Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2016-2017

Take Home Exam 5

Due date: 9 January 2017, Monday, 23:59

Question 1

Prove the following related to graph theory:

a) Show that an edge in a simple graph is a cut edge if and only if this edge is not part of any simple circuit in the graph. ()
b) Show that in any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.
c) Given below the initial chessboard configuration, prove of disprove whether the other two boards can be independently generated by moving one knight ((W)hite, (B)lack) at a time in finite number of steps ? Then show that there is no *knight's tour* on one of the chessboards. (exhaustive search is forbidden), (*knight's tour is a sequence of L-moves by a knight starting at some square and visiting each square once*).

start state

W1	W2	W3
B1	B2	B3

final state 1

B2	B1	B3
W1	W3	W2

final state 2

B1	B2	B3
W1	W2	W3

Question 2

In a court where defendants, $A_1, A_2, \ldots A_n$ are accused of bribery, con and thievery, some of which know each other i.e. friends. Any of two of the defendants who do not know each other, have exactly 2 common friends. If A_1, A_2 have known each other, i.e. friends, and don't have any friends in common, prove that A_1, A_2 know same number of defendants.

Question 3

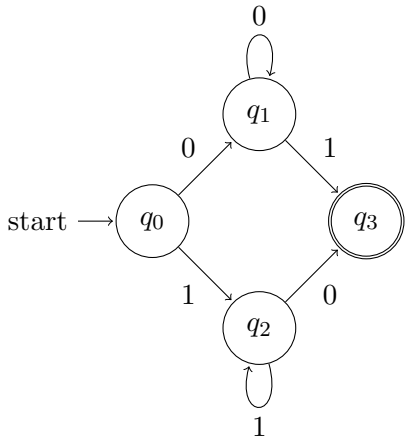
There exists 3 schools each having n students. If every student knew $n+1$ students in both of the other 2 schools, show that there exists 3 students, each from different school, that know each other.

1 Regulations

1. A simple code for inserting an automaton is :

```
\begin{tikzpicture}[shorten >=1pt,node distance=2cm,on grid,auto]
   \node[state,initial] (q_0)   {$q_0$};
   \node[state] (q_1) [above right=of q_0] {$q_1$};
   \node[state] (q_2) [below right=of q_0] {$q_2$};
   \node[state,accepting](q_3) [below right=of q_1] {$q_3$};
    \path[->]
    (q_0) edge  node {0} (q_1)
          edge  node [swap] {1} (q_2)
    (q_1) edge  node  {1} (q_3)
          edge [loop above] node {0} ()
    (q_2) edge  node [swap] {0} (q_3)
          edge [loop below] node {1} ();
\end{tikzpicture}
```

   which will give ;



2. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

3. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

4. **Late Submission: Not Allowed**

5. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

6. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

7. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the4.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the5.tex
```