

Formulaire mathématique

Opérateurs - Coordonnées cartésiennes

Le vecteur nabla permet de retrouver les autres opérateurs en coordonnées cartésiennes

uniquement : $\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$. On a ainsi : $\begin{cases} \vec{\nabla} f = \vec{\text{grad}}(f) \\ \vec{\nabla} \cdot \vec{f} = \text{div}(\vec{f}) \\ \vec{\nabla} \wedge \vec{f} = \vec{\text{rot}}(\vec{f}) \\ \vec{\nabla} \cdot \vec{\nabla} f = \Delta f \text{ Attention, laplacien scalaire} \end{cases}$.

Opérateurs - Fondamentaux

$$\begin{aligned} \vec{\text{grad}}(f.g) &= f.\vec{\text{grad}}(g) + g.\vec{\text{grad}}(f) \\ \text{div}(f.\vec{g}) &= f.\text{div}(\vec{g}) + \vec{g} \cdot \vec{\text{grad}}(f) \\ \vec{\text{rot}}(f.\vec{g}) &= f.\vec{\text{rot}}(\vec{g}) + \vec{g} \wedge \vec{\text{grad}}(f) \end{aligned}$$

$$\begin{aligned} \text{div}(\vec{\text{grad}} f) &= \Delta f \\ \vec{\text{grad}}(\text{div} \vec{f}) - \vec{\text{rot}}(\vec{\text{rot}} \vec{f}) &= \Delta \vec{f} \\ \text{div}(\vec{\text{rot}}) &= 0 \\ \vec{\text{rot}}(\vec{\text{grad}}) &= \vec{0} \end{aligned}$$

Opérateurs - Coordonnées cylindriques

$$\vec{\text{grad}}(f) = \begin{pmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r} \frac{\partial f}{\partial \theta} \\ \frac{\partial f}{\partial z} \end{pmatrix}; \quad \text{div}(\vec{f}) = \frac{1}{r} \frac{\partial(r f_r)}{\partial r} + \frac{1}{r} \frac{\partial f_\theta}{\partial \theta} + \frac{\partial f_z}{\partial z}; \quad \vec{\text{rot}}(\vec{f}) = \begin{pmatrix} \frac{1}{r} \frac{\partial f_z}{\partial \theta} - \frac{\partial f_\theta}{\partial z} \\ \frac{\partial f_r}{\partial z} - \frac{\partial f_z}{\partial r} \\ \frac{1}{r} \left(\frac{\partial(r f_\theta)}{\partial r} - \frac{\partial(r f_r)}{\partial \theta} \right) \end{pmatrix}$$

$$\Delta f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Opérateurs - Coordonnées sphériques

$$\vec{\text{grad}}(f) = \begin{pmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r} \frac{\partial f}{\partial \theta} \\ \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \end{pmatrix}; \quad \text{div}(\vec{f}) = \frac{1}{r^2} \frac{\partial(r^2 f_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\sin \theta f_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial f_\varphi}{\partial \varphi}; \quad \vec{\text{rot}}(\vec{f}) = \begin{pmatrix} \frac{1}{r \sin \theta} \left(\frac{\partial(\sin \theta f_\varphi)}{\partial \theta} - \frac{\partial f_\theta}{\partial \varphi} \right) \\ \frac{1}{r \sin \theta} \left(\frac{\partial f_r}{\partial \varphi} - \frac{\partial(r \sin \theta f_\varphi)}{\partial r} \right) \\ \frac{1}{r} \left(\frac{\partial(r f_\theta)}{\partial r} - \frac{\partial f_r}{\partial \theta} \right) \end{pmatrix};$$

$$\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2}$$

Opérateurs de second ordre - Coordonnées cartésiennes

Soit \vec{u} un vecteur et T un tenseur.

$$\vec{\text{grad}}(\vec{u}) = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}; \quad \vec{\text{div}}(\vec{T}) = \begin{pmatrix} \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \\ \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \\ \frac{\partial T_{zx}}{\partial x} + \frac{\partial T_{zy}}{\partial y} + \frac{\partial T_{zz}}{\partial z} \end{pmatrix}; \quad \Delta(\vec{u}) = \begin{pmatrix} \frac{\partial^2 u_x}{\partial x^2} & \frac{\partial^2 u_x}{\partial y^2} & \frac{\partial^2 u_x}{\partial z^2} \\ \frac{\partial^2 u_y}{\partial x^2} & \frac{\partial^2 u_y}{\partial y^2} & \frac{\partial^2 u_y}{\partial z^2} \\ \frac{\partial^2 u_z}{\partial x^2} & \frac{\partial^2 u_z}{\partial y^2} & \frac{\partial^2 u_z}{\partial z^2} \end{pmatrix}$$

Opérateurs de second ordre - Coordonnées cylindriques

Soit \vec{u} un vecteur et T un tenseur.

$$\vec{\text{grad}}(\vec{u}) = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r} \left(\frac{\partial u_r}{\partial \theta} - u_\theta \right) & \frac{\partial u_r}{\partial z} \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right) & \frac{\partial u_\theta}{\partial z} \\ \frac{\partial u_z}{\partial r} & \frac{1}{r} \frac{\partial u_z}{\partial \theta} & \frac{\partial u_z}{\partial z} \end{pmatrix}; \quad \vec{\text{div}}(\vec{T}) = \begin{pmatrix} \frac{\partial T_{rr}}{\partial r} + \frac{1}{r} \frac{\partial T_{r\theta}}{\partial \theta} + \frac{\partial T_{rz}}{\partial z} + \frac{T_{rr} - T_{\theta\theta}}{r} \\ \frac{\partial T_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{\partial T_{\theta z}}{\partial z} + \frac{T_{r\theta} + T_{\theta r}}{r} \\ \frac{\partial T_{zr}}{\partial r} + \frac{1}{r} \frac{\partial T_{z\theta}}{\partial \theta} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{zr}}{r} \end{pmatrix}$$

Opérateurs de second ordre - Coordonnées sphériques

Soit \vec{u} un vecteur et T un tenseur.

$$\vec{\text{grad}}(\vec{u}) = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r} \left(\frac{\partial u_r}{\partial \theta} - u_\theta \right) & \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial u_r}{\partial \varphi} - u_\varphi \right) \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right) & \frac{1}{r \sin \theta} \left(\frac{\partial u_\theta}{\partial \varphi} - \cos \theta u_\varphi \right) \\ \frac{\partial u_\varphi}{\partial r} & \frac{1}{r} \frac{\partial u_\varphi}{\partial \theta} & \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial u_\varphi}{\partial \varphi} + u_r + \frac{1}{\tan \theta} u_\theta \right) \end{pmatrix}; \quad \vec{\text{div}}(\vec{T}) = \begin{pmatrix} \frac{\partial T_{rr}}{\partial r} + \frac{1}{r} \frac{\partial T_{r\theta}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial T_{r\varphi}}{\partial \varphi} + \frac{2T_{rr} - T_{\theta\theta} - T_{\varphi\varphi}}{r} + \frac{1}{r \tan \theta} T_{r\theta} \\ \frac{\partial T_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial T_{\theta\varphi}}{\partial \varphi} + \frac{3T_{r\theta}}{r} + \frac{1}{r \tan \theta} (T_{\theta\theta} - T_{\varphi\varphi}) \\ \frac{\partial T_{\varphi r}}{\partial r} + \frac{1}{r} \frac{\partial T_{\varphi\theta}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial T_{\varphi\varphi}}{\partial \varphi} + \frac{3T_{r\varphi}}{r} + \frac{2}{r \tan \theta} T_{\theta\varphi} \end{pmatrix}$$