Explanation Catalog explained

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1 Disclaimer

The explanation generator is a prototype. The decompositions are chosen to experiment its capabilities. One should not use these decompositions nor these explanations without care.

2 AllDifferent($\{X_1, \ldots, X_n\}$)

2.1 Decomposition

$$
\begin{aligned}
X_i = t &\iff b_{it} && \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
\sum_{i \in [\![1,n]\!]} b_{it} &\leq 1 && \forall t \in [\![1, m]\!]
\end{aligned}
$$

2.2 Explanation

$$\langle X_i \neq t \rangle \rightarrow \langle X_{i'} = t \rangle_{\forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!]}$$

A value is removed of a domain because an other variable is instantiated to it.

3 AllEqual($\{X_1, \ldots, X_n\}$)

3.1 Decomposition

$$
\begin{aligned}
X_i = t &\iff b_{it} && \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
\bigwedge_{i \in [\![1,n]\!]} b_{it} & && \forall t \in [\![1, m]\!]
\end{aligned}
$$

3.2 Explanation

$$\langle X_i = t \rangle \rightarrow \langle X_i = t \rangle_{\exists i,\ i \in [\![1,n]\!]}$$

$$\langle X_i \neq t \rangle \rightarrow \langle X_i \neq t \rangle_{\exists i,\ i \in [\![1,n]\!]}$$

A value instantiation or removal is due to an other variable same event.

3.3 Decomposition

$$
\begin{aligned}
X_i \geq t &\iff b_{it} && \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
\bigwedge_{i\in[\![1,n]\!]} b_{it} &\iff b1_t && \forall t \in [\![1,m]\!] \\
\bigwedge_{i\in[\![1,n]\!]} \neg b_{it} &\iff b2_t && \forall t \in [\![1,m]\!] \\
(b1_t \vee b2_t) & && \forall t \in [\![1,m]\!]
\end{aligned}
$$

3.4 Explanation

$$\langle X_i \geq t\rangle \rightarrow \langle X_i \geq t\rangle_{\exists i,\ i\in[\![1,n]\!]}$$

$$\langle X_i < t\rangle \rightarrow \langle X_i < t\rangle_{\exists i,\ i\in[\![1,n]\!]}$$

A modified bound is due to an other variable having the same bound.

4 NValue$(N, \{X_1, \ldots, X_n\})$

4.1 Decomposition

$$
\begin{aligned}
X_i = t &\iff b_{it} && \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
\bigvee_{i\in[\![1,n]\!]} b_{it} &\iff b2_t && \forall t \in [\![1,m]\!] \\
\sum_{t\in[\![1,m]\!]} b2_t = p &\iff b3_p && \forall t \in [\![1,m]\!] \forall p \in [\![1,n]\!] \\
N = p &\iff b3_p && \forall p \in [\![1,n]\!]
\end{aligned}
$$

4.2 Explanation

$$\langle X_i = t\rangle \rightarrow \langle X_i \neq t\rangle_{\forall i,\ i\in[\![1,n]\!],\ \forall t,\ t\in[\![1,m]\!]}\ \langle N = p\rangle_{\forall p,\ p\in[\![1,n]\!]}$$

An instantiation is explained by N instantiated and all the variables that cannot take the value t.

$$\langle X_i \neq t\rangle \rightarrow \langle X_i = t\rangle_{\exists i,\ i\in[\![1,n]\!],\ \forall t,\ t\in[\![1,m]\!]}\ \langle N = p\rangle_{\forall p,\ p\in[\![1,n]\!]}$$

An value removal is explained by N instantiated and all the variables that are instantiated to the value t.

$$\langle N = p\rangle \rightarrow \langle X_i \neq t\rangle_{\forall i,\ i\in[\![1,n]\!],\ \forall t,\ t\in[\![1,m]\!]}\ \langle X_i = t\rangle_{\forall t,\ t\in[\![1,m]\!],\ \forall i,\ i\in[\![1,n]\!]}$$

A N instantiation is explained by all the variables that takes the value t or not.

$$\langle N \neq p\rangle \rightarrow \langle X_i = t\rangle_{\forall t,\ t\in[\![1,m]\!],\ \forall i,\ i\in[\![1,n]\!]}$$

$$\langle N \neq p\rangle \rightarrow \langle X_i \neq t\rangle_{\forall t,\ t\in[\![1,m]\!],\ \forall i,\ i\in[\![1,n]\!]}$$

A N value removal can be explained either by all the variables that are instantiated to t or all the variables that does not have it in their domain.

