Opérateurs - Coordonnées cartésiennes

Le vecteur nabla permet de retrouver les autres opérateurs **<u>en coordonnées cartésiennes</u>** **<u>uniquement</u>** : $\overrightarrow{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$. On a ainsi : $\begin{cases} \overrightarrow{\nabla} f = \overrightarrow{grad}(f) \\ \overrightarrow{\nabla} \cdot \overrightarrow{f} = div(\overrightarrow{f}) \\ \overrightarrow{\nabla} \wedge \overrightarrow{f} = \overrightarrow{rot}(\overrightarrow{f}) \\ \overrightarrow{\nabla} \cdot \overrightarrow{\nabla} f = \Delta f \text{ Attention, laplacien } \underline{\text{scalaire}} \end{cases}$.

Opérateurs - Fondamentaux

$$\overrightarrow{grad}(f.g) = f.\overrightarrow{grad}(g) + g.\overrightarrow{grad}(f)$$

$$div(f.\overrightarrow{g}) = f.div(\overrightarrow{g}) + \overrightarrow{g} \cdot \overrightarrow{grad}(f)$$

$$\overrightarrow{rot}(f.\overrightarrow{g}) = f.\overrightarrow{rot}(\overrightarrow{g}) + \overrightarrow{g} \wedge \overrightarrow{grad}(f)$$

$$div(\overrightarrow{grad} f) = \Delta f$$

$$\overrightarrow{grad}(div\,\overrightarrow{f}) - \overrightarrow{rot}(\overrightarrow{rot}\,\overrightarrow{f}) = \Delta \overrightarrow{f}$$

$$div(\overrightarrow{rot}) = 0$$

$$\overrightarrow{rot}(\overrightarrow{grad}) = \overrightarrow{0}$$

Opérateurs - Coordonnées cylindriques

$$\overrightarrow{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r}\frac{\partial f}{\partial \theta} \\ \frac{\partial f}{\partial z} \end{pmatrix} \quad ; \quad div(\overrightarrow{f}) = \frac{1}{r}\frac{\partial (r f_r)}{\partial r} + \frac{1}{r}\frac{\partial f_\theta}{\partial \theta} + \frac{\partial f_z}{\partial z}; \quad \overrightarrow{rot}(\overrightarrow{f}) = \begin{pmatrix} \frac{1}{r}\frac{\partial f_z}{\partial \theta} - \frac{\partial f_\theta}{\partial z} \\ \frac{\partial f_r}{\partial z} - \frac{\partial f_z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial (r f_\theta)}{\partial r} - \frac{\partial (r f_r)}{\partial \theta}\right) \end{pmatrix}$$

$$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Opérateurs - Coordonnées sphériques

$$\overrightarrow{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r}\frac{\partial f}{\partial \theta} \\ \frac{1}{r.sin\theta}\frac{\partial f}{\partial \varphi} \end{pmatrix} ; \quad div(\overrightarrow{f}) = \frac{1}{r^2}\frac{\partial (r^2 f_r)}{\partial r} + \frac{1}{r.sin\theta}\frac{\partial (sin\theta.f_\theta)}{\partial \theta} + \frac{1}{r.sin\theta}\frac{\partial f_\varphi}{\partial \varphi} ; \quad \overrightarrow{rot}(\overrightarrow{f}) = \begin{pmatrix} \frac{1}{r.sin\theta}\left(\frac{\partial (sin\theta.f_\varphi)}{\partial \theta} - \frac{\partial f_\theta}{\partial \varphi}\right) \\ \frac{1}{r.sin\theta}\left(\frac{\partial f_r}{\partial \varphi} - \frac{\partial (r.sin\theta.f_\varphi)}{\partial r}\right) \\ \frac{1}{r}\left(\frac{\partial (r f_\theta)}{\partial r} - \frac{\partial f_r}{\partial \theta}\right) \end{pmatrix} ;$$

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2.sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2.sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

