University of Waterloo
CS240 Fall 2017
Tutorial 4

Monday, October 16

Problem 1

Insert the numbers 12, 11, 13, 10, 20 into an empty skip list using the coin flips HHTHTHTTHHHT.
Afterwards, delete 13 from the resulting skip list.

Problem 2

In this problem, we will show that deleting a single node in an AVL-tree of height h might require $\Theta(h)$ rotations. First, we define a family $(T_h)_{h \geq 1}$ recursively in the following manner: T_{-1} is empty and T_0 is a single node. To form T_h, we start with a single node and take a copy of T_{h-2} and a copy of T_{h-1} as the left and the right children of the root, respectively.

a) For $h \geq 0$, what is the height of T_h? Prove your claim.

b) Prove that for $h \geq 0$, T_h satisfies the height requirements of an AVL tree.

c) On T_3, what are the leaves which require $\lfloor 3/2 \rfloor = 1$ rotation upon deletion? Pick one and show the resulting tree.

d) Same question with T_4, but now with $\lfloor 4/2 \rfloor = 2$ rotations.

e) Prove by induction that the above construction of T_h results in trees for which there is a node that requires $\lfloor h/2 \rfloor$ rotations upon deletion.

Problem 3

Let L be a list of n elements. Give an sequence of m searches $(m \in \Omega(n))$ such that: (a) the average cost of a search under the MTF heuristic is $O(1)$ and (b) the average cost of a search under the Transpose heuristic is $O(n)$.