Lecture 4: VI Measurements, Wheatstone Bridge, Delta-to-Wye Equivalent Circuits

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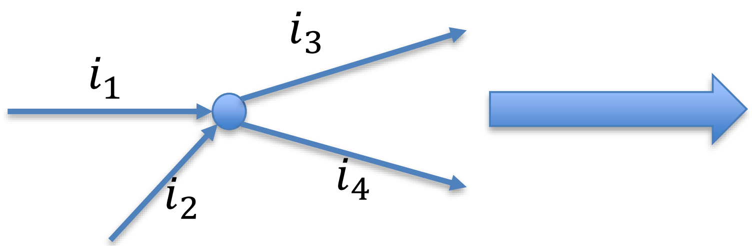
ECEN 214 – Electrical Circuit Theory (Spring 2020)

Outline

- Voltage & Current Measurements
- Wheatstone Bridge
- Delta-to-Wye Equivalent Circuit

Highlights from Last Lecture

- Kirchhoff's Current Law (KCL)



Assign current entering as +ive and current leaving as -ive:

$i_1 + i_2 + (-i_3) + (-i_4) = 0$ (1)

Or Assign current entering as –ive and current leaving as +ive:

$(-i_1) + (-i_2) + i_3 + i_4 = 0$ (2)

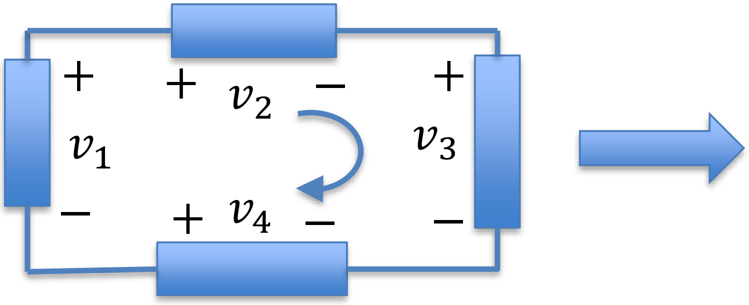
Or simply equate the sum of current entering to sum of current leaving

$i_1 + i_2 = i_3 + i_4$ (3)

Equations (1), (2) and (3) are exactly equivalent

Highlights from Last Lecture Cont'd

- Kirchhoff's Voltage Law (KVL)



As you move through the loop in clockwise direction, use the sign of the first polarity you come across in each element:

$(-v_1) + v_2 + v_3 + (-v_4) = 0$ (1)

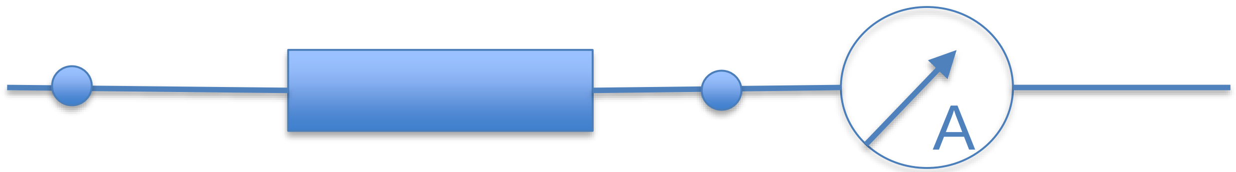
Or Use the sign of the second polarity you come across in each element:

$v_1 + (-v_2) + (-v_3) + v_4 = 0$ (2)

Equations (1) and (2) are exactly equivalent as equation (1) multiplied through by -1 will give equation (2)

