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Let $\mathcal{F}$ denote the discrete Fourier transform.

[this is just a linear map $\mathbb{C}^n \to \mathbb{C}^n$ so it is matrix multiplication, by the matrix here: `https://en.wikipedia.org/wiki/Discrete_Fourier_transform#The_unitary_DFT`.]

Notably the matrix is invertible so the DFT is 1-to-1.

Claim: The image of $\mathbb{R}^n$ under $\mathcal{F}$ is exactly the set of $x = (x_0, ..., x_{n-1}) \in \mathbb{C}^n$ that satisfy

$$x_i = x^*_{-i} \tag{1}$$

where the indices are taken modulo n.

Proof. Good maths problem. □

Then this means that when we take $\mathcal{F}^{-1}(x)$ and know that we're getting a real sequence out, it suffices to specify (complex numbers)) $x_0, x_1, ..., x_{n/2}$, as then we can determine $x_{n/2+1}, ..., x_{n-1}$ from these. However, the imaginary parts of x_0 and $x_{n/2}$ must be 0. This is the reason for the 113 in (1,3,224,113,2), since it specifies all the information required apply $\mathcal{F}^{-1}$ and get a real sequence. Moreover, I guess the entries [:,:,:,0,1] and [:,:,:,112,1] are redundant, since as mentioned for DFT to return something real these imaginary parts need be 0.