BINARY SEARCH TREES

Problem Solving with Computers-II

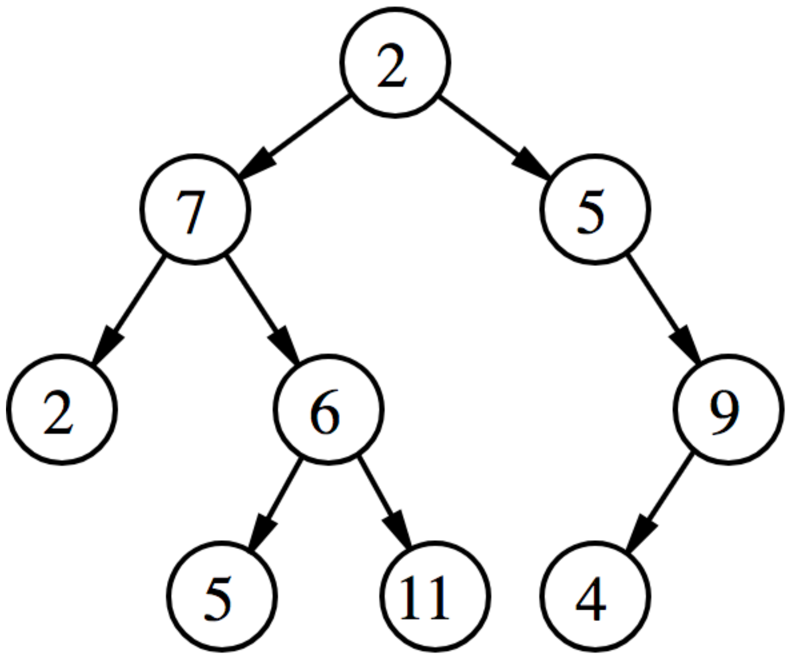
Binary Search Trees

- What are the operations supported?
- What are the running times of these operations?
- How do you implement the BST i.e. operations supported by it?

Operations supported by Sorted arrays and Binary Search Trees (BST)

Operations	Sorted Array	BST
Min		
Max		
Successor		
Predecessor		
Search		
Insert		
Delete		
Print elements in order		

Trees

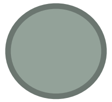


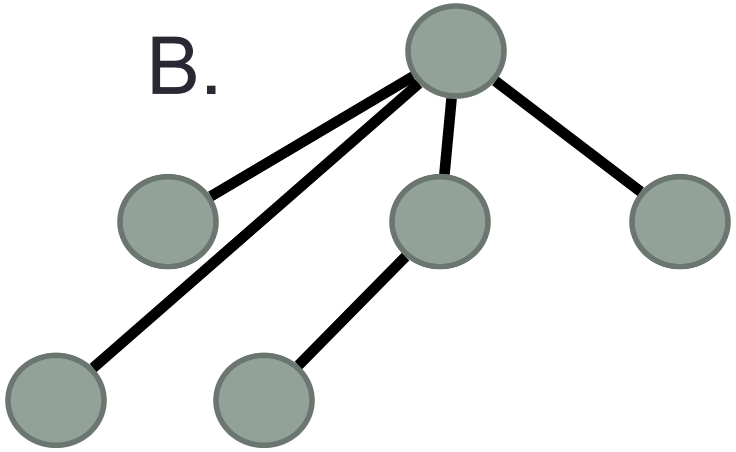
A tree has following general properties:

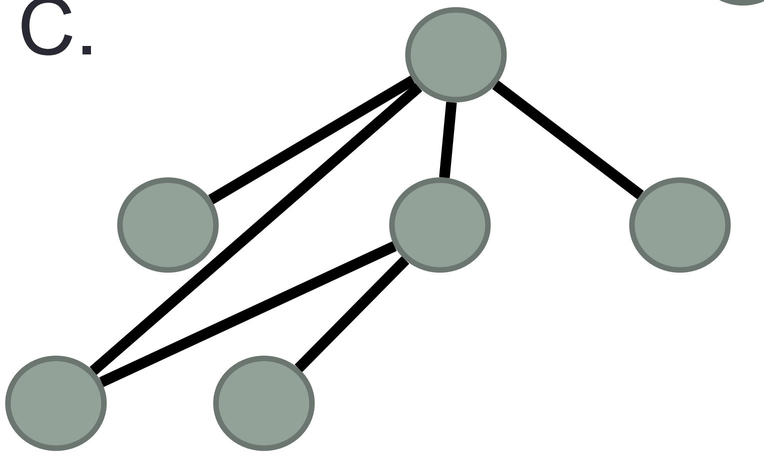
- One node is distinguished as a **root**;
- Every node (exclude a root) is connected by a directed edge *from* exactly one other node;

  A direction is: *parent -> children*
- *Leaf node: Node that has no children*

Which of the following is/are a tree?

A. 

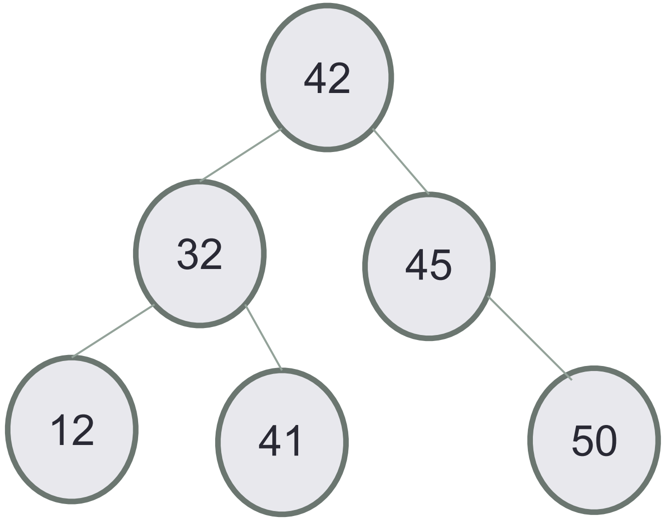
B. 

C. 

D. A & B

E. All of A-C

Binary Search Tree – What is it?



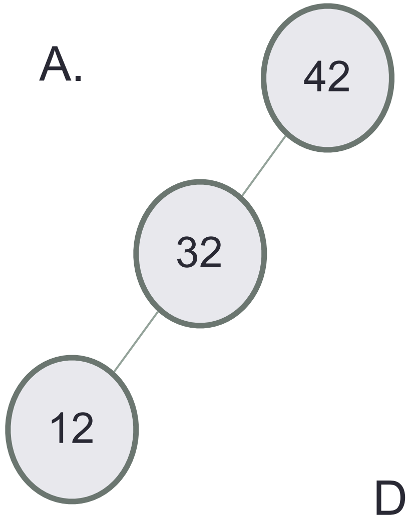
- Each node:
  - stores a key (k)
  - has a pointer to left child, right child and parent (optional)
  - Satisfies the Search Tree Property

For any node,
Keys in node's left subtree <= Node's key
Node's key < Keys in node's right subtree

Do the keys have to be integers?

