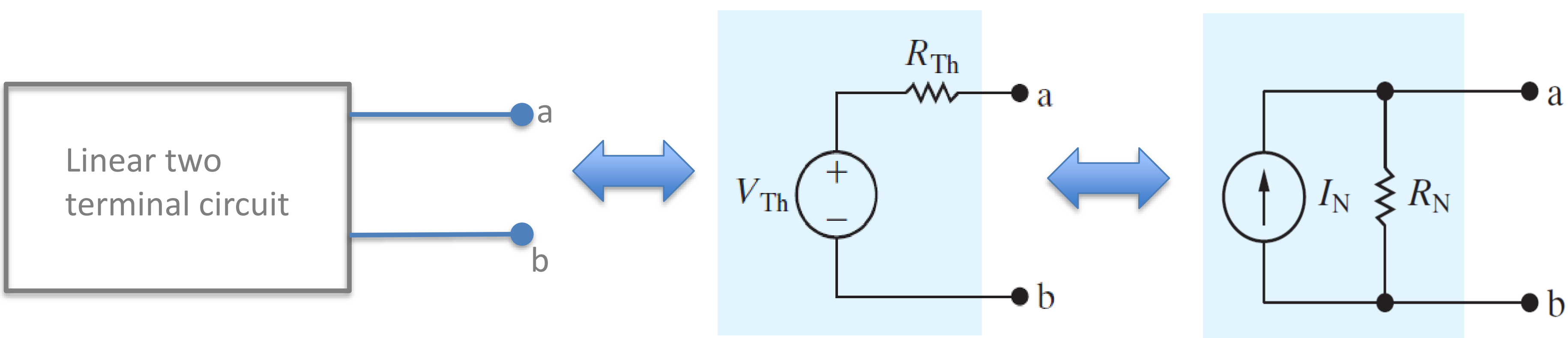
Norton's Theorem

- Northon's theorem states any linear two terminal circuit can be represented by an equivalent circuit made up of a current source, I_N, in parallel with a resistor, R_N
- This theorem was derived by E. L. Norton, an American engineer at Bell Telephone Laboratories
- The Norton's theorem is a dual of the Thevenin's theorem. **Both Norton and Thevenin equivalent circuits are equivalent and are source transformation of each other i.e.** $R_N = R_{Th}$, $V_{Th} = I_N R_{Th}$



Finding Thevenin Equivalent Voltage, V_{Th}, Norton Equivalent Current, I_N and Resistance, R_{th}/R_N

- If the linear two terminal circuit is equivalent to the Thevenin's and Norton's equivalents, then their open circuit (terminals a and b open) and short circuit (terminals a and b shorted) characteristics should be the same

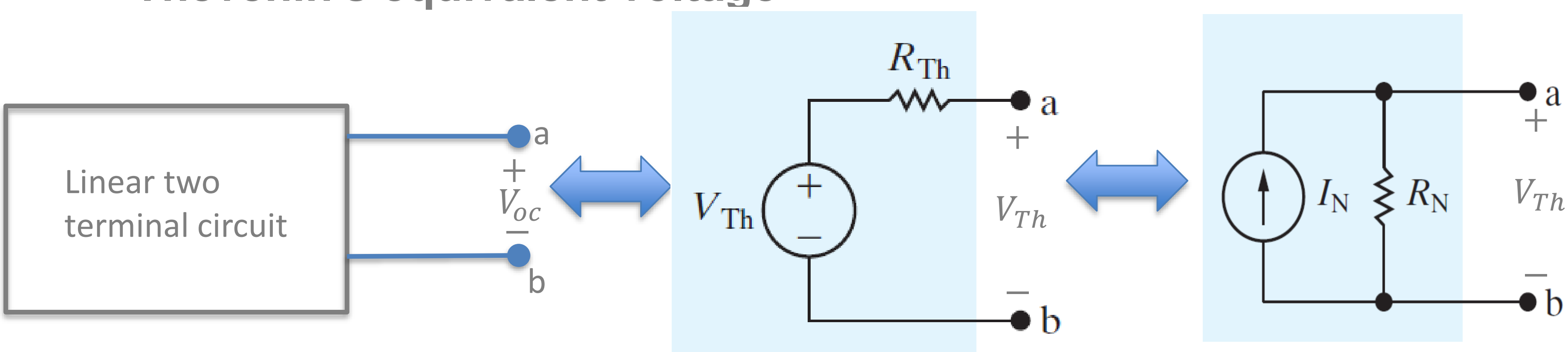


Finding Thevenin Equivalent Voltage, V_{th}

- Open circuit characteristics:

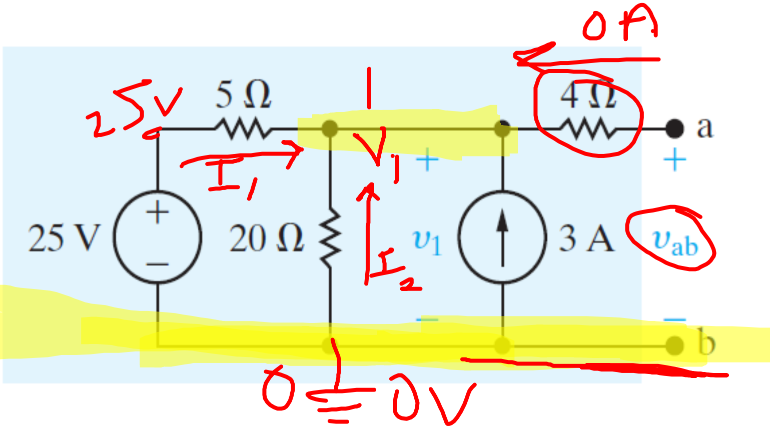
Open circuit voltage across terminals a and b of the Thevenin and Norton equivalent circuits is V_{th} and by hypothesis, should be equal to the open circuit voltage, V_{oc}, of the original circuit (i.e. the linear two terminal circuit)

- In other words, $V_{Th} = V_{oc}$, **solve for the open circuit voltage across terminals a and b of the linear two terminal circuit to get Thevenin's equivalent voltage**



Example 1A (Finding Thevenin Voltage)

Find the Thevenin voltage with reference to terminals a and b



Two essental node 0, 1

KCL at node 1

$$I_1 + I_2 + 3 + 0 = 0$$

$$\frac{25 - V_1}{5} + \frac{0 - V_1}{20} + 3 = 0, \Rightarrow V_1 = 32V$$

$$\Rightarrow V_a = 32V$$

$$V_{ab} = V_a - V_b = 32 - 0 = 32\ V$$

$$V_{Th} = V_{ab} = 32V$$

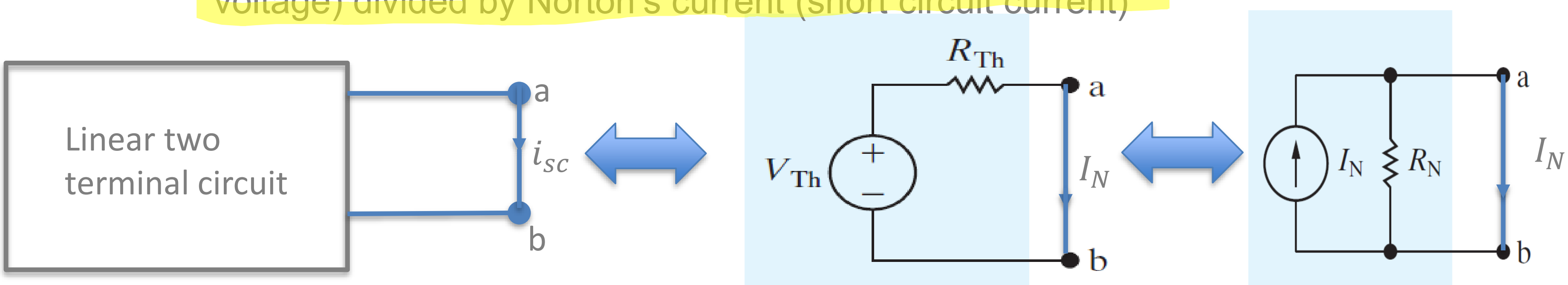
Finding Norton Equivalent Current, I_N and Thevenin/Norton Equivalent Resistance R_{Th}/R_N

- Short Circuit Characteristics:

Short circuit current across terminals a and b of the Thevenin's and Norton's equivalent circuit is I_N and by hypothesis, should be equal to the short circuit current, i_{sc}, of the original circuit (i.e. the linear two terminal circuit)

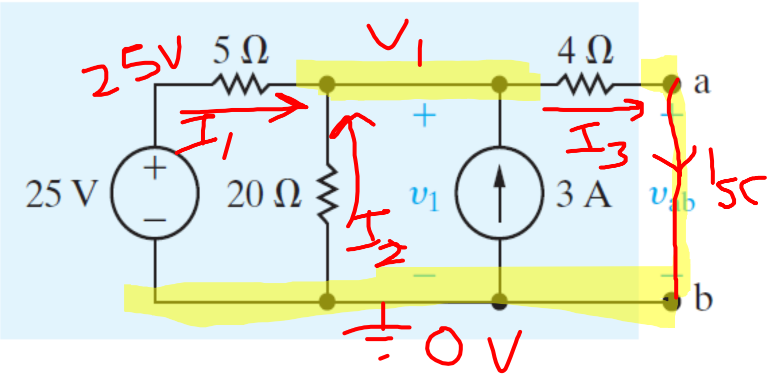
$I_N = i_{sc} = \frac{v_{Th}}{R_{Th}} \rightarrow R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{v_{Th}}{I_N}$ Note that $R_N = R_{Th}$

- In other words,
  - the Norton equivalent current is equal to the short circuit current of the original circuit
  - The Thevenin's/Norton's resistance is equal to the Thevenin's voltage (open circuit voltage) divided by Norton's current (short circuit current)