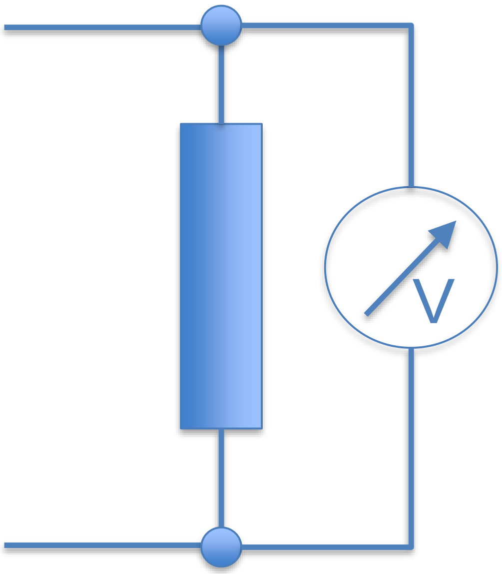
Current Measurement

- Current is measured with the use of an ammeter
- An ammeter is connected in series with the circuit element whose current is to be measured
- To ensure that the ammeter adds minimal distortion on the circuit it is connected to, it is desired that its internal resistance is very small ($R \rightarrow 0$) compared to the resistances in the circuit



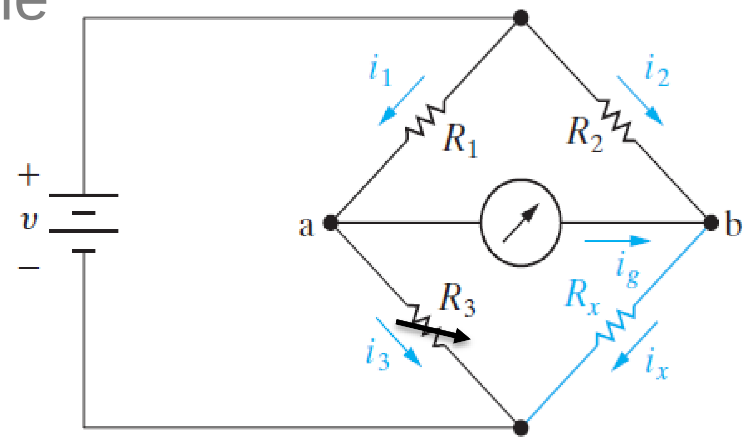
Voltage Measurement

- Voltage is measured with the use of a voltmeter
- A voltmeter is usually connected in parallel with the circuit element whose voltage is to be measured
- To ensure that the voltmeter adds minimal distortion on the circuit it is connected to, it is desired that its internal resistance is very large ($\rightarrow \infty$, i.e. open circuit) compared to the resistances in the circuit



The Wheatstone Bridge (Resistance Measurement)

- Resistance can be measured using the Wheatstone bridge shown on the right
- R_x is the unknown resistance
- The resistance of R_3 is varied until the current through the galvanometer reads zero current $(i_g = 0)$
- When $i_g = 0$ the bridge is said to be balanced



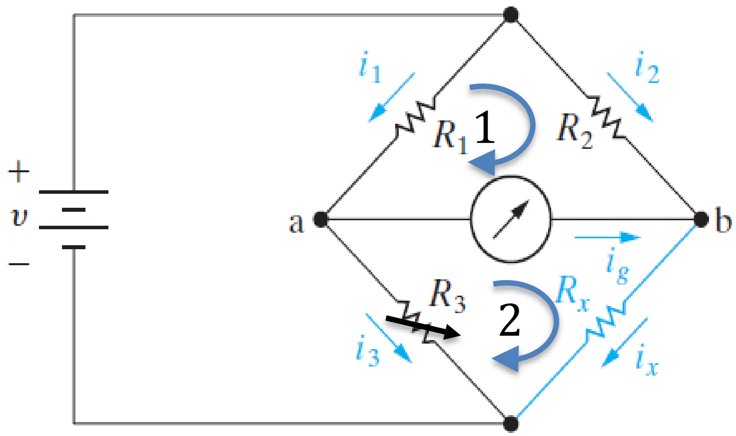
The Wheatstone Bridge (Resistance Measurement)

- $i_g = 0$ implies that point a and b are at the same potential, i.e., $v_{ab} = 0$
- By KCL, given $i_g = 0$, then $i_1 = i_3$ and $i_2 = i_x$
- Apply KVL and Ohm's law to loop 1 and 2:

$0 - i_1 R_1 + i_2 R_2 = 0, \rightarrow i_1 R_1 = i_2 R_2$ (1)

Similarly, $i_3 R_3 = i_x R_x$ (2)