Which of the following is/are a binary search tree?

A.



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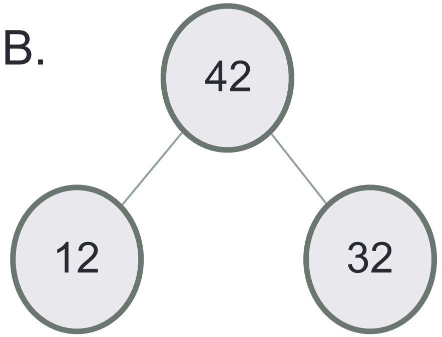

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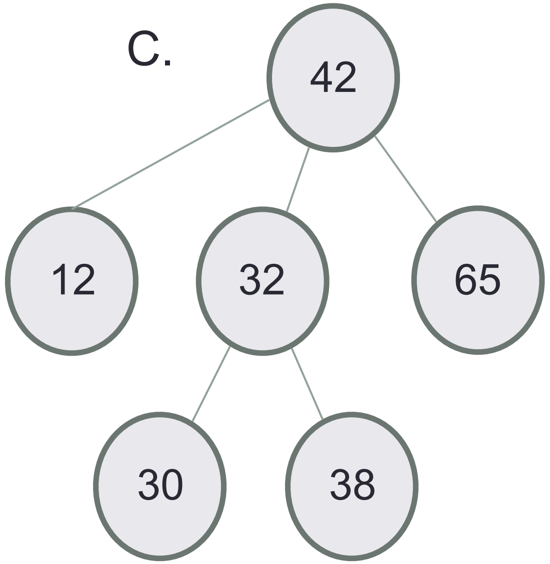
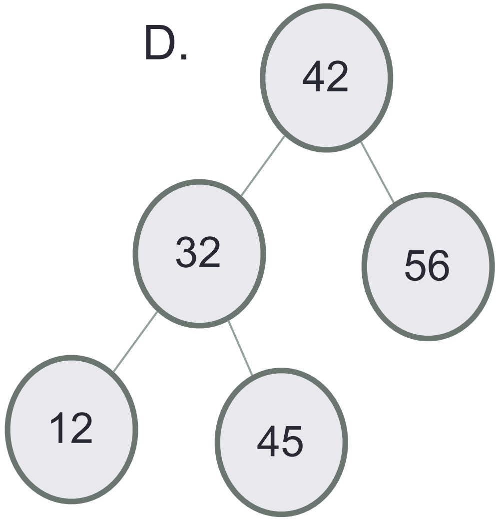
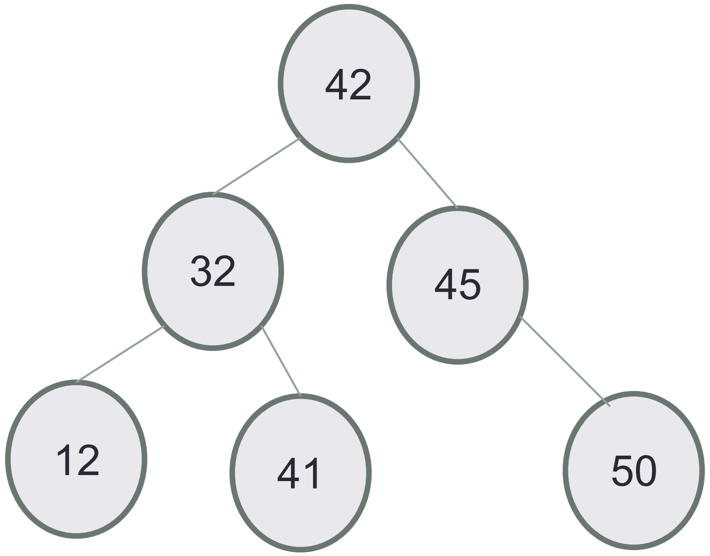


D.



E. More than one of these

BSTs allow efficient search!



- Start at the root;
- Trace down a path by comparing **k** with the key of the current node x:
  - If the keys are equal: we have found the key
  - If **k** < key[x] search in the left subtree of x
  - If **k** > key[x] search in the right subtree of x

Search for 41, then search for 53

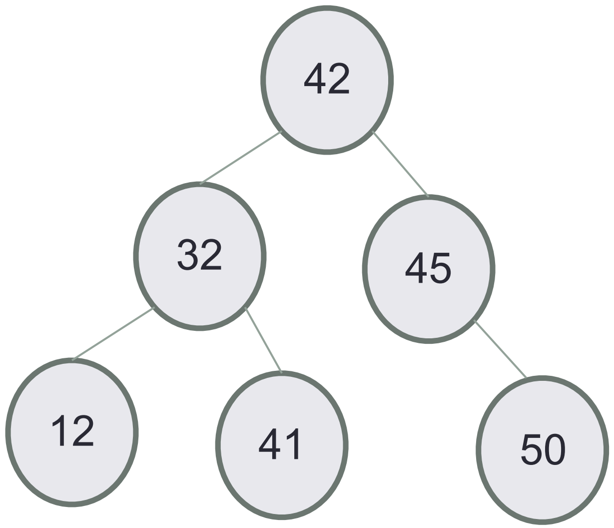
A node in a BST

```
class BSTNode {

public:
   BSTNode* left;
   BSTNode* right;
   BSTNode* parent;
   int const data;

   BSTNode( const int & d ) : data(d) {
      left = right = parent = 0;
   }
};
```

Define the BST ADT

Operations
Min
Max
Successor
Predecessor
Search
Insert
Delete
Print elements in order

Insert



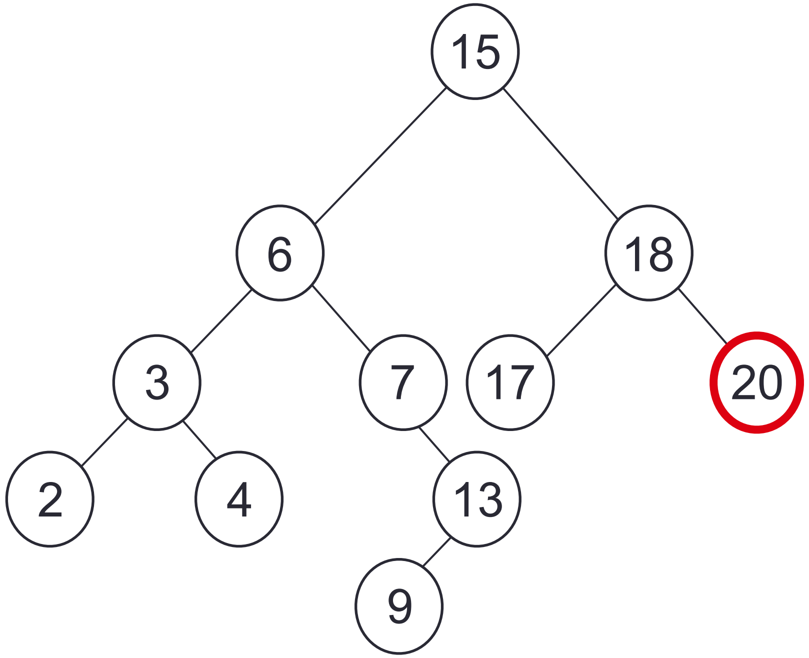
- Insert 40
- Search for the key
- Insert at the spot you expected to find it