5 AtLeastNValue$(N, \{X_1, \ldots, X_n\})$

5.1 Decomposition

$$
\begin{aligned}
X_i = t &\iff b_{it} & \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
\bigvee_{i \in [\![1,n]\!]} b_{it} &\iff b2_t & \forall t \in [\![1,m]\!] \\
\sum_{t \in [\![1,m]\!]} b2_t \geq p &\iff b3_p & \forall t \in [\![1,m]\!] \\
N \geq p &\iff b3_p & \forall t \in [\![1,m]\!]
\end{aligned}
$$

5.2 Explanation

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t' \rangle_{\forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!]} \ \langle N \geq p \rangle_{\forall p,\ p \in [\![1,n]\!]}$$

An instantiation is explained by the upper bound of N and all the variables that cannot take the value t.

$$\langle X_i \neq t \rangle \rightarrow \langle X_i = t' \rangle_{\forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!]} \ \langle N < p \rangle_{\forall p,\ p \in [\![1,n]\!]}$$

An value removal is explained by the lower bound of N and all the variables that are instantiated to the value t.

$$\langle N \geq p \rangle \rightarrow \langle X_i = t \rangle_{\forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}$$

The lower bound modification of N is explained by all the variables instantiated to t

$$\langle N < p \rangle \rightarrow \langle X_i \neq t \rangle_{\forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}$$

The upper bound modification of N is explained by all the variables that cannot be affected to the value t.

6 AtMostNValue$(N, \{X_1, \ldots, X_n\})$

6.1 Decomposition

$$
\begin{aligned}
X_i = t &\iff b_{it} & \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
\bigvee_{i \in [\![1,n]\!]} b_{it} &\iff b2_t & \forall t \in [\![1,m]\!] \\
\sum_{t \in [\![1,m]\!]} b2_t < p &\iff b3_p & \forall t \in [\![1,m]\!] \\
N \geq p &\iff b3_p & \forall t \in [\![1,m]\!]
\end{aligned}
$$

6.2 Explanation

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t' \rangle_{\forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!]} \ \langle N < p \rangle_{\forall p,\ p \in [\![1,n]\!]}$$

$$\langle X_i \neq t \rangle \rightarrow \langle X_i = t' \rangle_{\forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!]} \ \langle N \geq p \rangle_{\forall p,\ p \in [\![1,n]\!]}$$

$$\langle N < p \rangle \rightarrow \langle X_i = t \rangle_{\forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}$$

$$\langle N \geq p \rangle \rightarrow \langle X_i \neq t \rangle_{\forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}$$

Same explanations as AtLeastNValues but inverted.

7 Cumulative$(\{X_1, \ldots, X_n\}, \{d_1, \ldots, d_n\}, c)$

7.1 Decomposition

$$X_i \geq t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$(b_{i(t-d_i)} \wedge \neg b_{it}) \iff b2_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$\sum_{i \in [\![1,n]\!]} b2_{it} \leq c \qquad \forall t \in [\![1,m]\!]$$

7.2 Explanation

$$\langle X_i \geq t \rangle \to \langle X_i \geq t' \rangle_{t'=t-d_i} \ \langle X_{i'} < t \rangle_{\forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!]} \langle X_{i'} \geq t' \rangle_{\forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!],\ t'=t-d_{i'}}$$

A lower bound modification is explained by all the tasks that saturate the time t and the previous lower bound because the filtering cannot make holes in the domain

$$\langle X_i < t \rangle \to \langle X_i < t' \rangle_{t'=t+d_i} \ \langle X_{i'} < t' \rangle_{\forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!],\ t'=t+d_i} \langle X_{i'} \geq t'' \rangle_{\forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!],\ t''=t'-d_{i'},\ t'=t+d_i}$$

Same idea for the upper bound

Except for the heights of the tasks it is the similar to the explanation in: `http://arxiv.org/abs/1208.3015`

8 Element$(I, \{X_1, \ldots., X_n\}, V)$

8.1 Decomposition

$$X_i = t \iff b^X_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$I = i \iff b^I_i \qquad \forall i \in [\![1,n]\!]$$
$$V = t \iff b^V_t \qquad \forall t \in [\![1,m]\!]$$
$$\neg b^V_t \wedge \neg b^I_i \wedge b^X_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$b^V_t \wedge \neg b^I_i \wedge \neg b^X_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$

8.2 Explanation

$$\langle X_i = t \rangle \to \langle I = i \rangle_{V=t}$$
$$\langle X_i \neq t \rangle \to \langle I = i \rangle_{V \neq t}$$
$$\langle I \neq i \rangle \to \langle X_i \neq t \rangle_{\forall t,\ t \in [\![1,m]\!]} \ \langle V = t \rangle_{\forall t,\ t \in [\![1,m]\!]}$$
$$\langle I \neq i \rangle \to \langle X_i = t \rangle_{\forall t,\ t \in [\![1,m]\!]} \ \langle V \neq t \rangle_{\forall t,\ t \in [\![1,m]\!]}$$
$$\langle V = t \rangle \to \langle X_i = t \rangle_{\forall i,\ i \in [\![1,n]\!]} \ \langle I = i \rangle_{\forall i,\ i \in [\![1,n]\!]}$$
$$\langle V \neq t \rangle \to \langle X_i \neq t \rangle_{\forall i,\ i \in [\![1,n]\!]} \ \langle I = i \rangle_{\forall i,\ i \in [\![1,n]\!]}$$

