

# FTML Exercices 7

Pour le 2 mai 2025

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### 1 REGRESSION LOGISTIQUE

We consider the logistic regression problem, in the following setting :

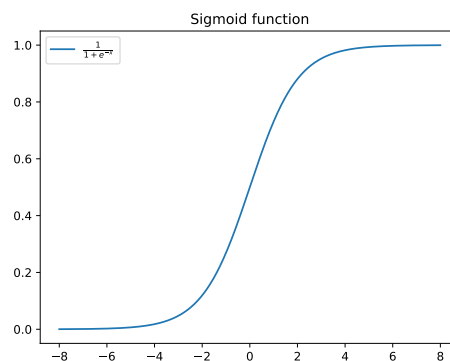
- $\mathcal{X} = \mathbb{R}^d$
- $\mathcal{Y} = \{-1, 1\}$
- Logistic loss :

$$l(\hat{y}, y) = \log(1 + e^{-\hat{y}y}) \quad (1)$$

We also define the following function :

**Definition 1.** Sigmoid function  
 $\sigma : \mathbb{R} \rightarrow \mathbb{R}$ .

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (2)$$



1] Show that  $\sigma$  is differentiable and that

$$\forall z, \sigma'(z) = \sigma(z)\sigma(-z) \quad (3)$$

2] Show that  $l(\hat{y}, y)$  is strictly convex in its first argument, which means for fixed  $y$ ,  $\hat{y} \mapsto l(\hat{y}, y)$  is strictly convex. Using properties that relate convexity and the derivative of a function, and using the sigmoid function will be helpful.

3] Without regularization, the empirical risk writes :

$$R_n(\theta) = \frac{1}{n} \sum_{i=1}^n l(x_i^T \theta, y_i) \quad (4)$$

- a** Show that  $\theta \rightarrow R_n(\theta)$  is convex
- b** Compute the gradient  $\nabla_{\theta} R_n(\theta)$  of the empirical risk  $R_n(\theta)$  in this setting.