Max

Goal: find the maximum key value in a BST

Following right child pointers from the root, until a leaf node is encountered. The least node has the max value

Alg: `int BST::max()`



Maximum = 20

Min

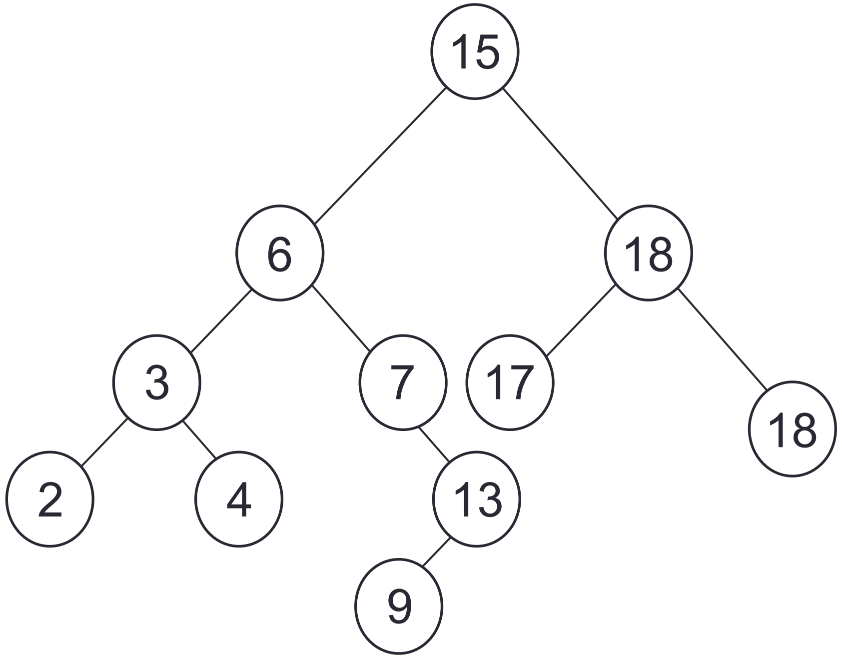
Goal: find the minimum key value in a BST

Start at the root.

Follow _________ child pointers from the root, until a leaf node is encountered

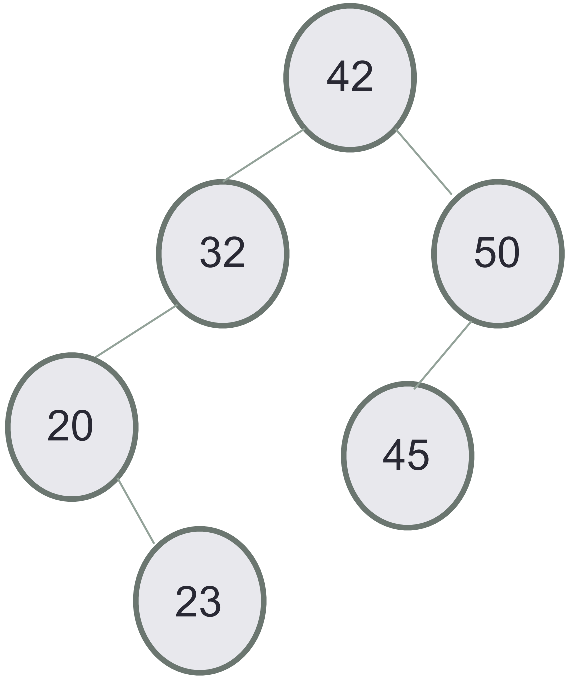
Leaf node has the min key value

Alg: `int BST::min()`



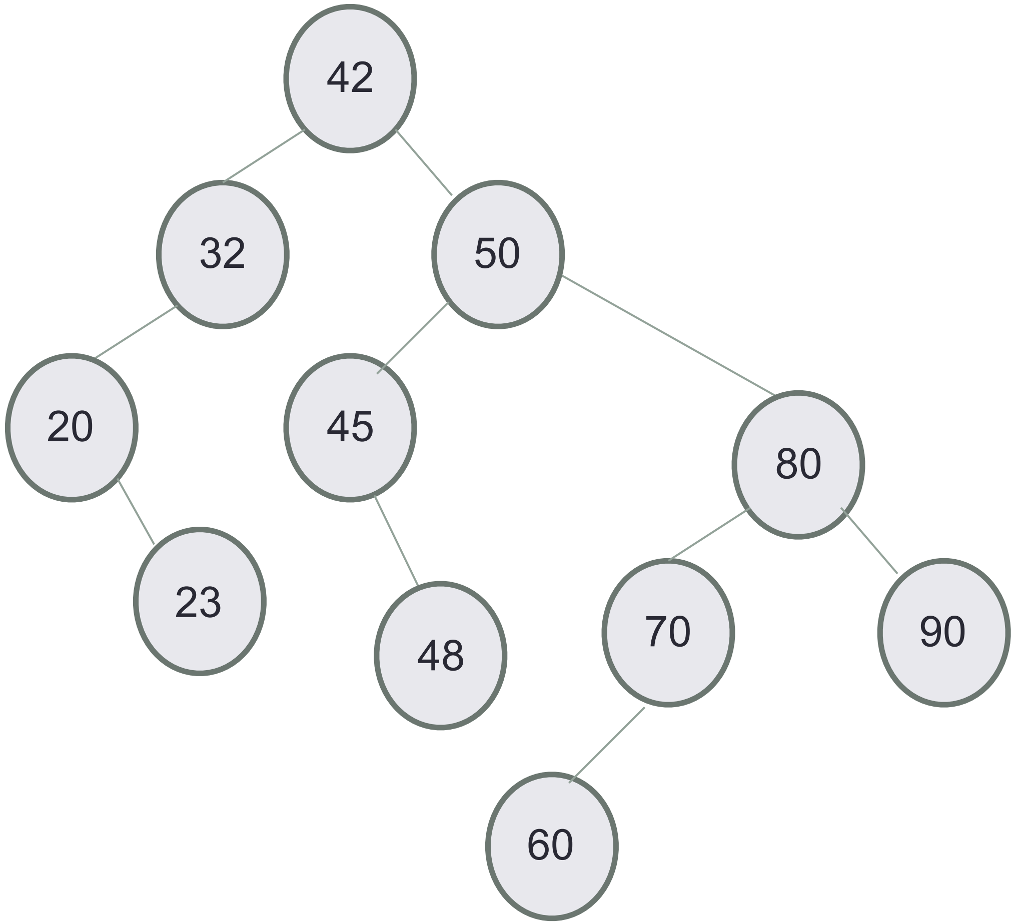
Min = ?