(2)/(1) gives $\frac{i_3 R_3}{i_1 R_1} = \frac{i_x R_x}{i_2 R_2}$

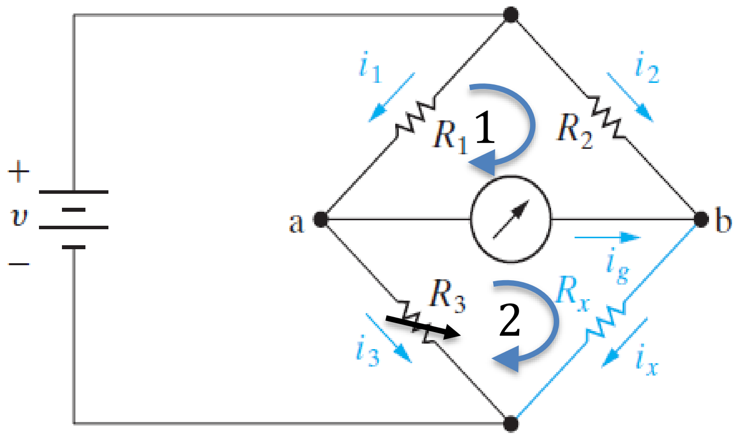


The Wheatstone Bridge (Resistance Measurement)

- since $i_1 = i_3$ and $i_2 = i_x$

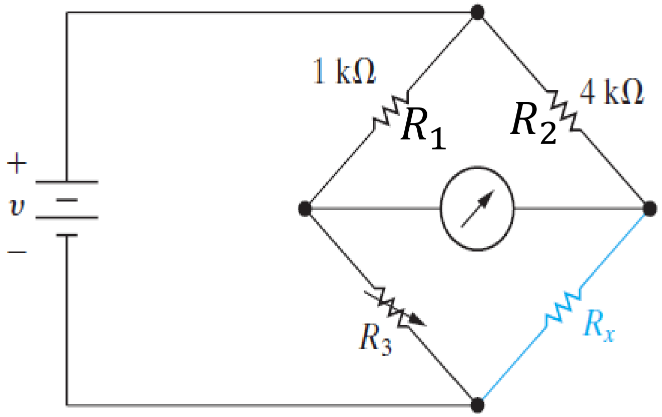
$$\frac{i_3 R_3}{i_1 R_1} = \frac{i_x R_x}{i_2 R_2} \rightarrow \frac{R_3}{R_1} = \frac{R_x}{R_2}$$

And $R_x = \dfrac{R_2 R_3}{R_1}$



Example 1 (Example 3.10 from textbook)

For the Wheatstone bridge shown on the right, R_3 can be varied from 10Ω to 2kΩ. What range of R_x can it measure?



$$R_x = \frac{R_2 R_3}{R_1}$$

$$R_{x,min} = \frac{R_2 R_{3,min}}{R_1} = 4000 \times \frac{10}{1000} = 40\Omega$$

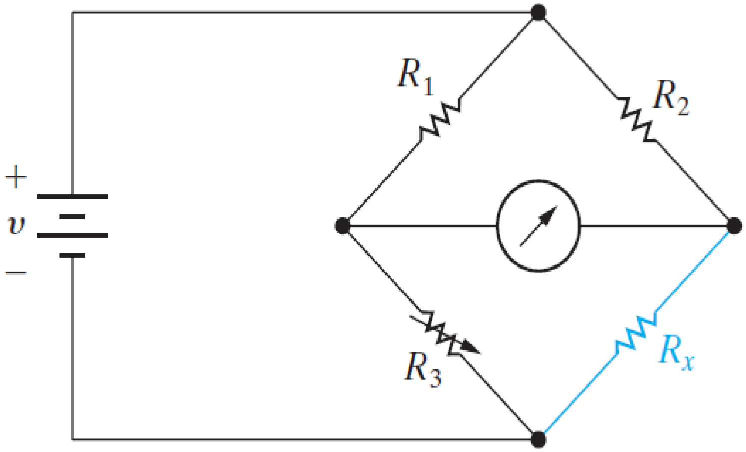
$$R_{x,max} = \frac{R_2 R_{3,max}}{R_1} = 4000 \times \frac{2000}{1000} = 8k\Omega$$