Example 1B (Finding Norton Current & Thevenin/Norton Resistance)

Find the Norton current and Thevenin/Norton resistance with reference to terminals a and b



KCL at node 1

$$I_1 + I_2 + 3 = I_3$$

$$\frac{25 - V_1}{5} + \frac{0 - V_1}{20} + 3 = \frac{V_1 - 0}{4}$$

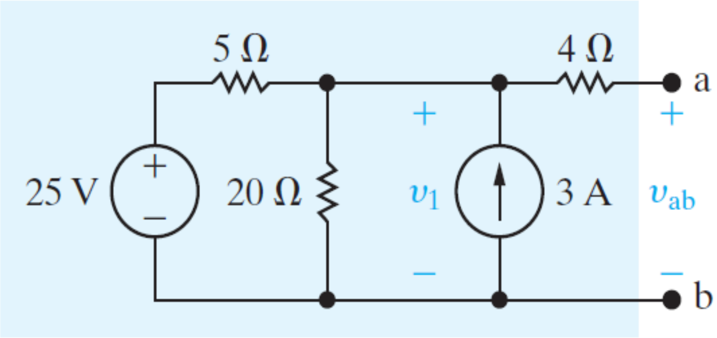
$$100 - 4V_1 - V_1 + 60 = 5V_1$$

$$V_1 = 16V$$

$$I_3 = \frac{V_1 - 0}{4} = 4A, \quad I_{sc} = I_N = I_3 = 4A$$

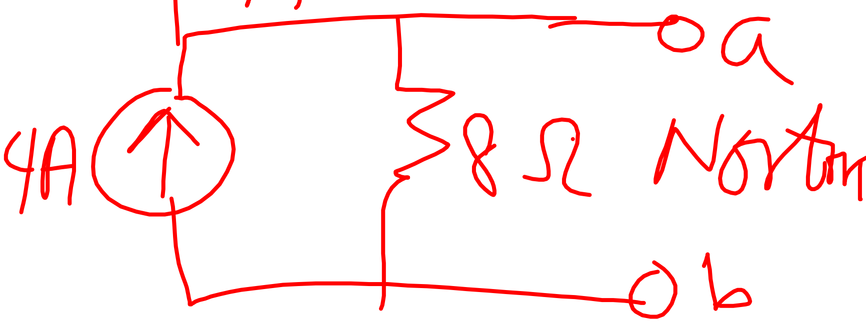
Example 1B Cont'd (Finding Norton Current & Thevenin/Norton Resistance)

Find the Norton current and Thevenin/Norton resistance looking through terminals a and b



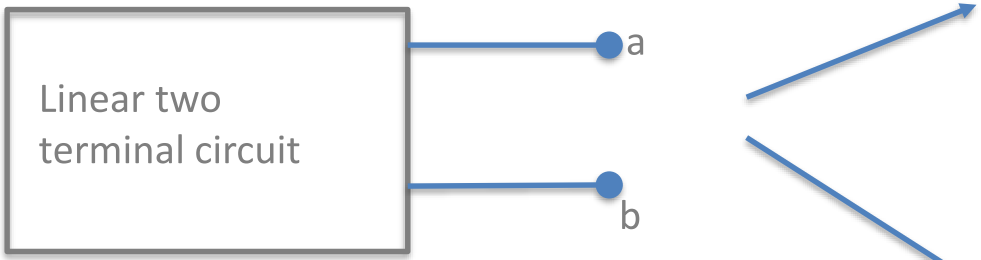
$$R_{Th} = R_N = \frac{V_{Th}}{I_N} = \frac{32V}{4A} = 8\,\Omega$$

↔

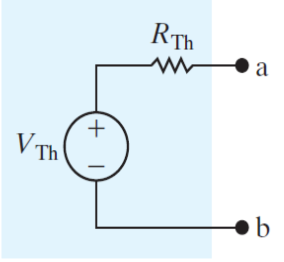


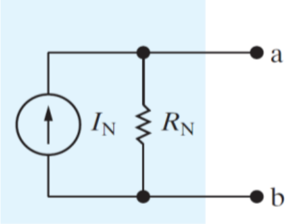
Finding Thevenin/Norton Equivalents by using Source Transformation

- Depending on the original linear circuit, it may be possible to apply a series of source transformations and parallel/series combination to get the Thevenin or Norton equivalent circuits. This approach works best when the original circuit does not contain dependent source



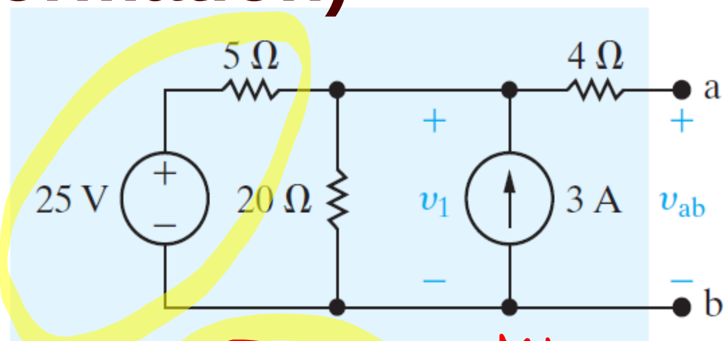
Apply source transformation to the original circuit until you get your desired equivalent form