Predecessor: Next smallest element



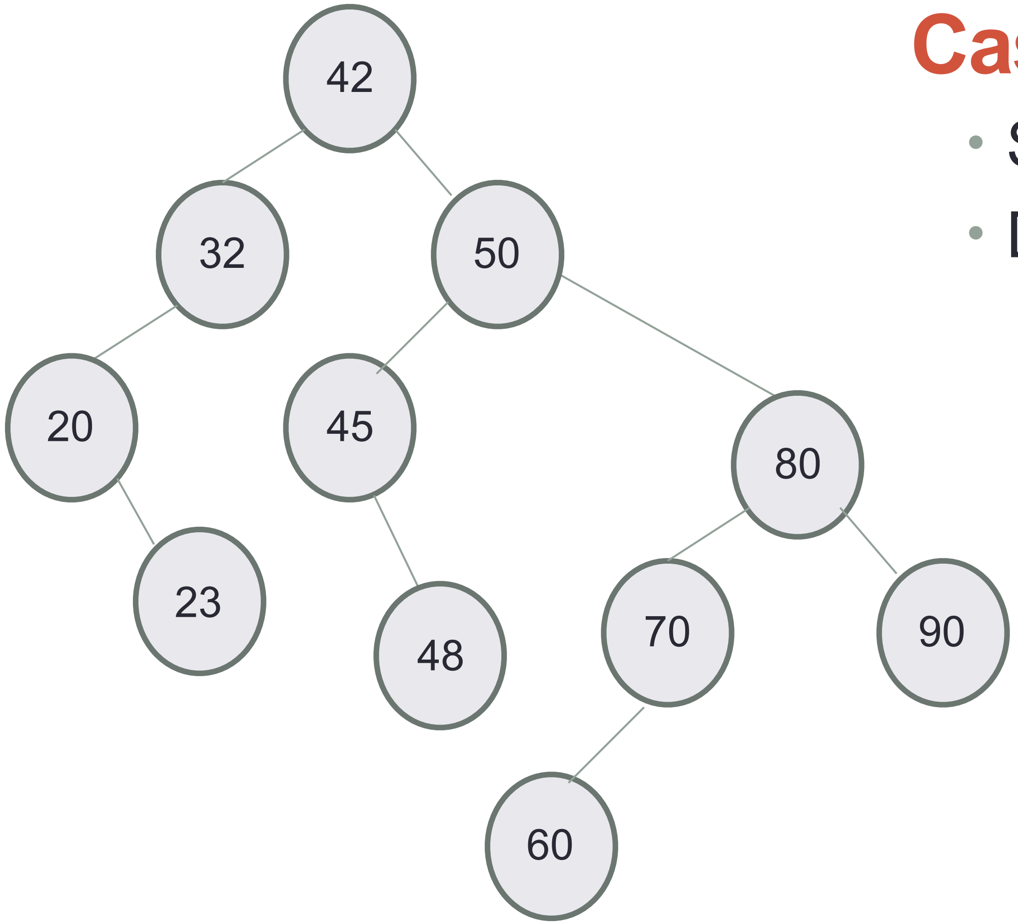
- What is the predecessor of 32?
- What is the predecessor of 45?