The range of resistance value it can measure is from 40Ω to 8kΩ

Example 2a (Assessment problem 3.7 from textbook)

The bridge circuit shown is balanced when $R_1 = 100\Omega, R_2 = 1000\Omega$, and $R_3 = 150\Omega$. $v = 5V$

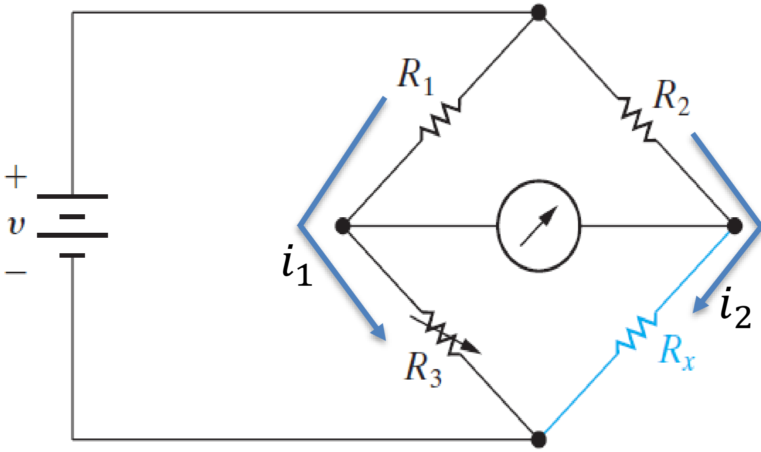
What is the value of R_x?



Example 2b (Assessment problem 3.7 from textbook)

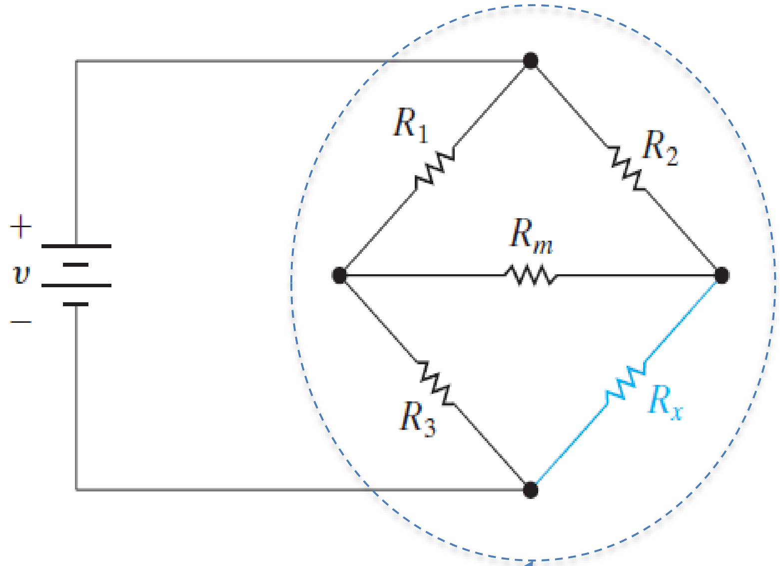
$v = 5V, R_1 = 100\Omega, R_2 = 1000\Omega$, and $R_3 = 150\Omega$, from part a $R_x = 1500\Omega$

Suppose each bridge resistor has a capacity of dissipating 250mW. Can the bridge be left in this balance state?



