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# Matriz Inversa

1)

$$\begin{pmatrix} x & 1 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 \\ y & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$x = 2$

$y = -5$

| | |
|---|---|
| $3x + y$ | $1$ |
| $-x + 2$ | $0$ |

$-x = -2$ $\quad$ $6 + y = 1$

$x = 2$ $\quad$ $y = -5$

$2 + (-5) = -3$

Letra C

# Matriz Inversa

(02)

$$A = \begin{pmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{pmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix}$$

$3 + 0 + k^2 - (1 + 3k)$

$2 - 3k + k^2$

$k^2 - 3k + 2 = 0$

$x' = 1$

$x^2 = 2$

Letra c

③

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$3a + 5c = 1$
$3b + 5d = 0$
$2a + 4c = 0$
$2b + 4d = 1$

$3a - 5 = 1$
$3a = 6$
$a = 2$

$2b + 4d = 1$
$3b + 5d = 0$

$3b + 6d = \frac{3}{2}$
$3b + 5d = 0$

$\begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix}$

Letra C

$3a + 5c = 1$
$2a + 4c = 0$

$1a + 2c = 0$
$3a + 6c = 0$

$-3a + 5c = 1$
$3a + 6c = 0$
$c = -1$

$d = \frac{3}{2}$

$3b + \frac{15}{2} = 0$

$b = -\frac{5}{2}$

④

$$\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix}$$

$x^2 + 20 + 6 - (20 + 2x + 3x)$

$x^2 - 5x + 6 = 0$

$x' = 2$

$x^2 = 3$

Otro A

⑤

$$\left(\begin{array}{ccc|ccc} -1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \cdot(-1) \\ \cdot(-2) \\ \cdot(-1) \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & -1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \cdot 2 \\ \cdot(-2) \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & -1 & 0 & 0 \\ 0 & -1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \\ \cdot(-1) \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \quad \begin{array}{l} \\ \cdot(-1) \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \cdot(-1) \\ \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$\begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$$

Letra B

⑥

$(X \cdot A)^t = B$

$X \cdot A = B^t$ – Para passar A para o outro lado da igualdade, é preciso inverti-lo.

$X = B^t \cdot A^{-1}$

Letra B

⑦

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x+5y \\ 5x+6y \end{pmatrix}$$

$ax + by = 4x + 5y$

$a = 4$
$b = 5$

$cx + dy = 5x + 6y$

$c = 5$
$d = 6$

$$A = \begin{pmatrix} 4 & 5 \\ 5 & 6 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & \frac{5}{4} & \frac{1}{4} & 0 \\ 5 & 6 & 0 & 1 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & \frac{5}{4} & \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & -\frac{5}{4} & 1 \end{array}\right) = \left(\begin{array}{cc|cc} 1 & \frac{5}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 5 & -4 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & 0 & -6 & 5 \\ 0 & 1 & 5 & -4 \end{array}\right) \rightarrow$$ Letra D

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$$\begin{pmatrix} 2 & k \\ -2 & 1 \end{pmatrix} = X$$

$|X| = 2 + 2k$

$$\begin{pmatrix} 2 & k \\ -2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2k+2} & \frac{-k}{2k+2} \\ \frac{1}{k+1} & \frac{1}{k+1} \end{pmatrix}$$

$$2 + 2k = \frac{1}{2k+2}$$

$(2+2k)(2k+2)$
$4k + 4 + 4k^2 + 4k$
$4k^2 + 8k + 4 = 1$
$4k^2 + 8k + 3 = 0$

$x^1 = -\frac{3}{2}$

$x^2 = -\frac{1}{2}$

$-\frac{3}{2} + \frac{-1}{2} = -2$

Letra B

⑨

A) $(A+B).(A-B)$

$A^2 - AB + BA - B^2$

A ordem dos fatores matriciais altera o produto, portanto, impossível anular.

b) $(A+B)^2 = (A+B).(A+B)$

$A^2 + AB + BA + B^2$

Se $AB = BA$, $(A+B)^2 = A^2 + 2AB + B^2$

Resposta: $AB = BA$

c)

$$\frac{|A|}{|-A|}$$

$$A = \begin{vmatrix} i & j \\ k & \ell \end{vmatrix} \qquad -A = \begin{vmatrix} -i & -j \\ -k & -\ell \end{vmatrix}$$

$$|A| = i\ell - kj$$
$$|-A| = i\ell - kj$$

$$\frac{i\ell - kj}{i\ell - kj} = 1$$

d) Assim como as matrizes são inversas, os determinantes delas também são inversos entre si, portanto:

$$\det B = \frac{1}{\det A}$$