9 Increasing$(\{X_1, \ldots, X_n\})$

9.1 Decomposition

$$X_i \geq t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$(\neg b_{(i-1)t} \vee b_{it}) \qquad \forall i \in [\![2,n]\!] \forall t \in [\![1,m]\!]$$

9.2 Explanation

$$\langle X_i \geq t \rangle \to \langle X_{i'} \geq t \rangle_{i'=i-1}$$
$$\langle X_i < t \rangle \to \langle X_{i'} < t \rangle_{i'=i+1}$$

10 Decreasing($\{X_1, \ldots, X_n\}$)

10.1 Decomposition

$$
\begin{aligned}
&X_i \geq t \iff b_{it} && \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
&(b_{(i-1)t} \vee \neg b_{it}) && \forall i \in [\![2,n]\!] \forall t \in [\![1,m]\!]
\end{aligned}
$$

10.2 Explanation

$$\langle X_i \geq t \rangle \rightarrow \langle X_{i'} \geq t \rangle_{i'=i+1}$$

$$\langle X_i < t \rangle \rightarrow \langle X_{i'} < t \rangle_{i'=i-1}$$

11 Among($c, \{X_1, \ldots., X_n\}, D_4$)

11.1 Decomposition

$$
\begin{aligned}
&X_i = t \iff b_{it} && \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!] \\
&\bigvee_{t \in D_4} b_{it} \iff b2_i && \forall i \in [\![1,n]\!] \\
&\sum_{i \in [\![1,n]\!]} b2_i = c
\end{aligned}
$$

11.2 Explanation

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t \rangle_{\forall t,\ t \in D_4,\ \forall i,\ i \in [\![1,n]\!]}$$

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t' \rangle_{\forall t',\ t' \neq t,\ t' \in D_4,\ t \in D_4}$$

$$\langle X_i \neq t \rangle \rightarrow \langle X_i = t \rangle_{\exists t,\ t \in D_4 \forall i,\ i \in [\![1,n]\!]}$$

12 Roots($\{X_1, \ldots, X_n\}, I, V$)

12.1 Decomposition

$$
\begin{aligned}
&X_i = t \iff b_{it} && \forall it \\
&\sum_{t \in V} b_{it} = 1 && \forall i \in I \\
&\sum_{t \in D \setminus V} b_{it} = 1 && \forall i \in [\![1,n]\!] \setminus I
\end{aligned}
$$

12.2 Explanation

$V = D_5$ and $D \setminus V = D_6$

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t \rangle_{\forall t,\ t \in D_5}$$

$$\langle X_i = t \rangle \rightarrow \langle X_i \neq t \rangle_{\forall t,\ t \in D_6}$$

$$\langle X_i \neq t \rangle \rightarrow \langle X_i = t \rangle_{\forall t,\ t \in D_5}$$

$$\langle X_i \neq t \rangle \rightarrow \langle X_i = t \rangle_{\forall t,\ t \in D_6}$$

13 Range($\{X_1, \ldots, X_n\}, I, V$)

13.1 Decomposition

$$\begin{aligned} &X_i = t \iff b_{it} && \forall it \\ &\sum_{i \in I} b_{it} \geq 1 && \forall t \in V \\ &\sum_{t \in V} b_{it} = 1 && \forall i \in I \end{aligned}$$

13.2 Explanation

$I = D_5$ and $V = D_6$

$$\langle X_i = t \rangle \to \langle X_i \neq t' \rangle_{\forall t',\ t' \neq t,\ t' \in D_5,\ t \in D_5}$$
$$\langle X_i = t \rangle \to \langle X_i \neq t \rangle_{\forall t,\ t \in D_6}$$
$$\langle X_i \neq t \rangle \to \langle X_i = t \rangle_{\forall t,\ t \in D_6}$$

14 Xor($b1, b2, b3$)

14.1 Decomposition

$$b1 = (b2 \neq b3)$$

14.2 Explanation

$$\langle b1 \rangle \to \langle b2 \rangle \langle \neg b3 \rangle$$
$$\langle b1 \rangle \to \langle b3 \rangle\ \langle \neg b2 \rangle$$
$$\langle \neg b1 \rangle \to \langle b2 \rangle\ \langle b3 \rangle$$
$$\langle \neg b1 \rangle \to \langle \neg b2 \rangle\ \langle \neg b3 \rangle$$

And conversly for $b2$ and $b3$