Delta-to-Wye (or Pi-to-Tee) Equivalent Circuit

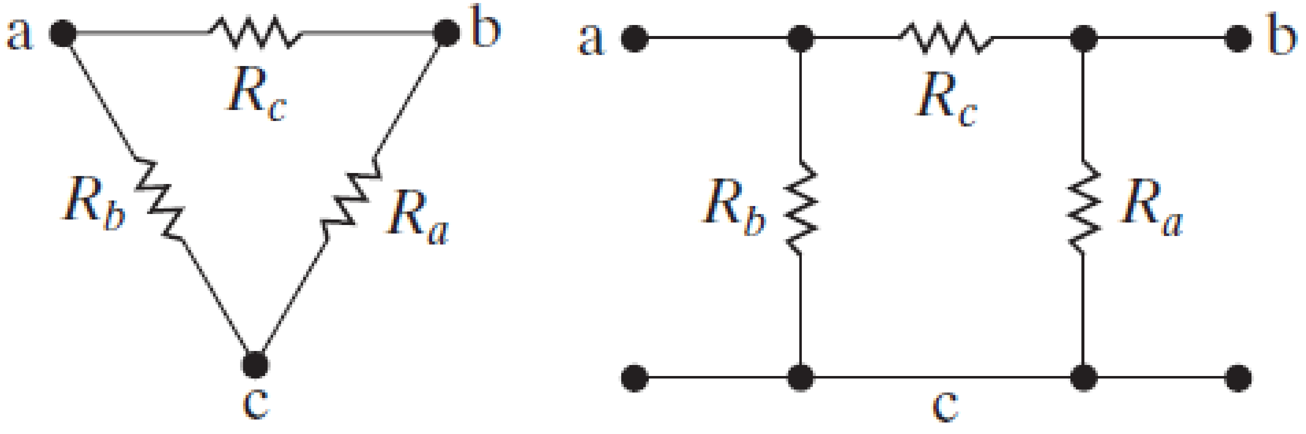
- There are resistive networks in which series and parallel evaluation of resistances alone is not sufficient to find their equivalent resistance
- Delta-to-Wye Transformation can be used to transform complex resistive network into forms where we can evaluate the equivalent resistance



Neither series nor parallel

Delta/Pi Configuration

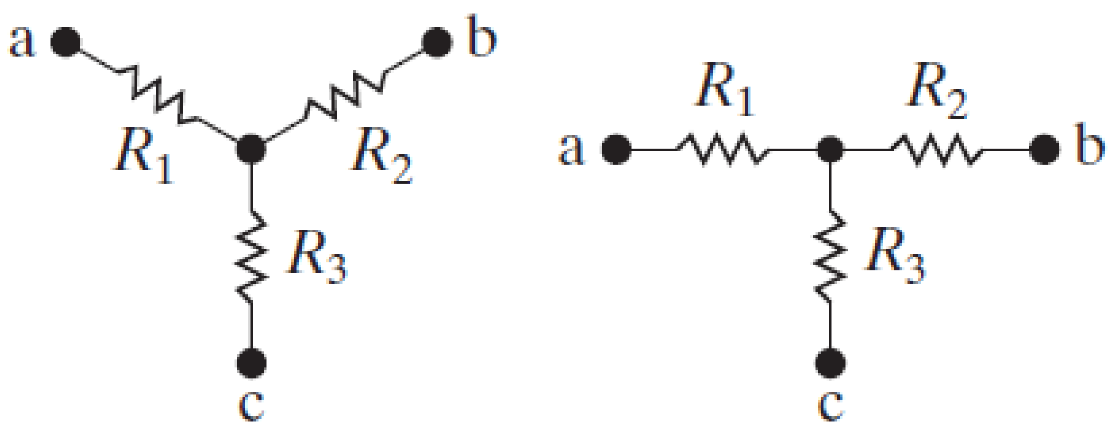
- Three nodes connecting three resistors in a cycle
- Both are equivalent



(a) Delta (Δ) Structure (b) Pi (π) Structure

Wye/T Configuration

- Four nodes – with center node connected to the other three nodes through a resistor
- Both are equivalent



(a) Wye (Y) Structure (b) T Structure

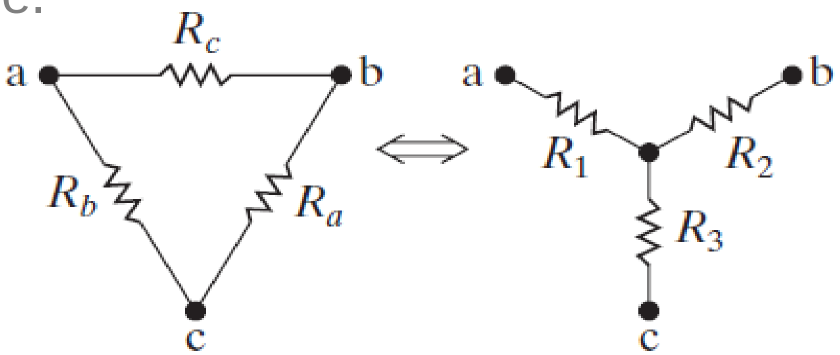
Δ-to-Y Transformation

Assuming the two circuits on the right are equivalent, then the equivalent resistance through any two terminal must be the same. Therefore,