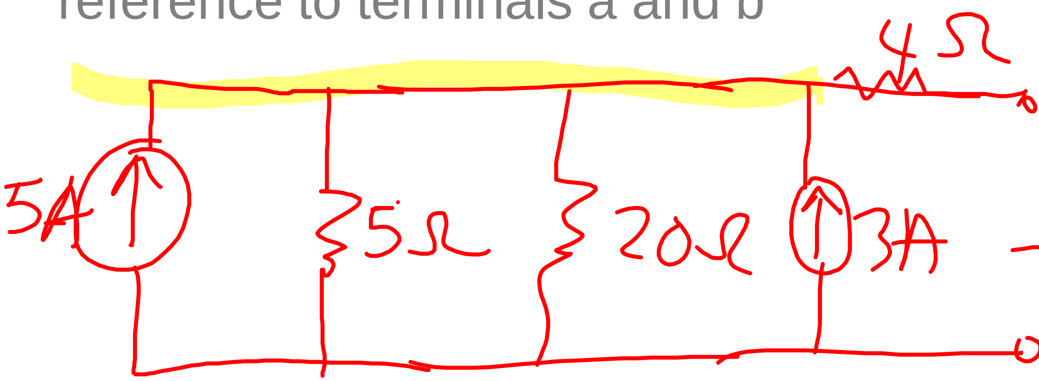


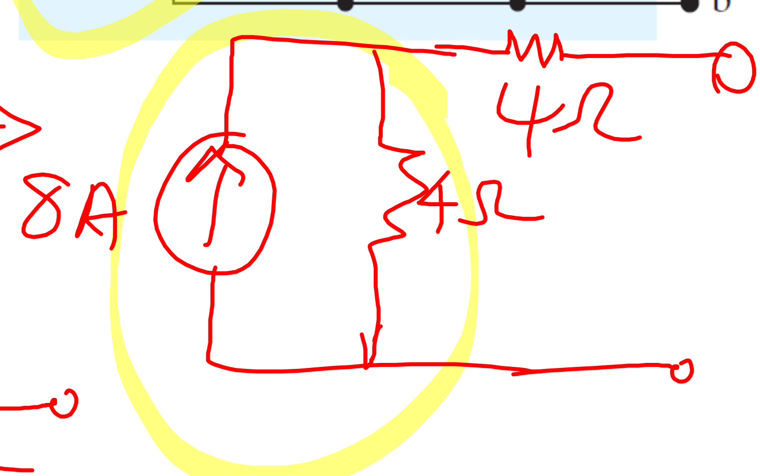


Example 1C (Thevenin and Norton Equivalents by source transformation)

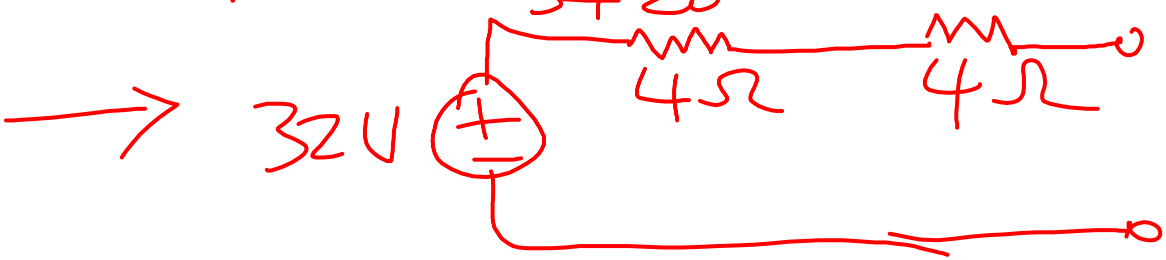
Find the Thevenin and Norton Equivalent Circuits by source transformation with reference to terminals a and b





