Feuille de travaux dirigés n° 2
Multi-objective Linear Programming

Exercise 2.1

We consider the following bi-objective linear programme

$$\begin{array}{lllr} \min & -2x_1 + x_2 & & \\ \min & -4x_1 - 3x_2 & & \\ \text{s. t.} & x_1 + 2x_2 & \leq & 10 \\ & x_1 & \leq & 5 \\ & x & \geqq & 0 \end{array}$$

Solve this problem using the parametric simplex method.

Exercise 2.2

We consider the following bi-objective linear programme

$$\begin{array}{lrll} \min & x_1 + 2x_2 & & \\ \min & -2x_2 & & \\ \text{s. t.} & -x_1 + x_2 & \leq & 3 \\ & x_1 + x_2 & \geq & 3 \\ & x & \geqq & 0 \end{array}$$

Solve this problem using the parametric simplex method. What happens ? What can you say about X_E ?

Exercise 2.3

Give examples of MOLPs with the following properties :

1. $p = 2$ objectives and an optimal solution of the weighted sum $LP(\lambda)$ wiht $\lambda \geq 0$ but $\lambda_i = 0$ for $i = 1$ or $i = 2$ is weakly efficient, but not efficient.
2. X_E is a singleton, although X is full-dimensional, i.e. dim $X = n$.
3. $X \neq \emptyset$ but $X_E = \emptyset$.

Exercise 2.4

Given a MOLP with objective matrix C.
Let $x^0 \in X = \{x \geqq 0 : Ax = b\}$. We consider the following LP

$$\begin{array}{llll} \max & e^T z & & \\ \text{s. t.} & Ax & = & b \\ & Cx + Iz & = & Cx^0 \\ & x, z & \geqq & 0 \end{array} \tag{1}$$

where $e = (1, \ldots, 1) \in \mathbb{R}^p$.

1. Show that if $(\hat{x}, \hat{z})$ is an optimal solution of (1) then $\hat{x}$ is an efficient solution of the MOLP.
2. Show that if (1) is unbounded then $X_E = \emptyset$.