$$R_{ab} = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} = R_1 + R_3$$



From the three equations above, we can solve for $R_1, R_2, \& R_3$ for the Y equivalence of a Δ network

or

we can solve for $R_a, R_b, \& R_c$ for the Δ equivalence of a Y network

Δ-to-Y Transformation

Solving for $R_1, R_2, \& R_3$ gives:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

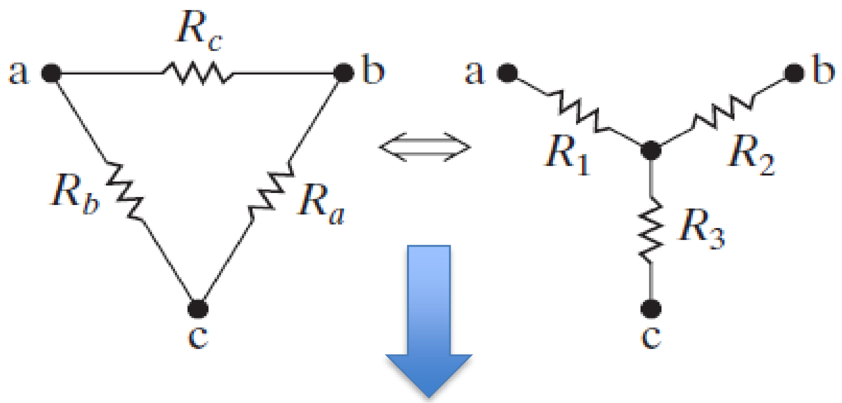
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

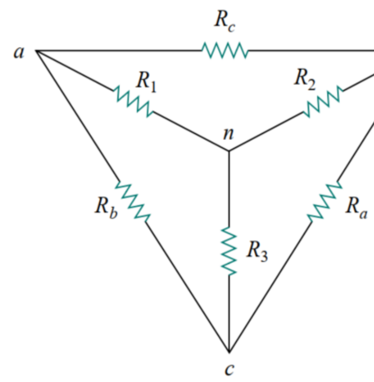
Solving for $R_a, R_b, \& R_c$ gives:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

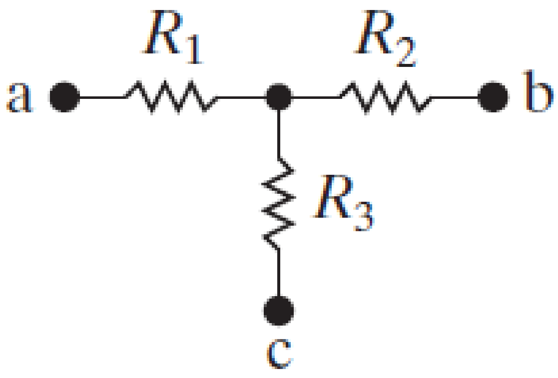


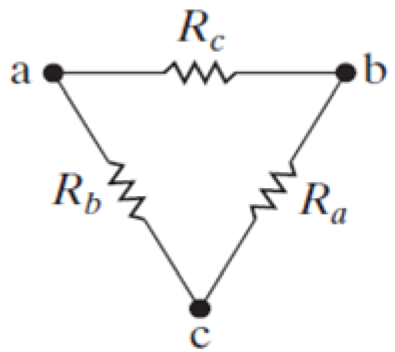


Superpositioning both network shows some form of visual relationship between the transformation equation and the network.

Example 3

Convert the following Y/T configuration (top right) to its equivalent delta configuration (bottom right) given $R_1 = 20\Omega, R_2 = 10\Omega, R_3 = 5\Omega$





Example 4

A)Find the equivalent resistance seen by the current source as shown in the figure on the top right

B)Find v