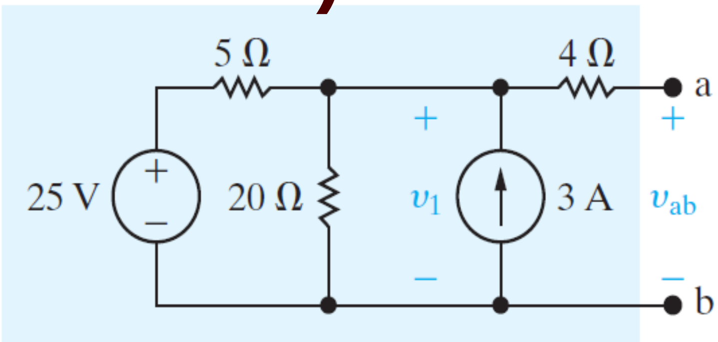


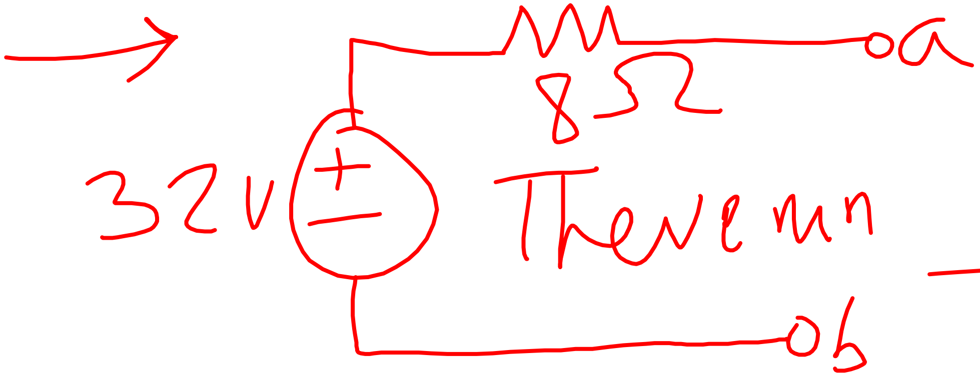
$$5 \,||\, 20 = \frac{5 \times 20}{5+20} = 4\Omega$$

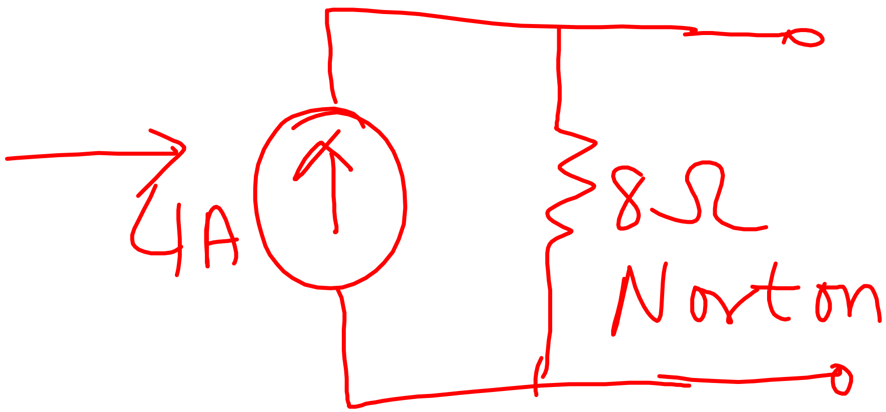


Example 1C Cont'd (Thevenin and Norton Equivalents by source transformation)

Find the Thevenin and Norton Equivalent Circuits by source transformation with respect to terminals a and b





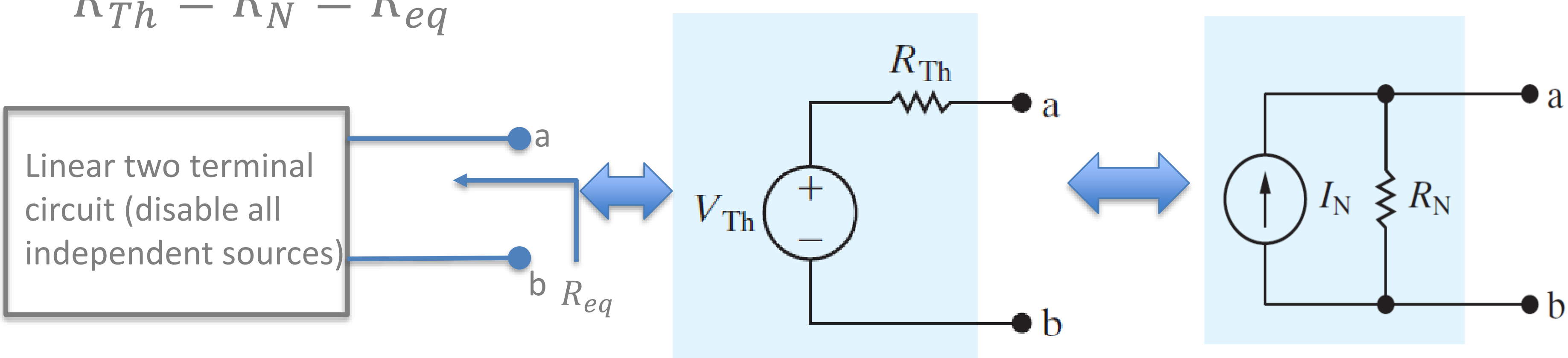


Alternative Approach 1 to Calculate Thevenin/Norton Equivalent Resistance R_{Th}/R_N

- Approach 1 (Circuits with no dependent source):