Successor: Next largest element



- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

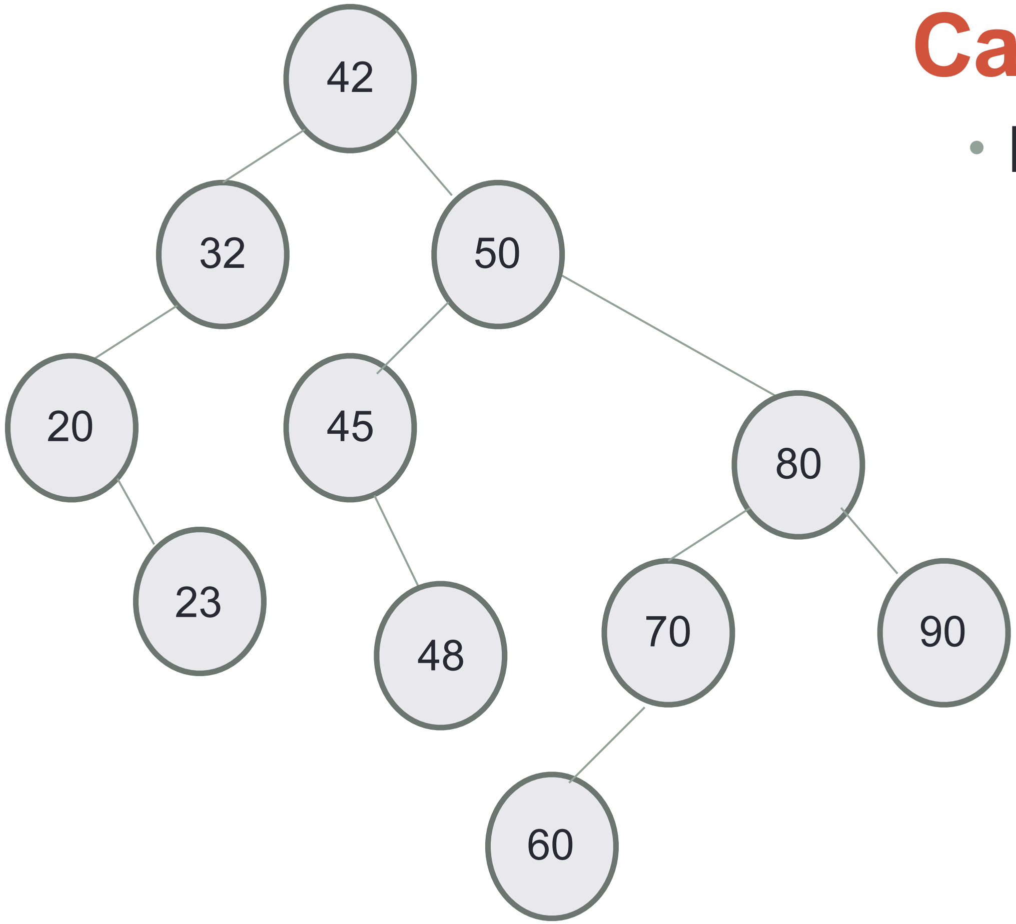
Delete: Case 1



Case 1: Node is a leaf node

- Set parent’s (left/right) child pointer to null
- Delete the node

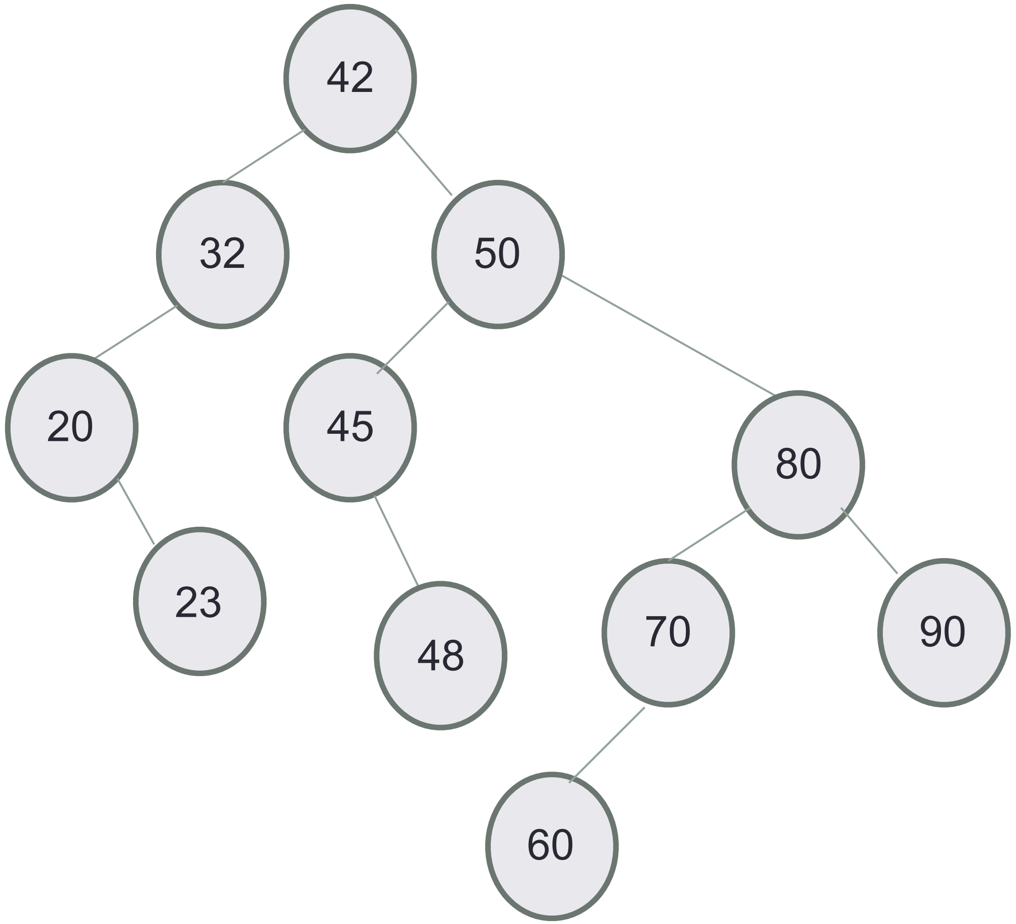
Delete: Case 2



Case 2 Node has only one child

- Replace the node by its only child

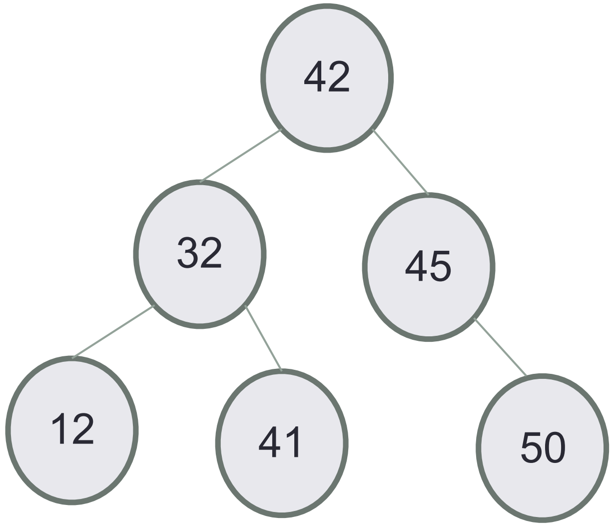
Delete: Case 3



Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?

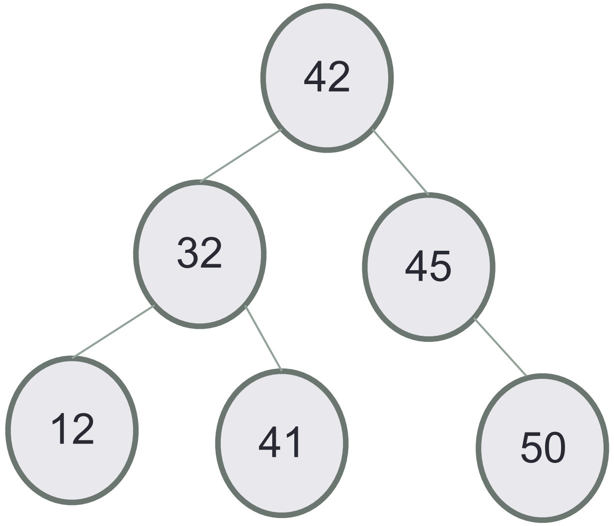
In order traversal: print elements in sorted order



Algorithm Inorder(tree)

1. Traverse the left subtree, i.e., call Inorder(left-subtree)
2. Visit the root.
3. Traverse the right subtree, i.e., call Inorder(right-subtree)

Pre-order traversal: nice way to linearize your tree!



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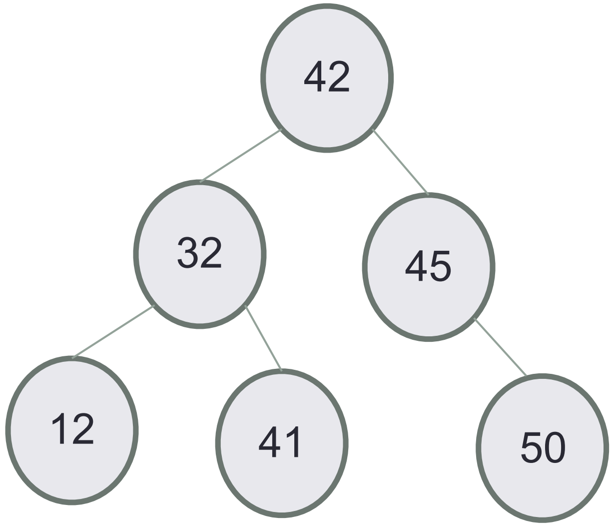
Post-order traversal: use in recursive destructors!



