

CENG 280

Formal Languages and Abstract Machines

Fall '2016-2017

Take Home Exam 3

Due date: 29 May 2017, 17:00

Question 1

Answer the following questions. Please justify your answer by giving a detailed explanation.

- Given a partition of languages $L_1, L_2, L_3, \dots, L_n$ that form Σ^* over alphabet $\Sigma = \{x, y, z\}$. Suppose that each L_k (where $1 \leq k \leq n$) is recursively enumerable. Prove or disprove that each L_k is recursive. (Hint: See the definition of the partition of a set)
- Let L_1 and $\overline{L_1}$ be recursively enumerable languages and L_2 be recursive. Prove or disprove that $L_2 \setminus L_1$ is recursive. If L_2 is recursively enumerable instead of recursive, what can you say about $L_2 \setminus L_1$? Explain.
- Let f be a computable, one to one and onto function. L_1, L_2 and L_3 are Turing-decidable languages. Prove or disprove that $f^{-1}(L_1 \setminus (L_2 L_3))$ is Turing-decidable.

Question 2

Give (unrestricted) grammars for the following languages.

- $L = \{a^{2^n} : n \geq 0\}$
- $L = \{wx^n y^m : w \in \{1, 2\}^* \text{ and } m \text{ is the number of 1's in } w \text{ and } n \text{ is the number of 2's in } w\}$

Question 3

Give high-level procedural description of Turing Machines that decide the following languages.

- a. $L = \{a^n b^m c^k x^t y^t z^t : k = n \times m, k, n, m, t \geq 1\}$
- b. $L = \{w_1 w_2 \dots w_k : w_k \in \{a, b\}^*, w_i \neq w_j \text{ for } i \neq j, \text{ and } k \geq 1\}$

Question 4

In each part, give a formal description for the Turing machines that computes the given function.

- a. $f(x) = 2x + 1$, where $x \geq 1$.
- b. $h(x) = \lceil \frac{x}{5} \rceil$, where $x \geq 1$.
- c. $(f \circ h)(x)$, where $x \geq 1$.

Numbers will be given to TM's in unary. Initially, tape is of the form ($\triangleright \sqcup 11 \dots 11 \sqcup \dots$) where number of 1's equal to x in parts (a), (b) and (c). For all parts, initial position of the head is the blank symbol which is between left end symbol and the input. When the machines halt, tape will be of the form ($\triangleright \sqcup 11 \dots 11 \sqcup \dots$) where number of 1's equal to the output of the function.

In your description, represent the TM's as quintuples $(K, \Sigma, \delta, s, H)$ where K is the finite set of states; Σ is the alphabet containing blank symbol \sqcup , left end symbol \triangleright , and any other symbol you need to compute the functions, but not containing symbols \rightarrow and \leftarrow ; δ is the transition function from $(K-H) \times \Sigma$ to $K \times \Sigma \times \{\rightarrow, \leftarrow\}$; s is the start state and H is the set of halting states.

Question 5

- a. $L = \{ \langle M \rangle \mid M = (K, \Sigma, \delta, s, H) \text{ is a Turing Machine which can not move its head (i.e. } \delta \subseteq ((K - H) \times \Sigma) \times (K \times \Sigma)) \text{ and eventually writes a non blank symbol on its tape. The initial configuration of } M \text{ is } (s, \triangleright \sqcup w) \text{ where } w \in \Sigma^* \}$

Is L decidable? Discuss whether this conflicts with the halting problem ?

- b. Give a TM M such that M is not a decider, but $L = L(M)$ is decidable.
- c. For any language L , if there exists a Turing Machine D_1 that always halts and accepts every string w , $w \in L$, and another Turing Machine D_2 that always halts and rejects every string w , $w \notin L$, then can we say that every language is decidable ? Explain.

Question 6

Are the following languages deterministic context-free or not? Explain briefly for each case.

- a. $L = \{ a^n b^{n+m} c^m \mid n, m \geq 1 \} \cup \{ a^n b^{2n} \mid n \geq 1 \}$
- b. $L = \{ x a^n b^{n+m} c^m \mid n, m \geq 1 \} \cup \{ y a^n b^{2n} \mid n \geq 1 \}$

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
4. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.
5. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the3.tex", when you finish your exam upload the .tex file with the same name to COW.

Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

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$ pdflatex the3.tex
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