Opérateurs de second ordre - Coordonnées cartésiennes

Soit $\overrightarrow{u}$ un vecteur et T un tenseur.

$$\overline{\overline{grad}}(\overrightarrow{u}) = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}; \qquad \overrightarrow{div}(\overline{\overline{M}}) = \begin{pmatrix} \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \\ \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \\ \frac{\partial T_{zx}}{\partial x} + \frac{\partial T_{zy}}{\partial y} + \frac{\partial T_{zz}}{\partial z} \end{pmatrix}; \qquad \Delta(\overrightarrow{u}) = \begin{pmatrix} \frac{\partial^2 u_x}{\partial x^2} & \frac{\partial^2 u_x}{\partial y^2} & \frac{\partial^2 u_x}{\partial z^2} \\ \frac{\partial^2 u_y}{\partial x^2} & \frac{\partial^2 u_y}{\partial y^2} & \frac{\partial^2 u_y}{\partial z^2} \\ \frac{\partial^2 u_z}{\partial x^2} & \frac{\partial^2 u_z}{\partial y^2} & \frac{\partial^2 u_z}{\partial z^2} \end{pmatrix}$$

Opérateurs de second ordre - Coordonnées cylindriques

Soit $\overrightarrow{u}$ un vecteur et T un tenseur.

$$\overline{\overline{grad}}(\overrightarrow{u}) = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) & \frac{\partial u_r}{\partial z} \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{\partial u_\theta}{\partial z} \\ \frac{\partial u_z}{\partial r} & \frac{1}{r}\frac{\partial u_z}{\partial \theta} & \frac{\partial u_z}{\partial z} \end{pmatrix}; \qquad \overrightarrow{div}(\overline{\overline{T}}) = \begin{pmatrix} \frac{\partial T_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{\partial T_{rz}}{\partial z} + \frac{T_{rr} - T_{\theta\theta}}{r} \\ \frac{\partial T_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{\partial T_{\theta z}}{\partial z} + \frac{T_{r\theta} + T_{\theta r}}{r} \\ \frac{\partial T_{zr}}{\partial r} + \frac{1}{r}\frac{\partial T_{z\theta}}{\partial \theta} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{zr}}{r} \end{pmatrix}$$

Opérateurs de second ordre - Coordonnées sphériques

Soit $\overrightarrow{u}$ un vecteur et T un tenseur.

$$\overline{\overline{grad}}(\overrightarrow{u}) = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) & \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial u_r}{\partial \varphi} - u_\varphi\right) \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{1}{r.sin\theta}\left(\frac{\partial u_\theta}{\partial \varphi} - cos\theta u_\varphi\right) \\ \frac{\partial u_\varphi}{\partial r} & \frac{1}{r}\frac{\partial u_\varphi}{\partial \theta} & \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + u_r + \frac{1}{tan\theta}u_\theta\right) \end{pmatrix}; \qquad \overrightarrow{div}(\overline{\overline{T}}) = \begin{pmatrix} \frac{\partial T_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{1}{r.sin\theta}\frac{\partial T_{r\varphi}}{\partial \varphi} + \frac{2T_{rr} - T_{\theta\theta} - T_{\varphi\varphi}}{r} + \frac{1}{r.tan\theta}T_{r\theta} \\ \frac{\partial T_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{1}{r.sin\theta}\frac{\partial T_{\theta\varphi}}{\partial \varphi} + \frac{3T_{r\theta}}{r} + \frac{1}{r.tan\theta}\left(T_{\theta\theta} - T_{\varphi\varphi}\right) \\ \frac{\partial T_{\varphi r}}{\partial r} + \frac{1}{r}\frac{\partial T_{\varphi\theta}}{\partial \theta} + \frac{1}{r.sin\theta}\frac{\partial T_{\varphi\varphi}}{\partial \varphi} + \frac{3T_{r\varphi}}{r} + \frac{2}{r.tan\theta}T_{\theta\varphi} \end{pmatrix}$$