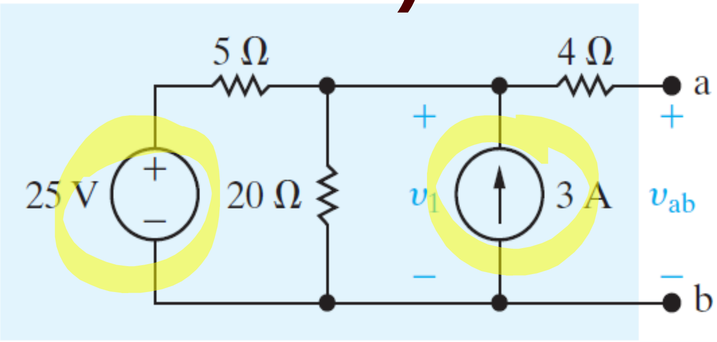
R_{Th} & R_N can be found by disabling all independent sources (i.e replace all voltage sources with short circuit and current sources with open circuits) and then calculate the equivalent resistance as seen from terminals a and b

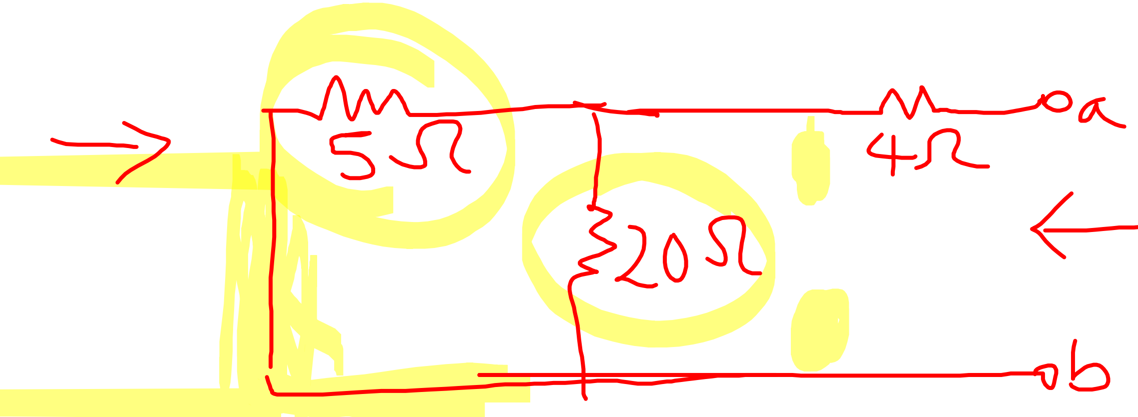
$R_{Th} = R_N = R_{eq}$

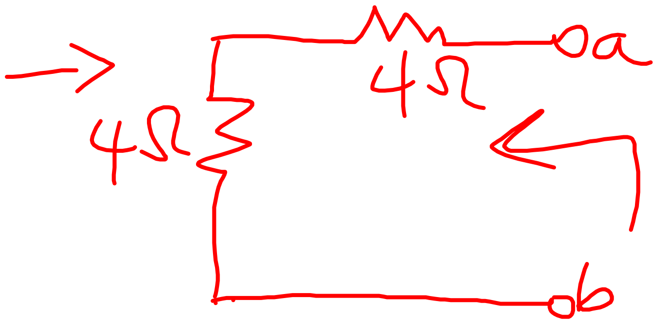


Example 1D (Alternative approach 1 for calculating Thevenin/Norton Resistance)

Find the Thevenin/Norton resistance with respect to terminals a and b by disabling all independent sources and finding the equivalent resistance







$5 \| 20\,\Omega = 4\,\Omega$

R_{eq}

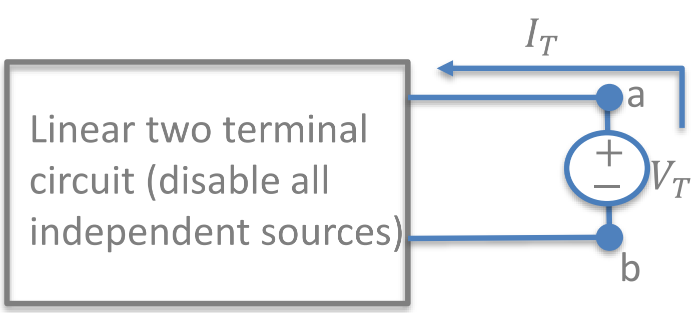
$R_{eq} = 8\,\Omega = R_{Th} = R_N$

Alternative Approach 2 to Calculate Thevenin/Norton Equivalent Resistance R_{Th}/R_N

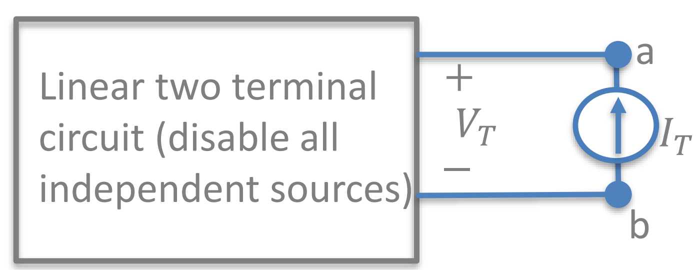
- Approach 2 (Circuits with dependent source):