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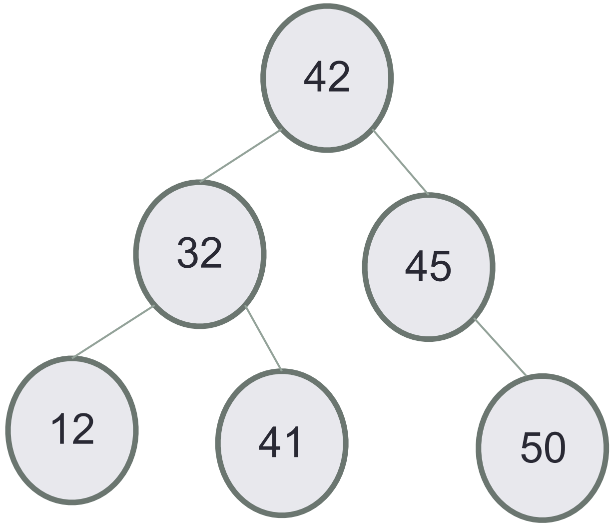

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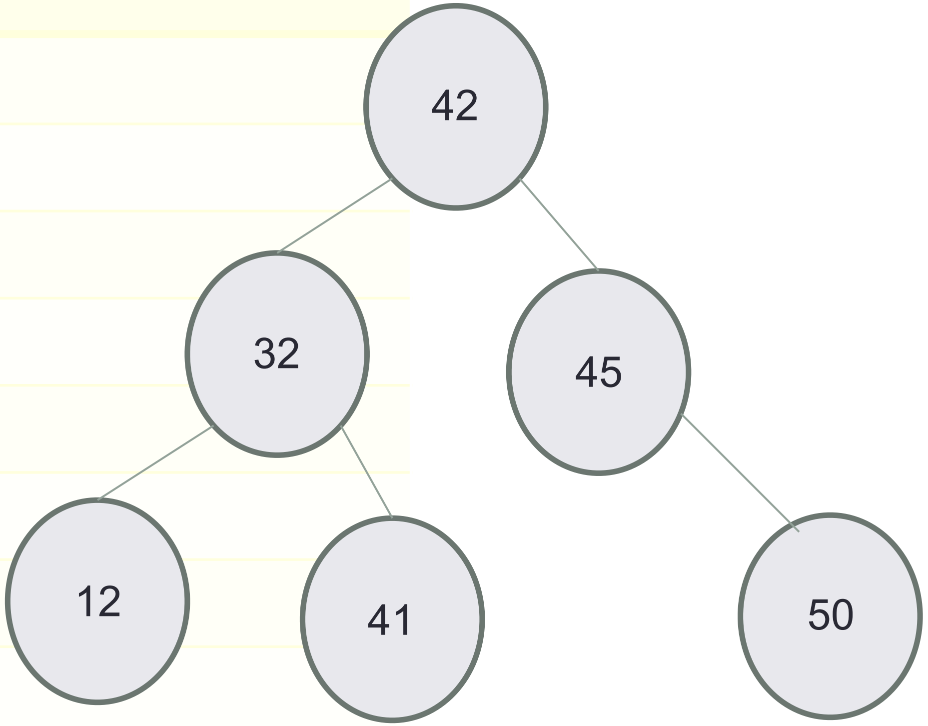
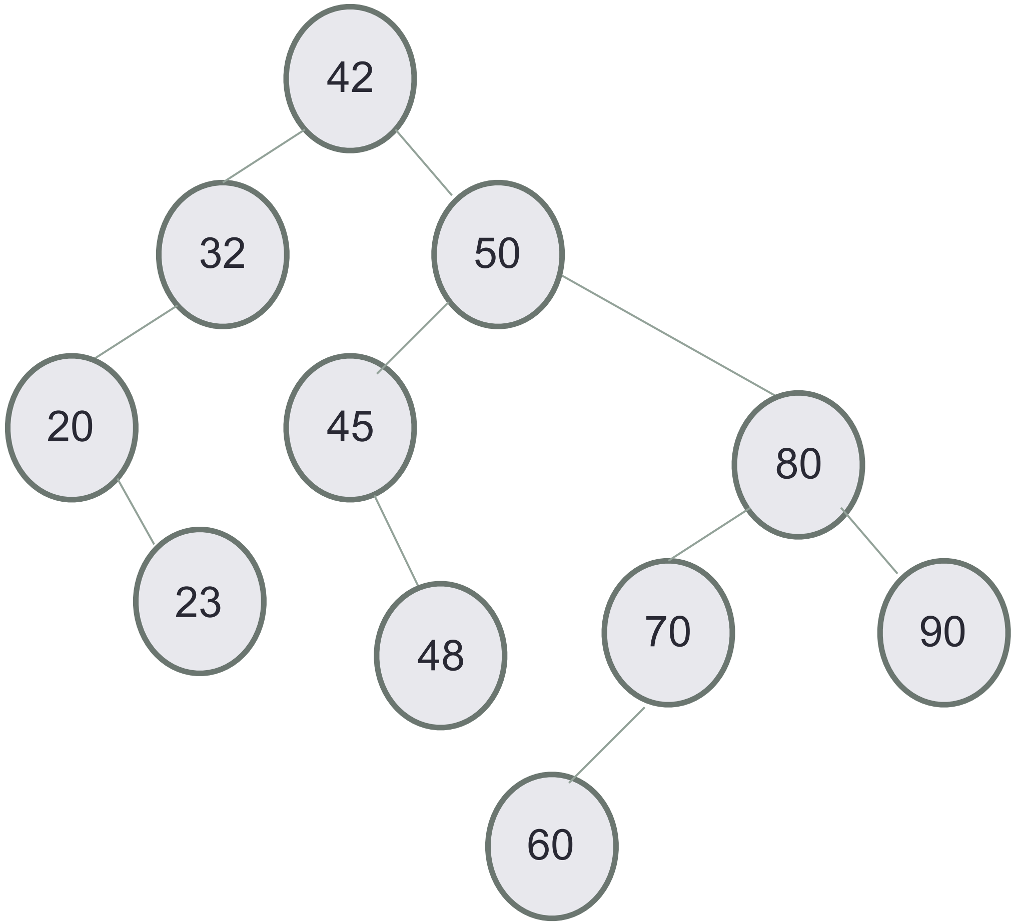


Successor: Next largest element



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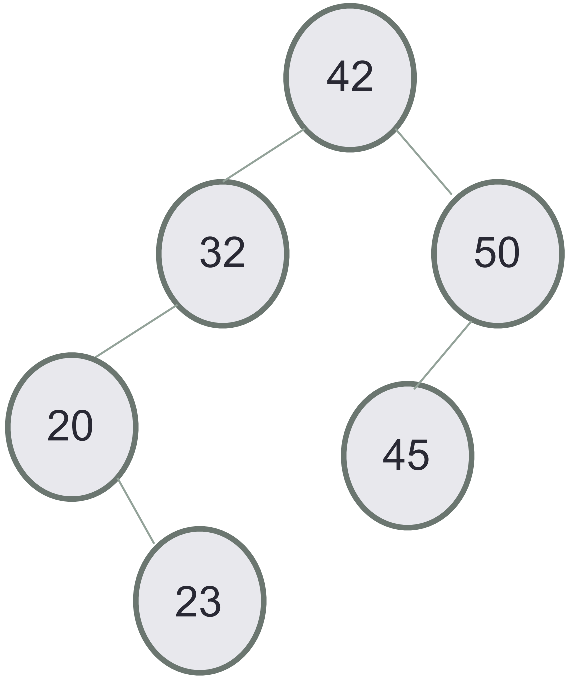
Predecessor: Next smallest element



