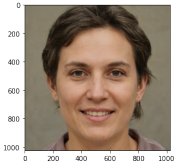
Generative adversarial neural networks for state-of-the-art image reconstruction.

Arthur Conmy and Dr Subhadip Mukherjee. *The following face does not exist: it was generated as the 'average' face from a pre-trained GAN. You can view your own synthetic faces (that no one will have seen before) at* `thispersondoesnotexist.com`*, and reload the page to generate a different face.*



Plan

1. Background context: machine learning for mathmos.
   - Neural nets.
   - Generative modelling.
   - GANs.

2. Our work.
   - Image reconstruction.
   - Our contribution.

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Neural networks

For the purposes of this talk, it will be useful to think of neural networks as black-boxes that are very good at representing high-dimensional functions that lie in very large function classes.

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$$f : [0,1]^{H\times W\times C} \to \{\mathrm{Dog}, \mathrm{Cat}\} \tag{1}$$

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- More exciting example to come:

$$\mathcal{F} = \{f : f : [0,1]^{H \times W \times C} \to \mathbb{R}, f \in \mathrm{Lip}(1)\} \tag{2}$$

Generative modelling.

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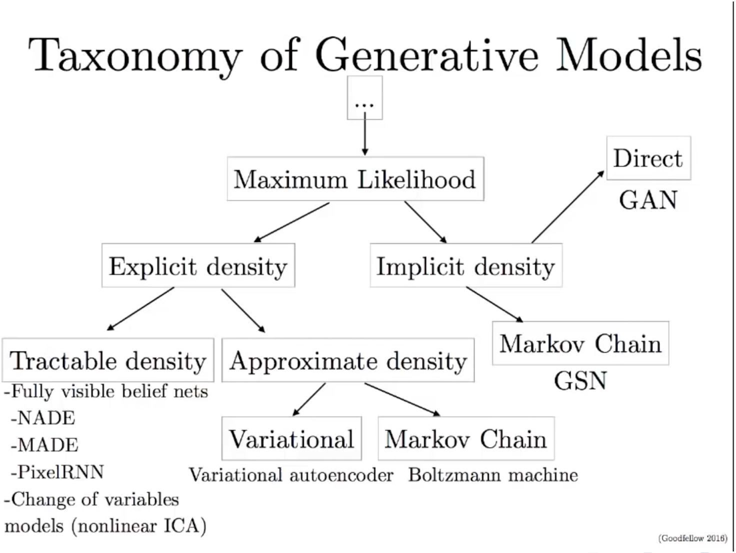
- Big question: what does 'similar' mean?

Taxonomy

From Goodfellow's 2016 NIPS GANs tutorial:



GANs, part one.

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- HOWEVER, significant advances have been made to GAN theory since this initial work, citing Cedric Villani's optimal transport theory to redesign the loss function between the distributions!

GANs, part two

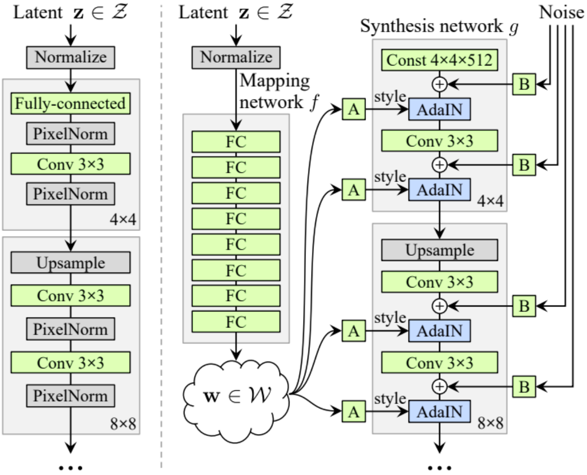
The most natural way to choose a loss function is the analogue of cross entropy loss in this case:

$$V(D, G) = \mathbb{E}_{p_d}[\log D(x)] + \mathbb{E}_{z \sim \mathcal{N}}[\log(1 - D(G(z)))].$$

A much better choice of loss function between distributions is the Wasserstein distance:

$$\mathrm{EMD}(p_d, p_g) = \sup_{\|f\|_{L \leq 1}} \mathbb{E}_{x \sim p_d} f(x) - \mathbb{E}_{x \sim p_g} f(x).$$

GANs, part three.



(a) Traditional (b) Style-based generator

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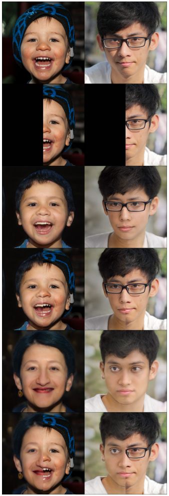
The resulting optimization reads $\min_{(z,w^+)} J(w^+, z)$, where

$$J(w^+, z) := \lambda_{\text{pix}} \|y - A\,G(w^+)\|_2^2 + \lambda_{\text{map}} \sum_{i=1}^{L} \|w_i - M(z)\|_2^2 + \lambda_{\text{vgg}} \|\varphi(y) - \varphi(A\,G(w^+))\|_2^2 + \lambda_{\text{lat}} \|z\|_2^2. \quad (5)$$



We changed the regularization term used to reconstruct images, and obtained strong results.

	Input	Mask	Model	LPIPS				SSIM	
Inpainting	256^2	Half	DeepGIN	1	0.303	2	0.303	**59**	**0.625**
			BRGM	47	0.226	**57**	**0.231**	21	0.606
			L-BRGM	**51**	**0.224**	40	0.240	19	0.594
	1024^2	Half	BRGM	17	0.495				
			L-BRGM	**82**	**0.460**				
	1024^2	Eyepatch	BRGM	10	0.428				
			L-BRGM	**89**	**0.393**				
	Input	**Output**	**Model**	LPIPS				SSIM	
Super-resolution	64^2	256^2	GFPGAN	**72**	**0.231**	**77**	**0.231**	**89**	**0.534**
			BRGM	6	0.294	9	0.295	10	0.480
			L-BRGM	21	0.277	13	0.294	0	0.404
	64^2	1024^2	BRGM	32	0.426				
			L-BRGM	**67**	**0.414**				
	32^2	1024^2	BRGM	33	0.470				
			L-BRGM	**66**	**0.445**				

Table 1. Quantatitive evaluation of L-BRGM, BRGM, DeepGIN and GFPGAN. The two columns marked with an asterix refer to minimal *final* values of those metrics, i.e on the 2000th optimization step for (L-)BRGM.