R_{Th} & R_N can be found by disabling all independent sources and then apply a test source, V_T or I_T, and calculate the resistance seen by the test source at terminals a and b

$$R_{Th} = R_N = R_{eq}$$



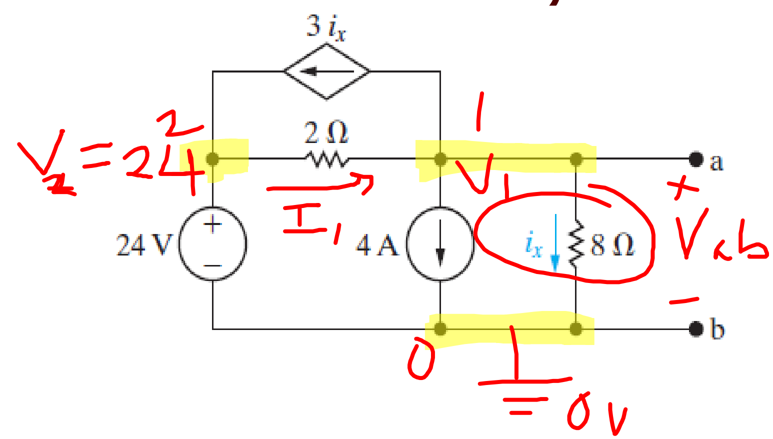
$$R_{Th}(or\ R_N) = V_T/I_T$$



$$R_{Th}(or\ R_N) = V_T/I_T$$

Example 2 (Circuits with dependent source, Assessment Problem 4.19)

Find the Thevenin & Norton Equivalent circuit with reference to terminals a and b



KCL at node 1

$$I_1 = 4 + i_x + 3i_x$$

$$\frac{24 - V_1}{2} = 4 + 4i_x$$

$$V_1 + 8i_x = 16 \quad \text{—— (1)}, \qquad i_x = \frac{V_1 - 0}{8} \rightarrow$$

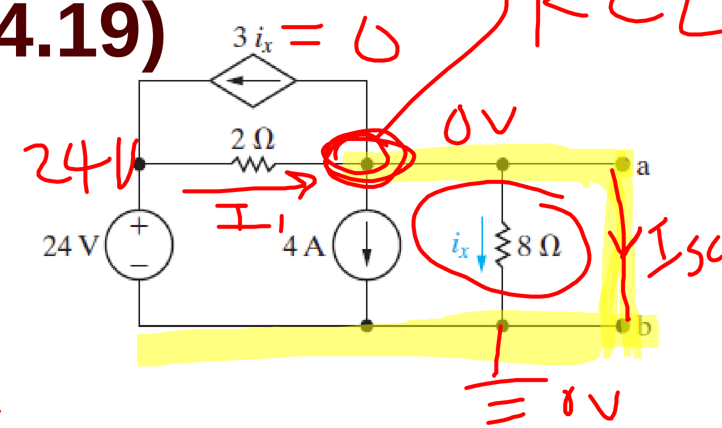
$$\rightarrow V_1 - 8i_x = 0 \quad \text{—— (2)}$$

$$V_1 = 8V, \quad i_x = 1A, \quad V_{ab} = \underset{V_1}{V_a} - \underset{0}{V_b} = 8 - 0 = 8V$$

$$V_{Th} = 8V$$

Example 2 Cont'd (Circuits with dependent source, Assessment Problem 4.19)

Find the Thevenin & Norton Equivalent circuit with reference to terminals a and b



