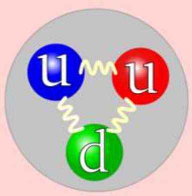
Nucleon mass and form factors from lattice quantum chromodynamics simulations

Andrew Connelly *Department of Physics, North Carolina State University, Raleigh, NC 27695, USA*
Meifeng Lin *Computational Science Initiative, Brookhaven National Laboratory, Upton, NY 11973, USA*

Introduction

Quantum Chromodynamics (QCD) is the study of matter at its smallest-understood scale, that of the fundamental particles. Fundamental particles are those which we do not know to be composed of any other particles. There are many methods for studying QCD, a very well-known one is perturbation theory, which breaks down in the QCD small-energy regime due to the asymptotic freedom of QCD. To learn more we need a method which does not have the same weakness as perturbation theory. One such method is Lattice QCD (LQCD). LQCD is the method of approximating reality with a fine grid of points placed in a 4-dimensional Euclidean space; three dimensions represent spatial coordinates and one, time. We are able to input our first-principles understanding of particle dynamics, and then use simulations to measure what they do and see if our input lines up with reality. The goal of my project is to analyze these simulation results in their raw form and produce meaningful information about observables from them.[1]



A diagram depicting a proton represented with its valence quarks visible. A proton is characterized by having one down- and two up valence quarks. The springs between the valence quarks are the gluons mediating the strong force. QCD studies systems like this, where quark-gluon dynamics are highly relevant.

Methods

There are two significant computational issues we deal with here, massive compute cost of the LQCD simulation itself and significant error propagation required to go from data to a physical observable.

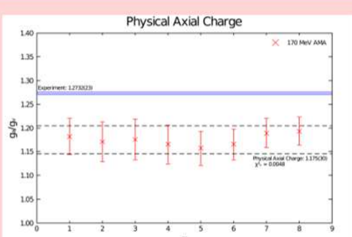
We address the compute cost issue with All-Modes-Averaging (AMA). This allows us to reduce compute cost during simulation with relatively little cost to accuracy due to a bias correction method.

For dealing with error propagation easily, we use the "Jackknife" resampling method for error analysis.[2] Jackknife replicates replace observed values of an observable in a set of data $\{\mathcal{O}_j\}_{j=1}^{n}$ like the following:

$$\hat{\mathbf{R}} = \left\{ \frac{1}{n-1} \sum_{j \neq i} \mathcal{O}_j \right\}_{i=1}^{n}$$

This makes error propagation easy, as our estimator is the average of the replicates, $\hat{\mathcal{O}}$, and we can at any time find the standard error with:

$$SE(\hat{\mathcal{O}})_{jack} = \left[\frac{n-1}{n} \sum_{i=1}^{n} (\hat{R}_i - \hat{\mathcal{O}})^2 \right]^{\frac{1}{2}}.$$



To the left is a plot of the physical axial charge as we have computed it compared with its experimentally-acquired value. Experiment provides us the ability to verify the accuracy of our results, as each of the Vector, Axial, and Scalar Charges are linked to certain fundamental processes which can be measured.

We also implement a Machine-Learning (ML) Model which estimates three-point correlation functions (and thus observables) from two-point correlation functions.[3] Here we evaluate its effectiveness against the entire data set as a control. We believe that this process, if accurate enough, may produce similar results to standard analysis with less computational cost.

We present calculated values of the isovector axial and vector charges as well as the isovector scalar coupling resulting from domain-wall fermion lattice simulations; The gauge ensembles are generated with an Iwasaki $\times$ dislocation-suppressing-determinant-ratio gauge action characterized by $\beta = 1.75$, and inverse lattice spacing $a^{-1} = 1.378(7)$ GeV. We compare these results to those obtained by processing Machine-Learning prediction data in order to evaluate the accuracy of such methods in the scope of Lattice observable prediction.

Results

Standard Approach

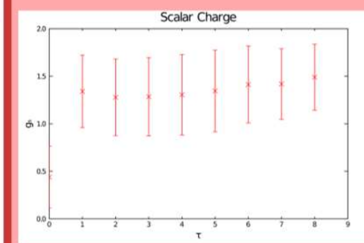


FIG. 1: Scalar Charge plotted as a result of standard analysis of the full data set.

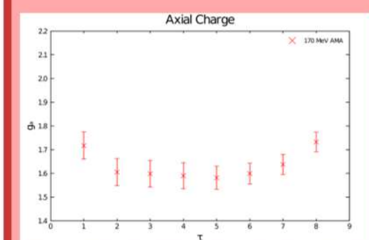


FIG. 2: Axial Charge plotted as a result of standard analysis of the full dataset

ML Comparison

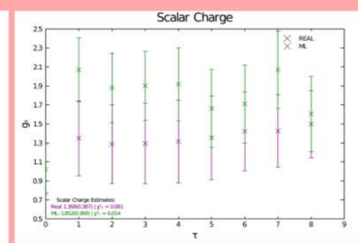


FIG. 3: In green, Scalar Charge as predicted by our ML implementation. In purple, Scalar Charge calculated using the true data that the ML implementation is meant to predict.

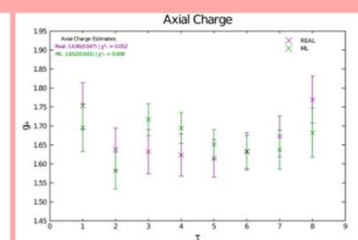


FIG. 4: In green, Axial Charge as predicted by our ML implementation. In purple, Axial Charge calculated using the true data that the ML implementation is meant to predict.

Conclusion & Further Work

We have been able to determine, as can be seen in the plots above, that this ML implementation is capable of accurately predicting the nucleon three-point correlation function from a given two-point function precisely enough to produce reasonable estimates for the isovector Vector and Axial charges. The scalar charge is less accurate, but this is expected due to its high statistical error to begin with. This work can be improved upon with the following actions:

- Performing statistical tests to determine the significance of the difference between real and ML-predicted observables
- Incorporating the tensor coupling constant into this prediction model
- implementing AMA bias correction

As we continue this work into the 2020-2021 school year, we intend to implement most, if not all of these improvements.

References

[1] R. Gupta, *Introduction to Lattice QCD* (1997), arXiv:hep-lat/9807028.

[2] A. McIntosh, "The jackknife estimation method," (2016), arXiv: 1606.00497.

[3] B. Yoon, T. Bhattacharya, and R. Gupta, Phys. Rev. D100, 014504 (2019).

Acknowledgements

This project was supported in part by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists (WDTS) under the Science Undergraduate Laboratory Internships Program (SULI).

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Connelly, Andrew