

Let  $L: \mathbb{R}_2 \rightarrow \mathbb{R}_2$  be a linear operator such that  $L(1,2) = (-2,3)$  and  $L(1,-1) = (5,2)$ . Find the value of  $L(7,11)$ .

Let  $(7,11)$  be the linear combination of  $(1,2)$  and  $(1,-1)$ :

$$a(1,2) + b(1,-1) = (7,11)$$

then we obtain this system of equation:

$$\begin{aligned} a + b &= 7 \\ 2a - b &= 11 \end{aligned}$$

Thus

$$\begin{aligned} a &= 6 \\ b &= 1 \end{aligned}$$

Given that  $L$  is linear operator:

$$\begin{aligned} L(7,11) &= L(6(1,2) + 1(1,-1)) \\ &= L(6(1,2)) + L(1(1,-1)) \\ &= 6 * L(1,2) + 1 * L(1,-1) \\ &= 6 * (-2, 3) + 1 * (5, 2) \\ &= (-12, 18) + (5, 2) \\ &= (-7, 20) \end{aligned}$$