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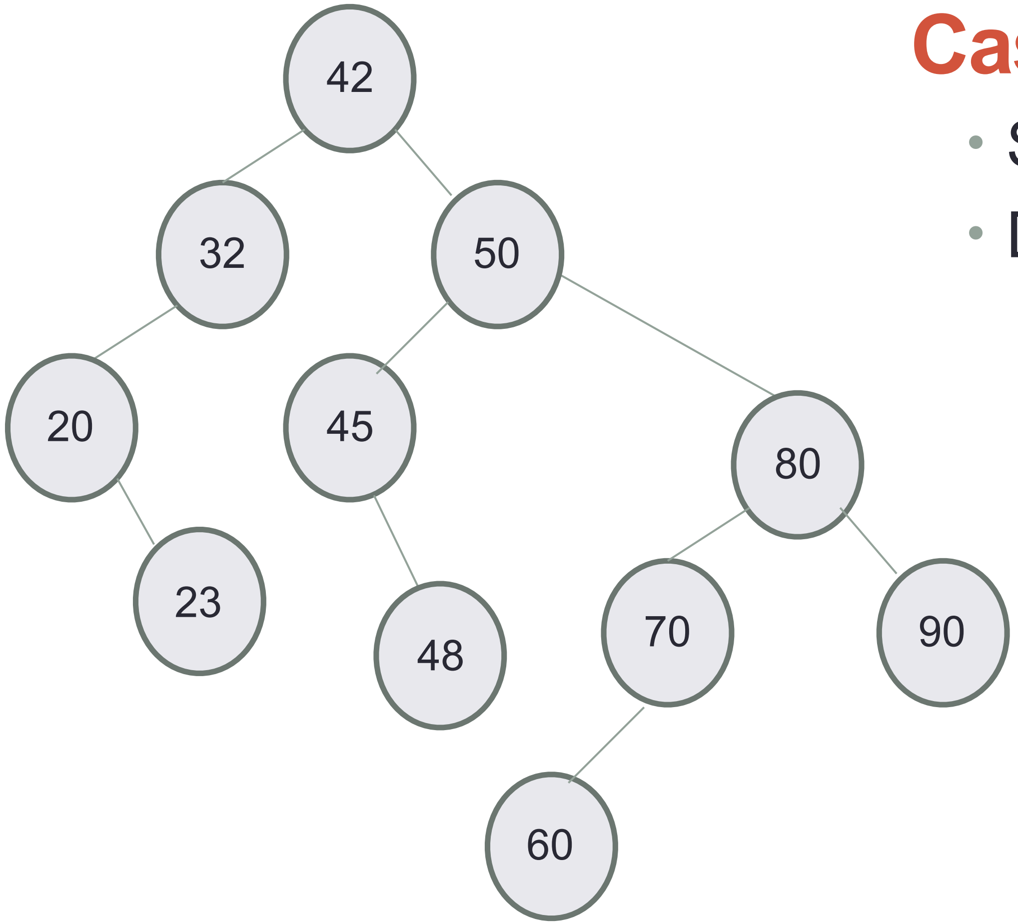

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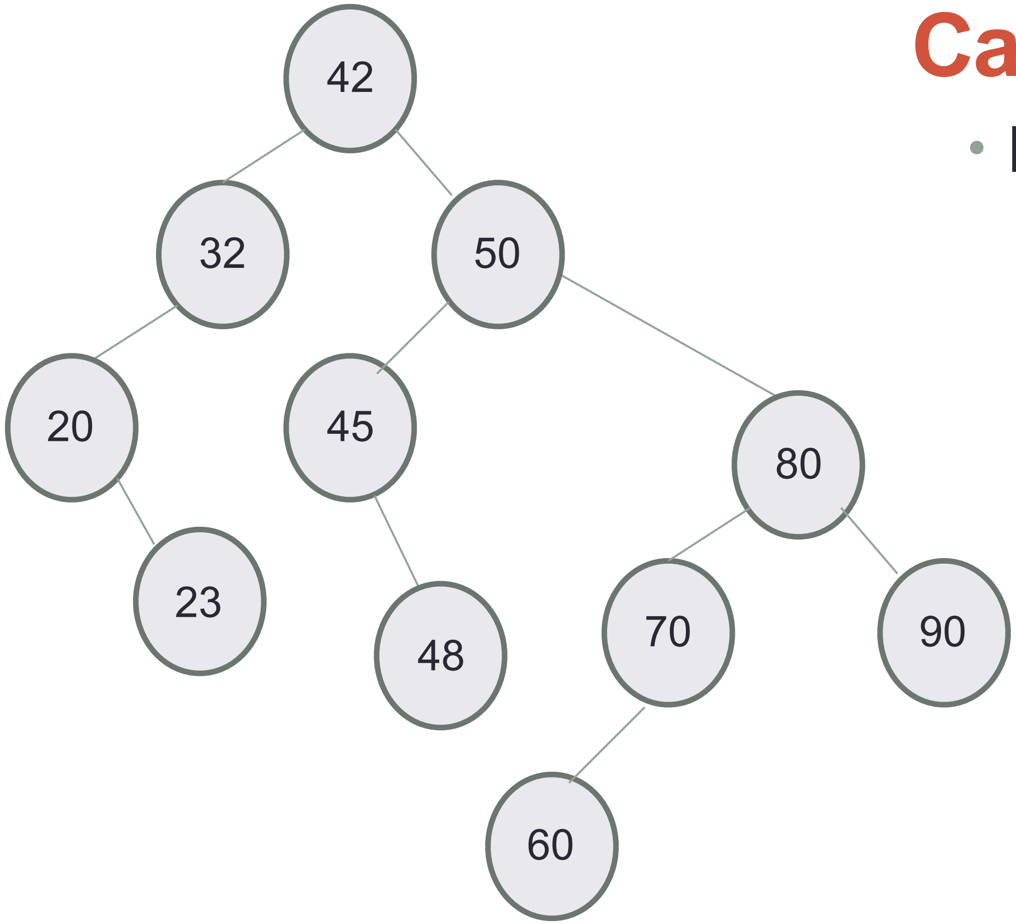