$$i_x = \frac{0-0}{8} = 0\ A$$

$$I_1 = 3i_x + 4 + i_x + I_{sc}$$

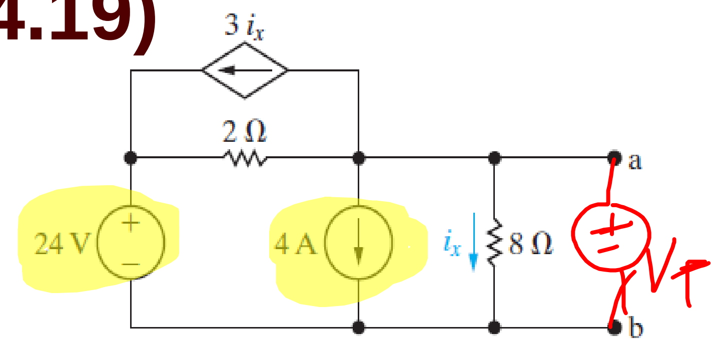
$3i_x \to 0$, $i_x \to 0$

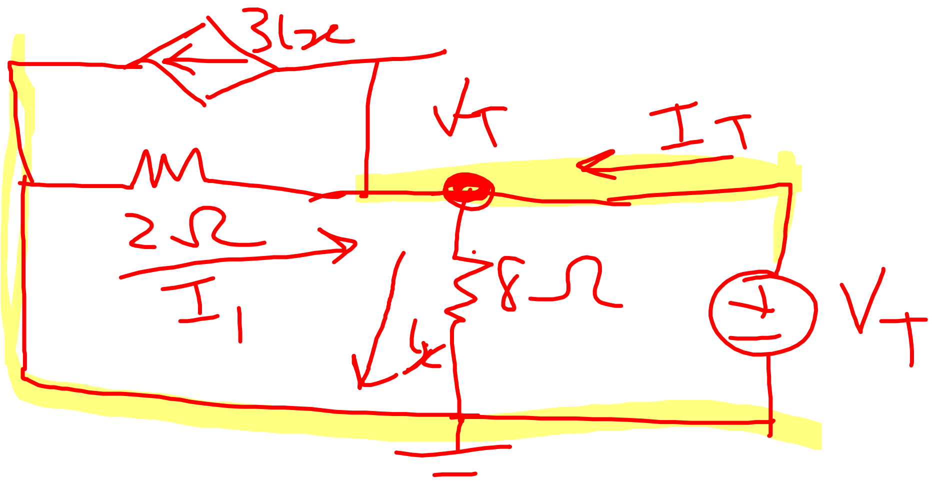
$$\frac{24-0}{2} = 0 + 4 + 0 + I_{sc}, \quad I_{sc} = 8A$$

$$I_N = 8A, \quad R_N = R_{Th} = \frac{V_{Th}}{I_N} = \frac{8V}{8A} = 1\Omega$$

Example 2 Cont'd (Circuits with dependent source, Assessment Problem 4.19)

Find the Thevenin & Norton Equivalent circuit with reference to terminals a and b





$$I_1 + I_T = 3i_x + i_x$$

$$\frac{0 - V_T}{2} + I_T = 4\,i_x$$

$$0 - \frac{V_T}{2} + I_T = 4\,\frac{V_T}{8}$$

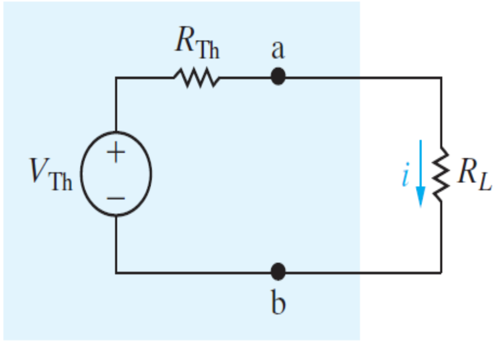
$$I_T = V_T$$

$$1 = \frac{V_T}{I_T} = 1\Omega$$

$$R_{Th} = R_N = 1\Omega$$

Maximum Power Transfer

- Many practical circuits are designed to deliver power to a load
- The Thevenin equivalent of the original circuit is useful in determining the maximum power that can be delivered to a load
- Consider the circuit on the right whose Thevenin equivalent parameters have been resolved. We desire to find what value of the load resistance, R_L, that yields maximum power



Maximum Power Transfer

- Power delivered to R_L, p is:

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

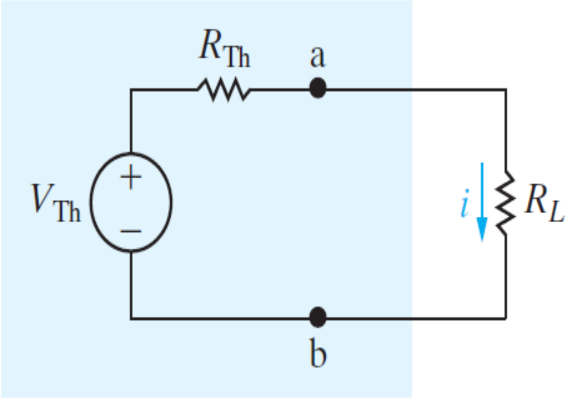
$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$

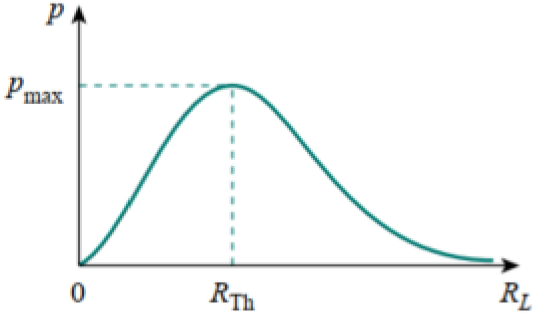
$$= V_{Th}^2 \left[\frac{R_{Th}+R_L-2R_L}{(R_{Th}+R_L)^3}\right] = 0$$

$$\rightarrow R_{Th} + R_L - 2R_L = 0,$$

$$\rightarrow R_L = R_{Th}$$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$





Example 3 (Same circuit as example 1)

Find the value of R_L for maximum power transfer. Find the maximum power.