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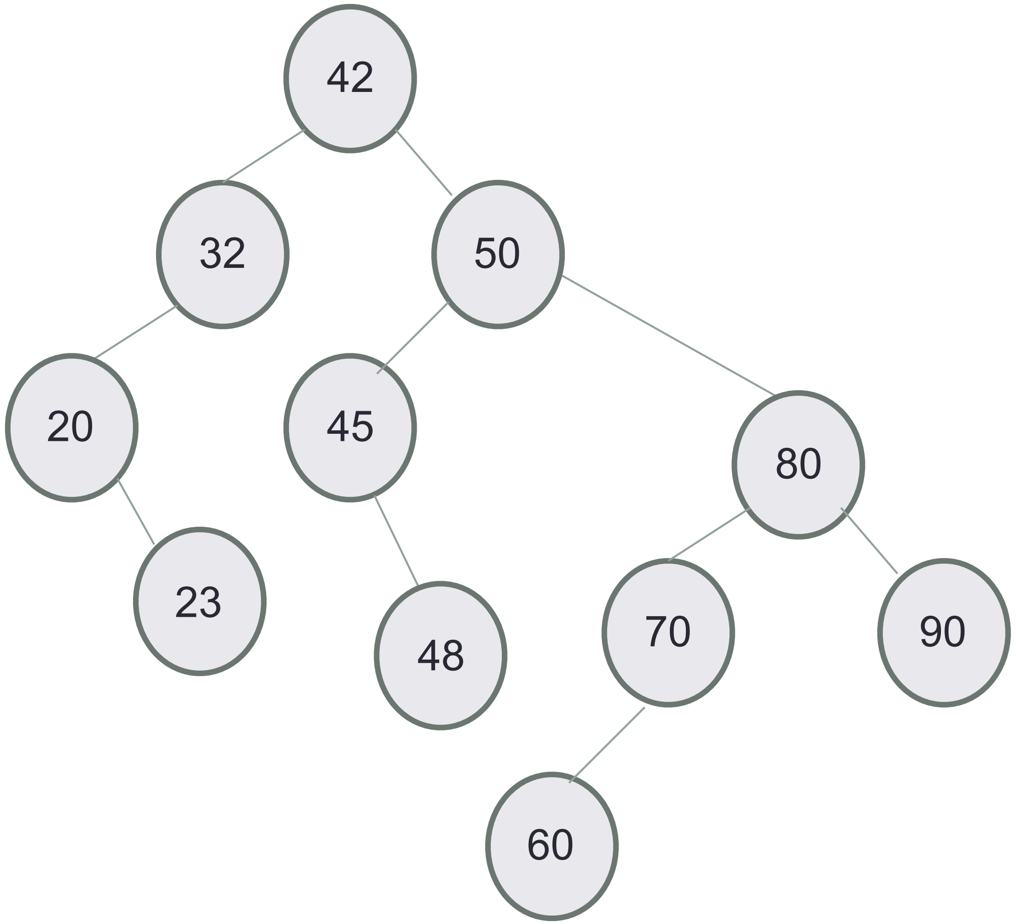

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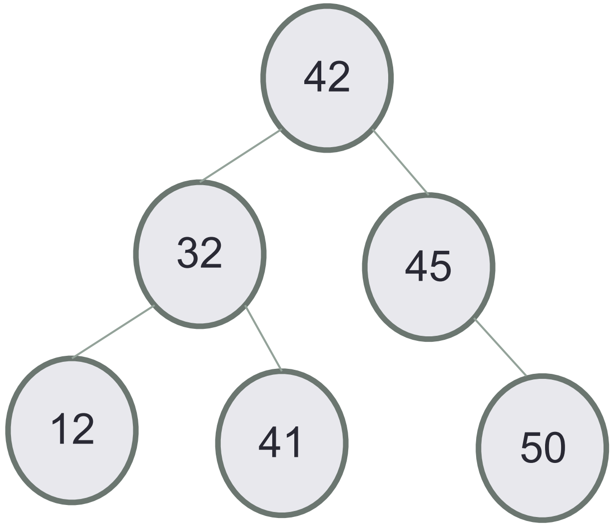

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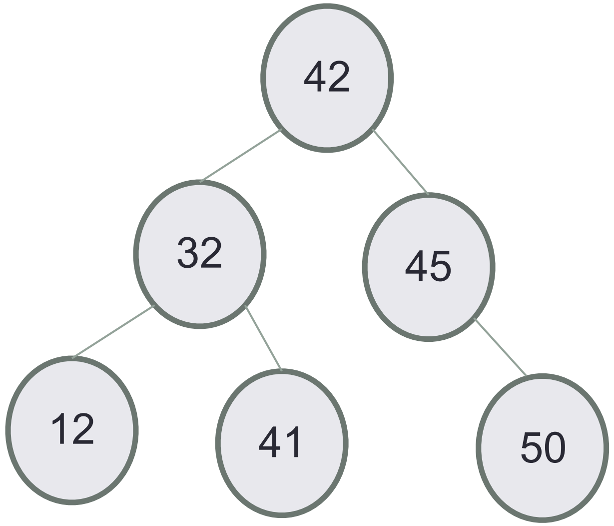

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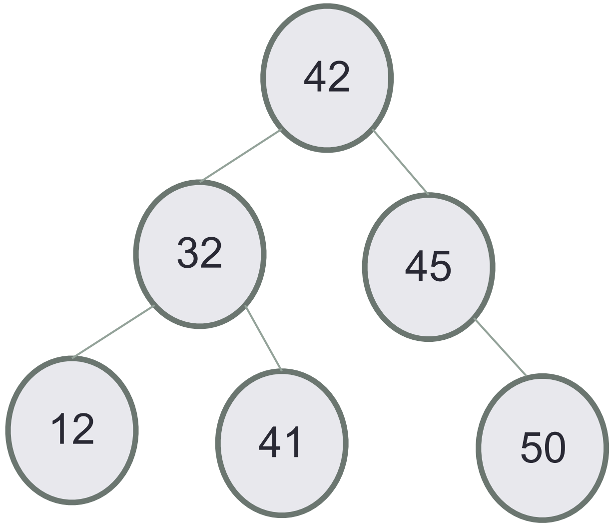

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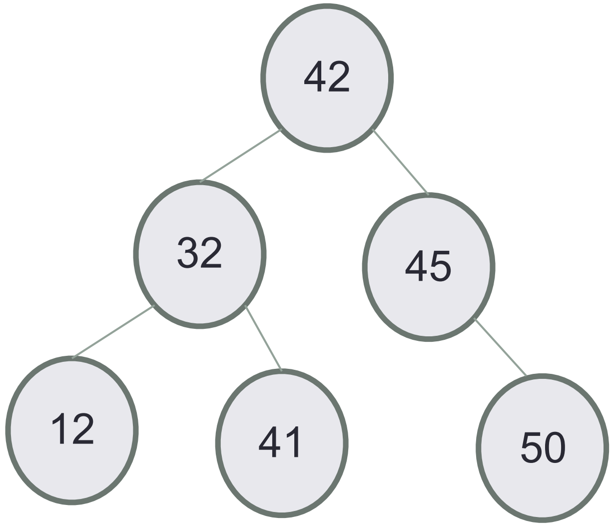

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