

Discussão de sistemas lineares - Sistemas homogêneos

Discussão de Sistemas Lineares

①

$$\begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$$

$$\begin{array}{cc|c} a & 4 & 1 \\ 1 & 2 & b \end{array} \quad \rightarrow \quad \begin{array}{cc|c} 1/a & 4/a & 1/a \\ -1/a & -2/a+4 & -ab+1 \end{array}$$

$$y = \frac{-ab+1}{-2a+4}$$

Se $a = 2$, o denominador irá ser zero.
Com o denominador zero, para termos
um resultado indeterminado, basta b
ter o valor de $\frac{1}{2}$. Portanto

Resposta = {Linha B}

(2)

$$\begin{cases} x + ky = 1 \\ kx + y = 1 - k \end{cases}$$

$$-k \rightarrow \begin{vmatrix} 1 & k & 1 \\ k & 1 & 1-k \end{vmatrix} = \begin{vmatrix} 1 & k \\ -1k^2 & -1k \end{vmatrix} = 1k + 1k^2$$

$$\begin{aligned} -1k^2 + 1 &= 1k + 1k^2 + 1 \\ x &= \frac{1k + 1k^2 + 1}{-1k^2 + 1} \end{aligned}$$

Porém
não possui
resposta única,
portanto

Resposta = ~~Seta D~~

$$x = \frac{k + k^2 + 1}{-k^2 + 1}$$

Não pode ser indeterminado
pois nunca resulta em 0

Nem sempre admite solução,
como no caso $k = 1$ ou
 $k = -1$

③

$$\begin{cases} x + 2y + cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$$

$$a) = \begin{bmatrix} x & 2y & cz & | & 1 \\ 0 & y & z & | & 2 \\ 3x & 2y & 2z & | & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & c & | & 1 \\ 0 & 1 & 1 & | & 2 \\ 3 & 2 & 2 & | & -1 \end{bmatrix}$$

$$\begin{array}{ccc|cc} 1 & 2 & c & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 3 & 2 & 2 & 3 & 2 \end{array}$$

$$(2 + 6 + 0) - (3c + 2)$$

$$8 - 3c - 2$$

$$D = 6 - 3c$$

b)

$$\begin{array}{ccccc|c} -2 & 1 & 2 & c & 1 & 1 \\ & 0 & 1 & 1 & 2 & 2 \\ -2 & 3 & 2 & 2 & -1 & -1 \end{array}$$

$$= \begin{cases} 1 + 0 - 2 + c = -3 \\ \end{cases}$$

~~scribbled out~~

$$3 + 0 + 0 = -5$$

$$y = \frac{4}{6-3c}$$

$$c \in \mathbb{R} - \{2\}$$

$$\begin{array}{ccc} + & -3x & +6-3c = 4 \\ & 3x & 0 = -5 \end{array}$$

★
+ todos os reais, exceto 2

$$(6-3c)y = 4$$

11
(4)

$$\begin{cases} x - y = k \\ 12x - ky + z = 1 \\ 36x + kz = 2 \end{cases}$$

~~$$\begin{cases} x - y = k \\ 12x - ky + z = 1 \\ 36x + kz = 2 \end{cases}$$~~

$$\begin{array}{cccc|c} 1 & -1 & 0 & 1 & k \\ 12 & -k & 1 & 1 & 1 \\ 36 & 0 & k & 1 & 2 \end{array}$$

~~$$\begin{array}{cccc|c} 1 & -1 & 0 & 1 & k \\ 12 & -k & 1 & 1 & 1 \\ 36 & 0 & k & 1 & 2 \end{array}$$~~

$$\begin{aligned} (-12k + 36) + k^2 &= 2 - k \\ \bullet 1 \quad -1 &= k \end{aligned}$$

$$\begin{aligned} + \quad & (-12k + 36) + k^2 = 2 - k \\ & (12k - 36) + (-12k + 36) = (12k^2 - 36k) \end{aligned}$$

$$y(-12k + 36 + k^2) = 12k^2 - 36k + 2 - k$$

$$y = \frac{12k^2 - 36k + 2 - k}{-12k + 36 + k^2}$$

Però 0 quando $k = 6$
Portanto Resposta =
Letra E

(5)

$$\begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ x + 2y - z = -5 \end{cases}$$

$$\begin{array}{r} \textcircled{1} \\ \begin{array}{rrrr} -1 & 1 & & = 6 \\ 2 & 1 & -1 & = -3 \\ 1 & 2 & -1 & = -5 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{rrrr} 0 & 3 & -3 & = -15 \\ 0 & 3 & -2 & = -11 \end{array} \end{array}$$

$$\begin{array}{l} 3y - 12 = -15 \\ 3y = -3 \\ y = -1 \end{array}$$

$$\begin{array}{l} 0 + 3 - 3 = -15 \\ 0 - 3 + 2 = 11 \end{array}$$

$$\begin{array}{l} z(-1) = -4 \\ z = 4 \end{array}$$

$$\begin{array}{l} x + 1 + 4 = 6 \\ x = 1 \end{array}$$

$$x \cdot y \cdot z = 1 \cdot -1 \cdot 4 = -4$$

Resultado:

Retra B

⑥

$$\begin{cases} x + y + z = k \\ kx + y + z = 1 \\ x + y - z = k \end{cases}$$

$$\begin{array}{ccc|c} \textcircled{1} & \textcircled{1} & 1 & = k \\ k & 1 & 1 & = 1 \\ 1 & 1 & -1 & = k \end{array}$$

~~Handwritten scribbles~~

$$(k-1) + 0 + 0 = 1-k$$

$$x(k-1) = 1-k$$

$$x = \frac{1-k}{k-1}$$

Se $k=1$, numerador e denominador são zero, e os resultados são múltiplos.

Seta D

7

$$\begin{aligned}x + y + z &= 1 \\mx - 2y + 4z &= 5 \\m^2x + 4y + 16z &= 25\end{aligned}$$

$$\begin{array}{ccc|c}1 & 1 & 1 & 1 \\m & -2 & 4 & 5 \\-4 & m^2 & 4 & 25\end{array}$$

$$(m+2) + 0 + 6 = 7$$

$$(m^2-4) + 0 + 12 = 21$$

$$(-2m-4) - 12 = -14$$

$$(m^2-4) + 12 = 21$$

$$m^2 - 4 - 2m - 4 = 14$$

$$X (m^2 - 2m - 8) = 14$$

$$m^2 - 2m - 8 = 0$$

$$x^1 = -2$$

$$x^2 = 4$$

$$-2 + 4 = 2$$

Setro B

Sistemas Lineares Homogêneos

①

$$\begin{pmatrix} 1 & 7 \\ 7 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = k \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x + 7y \\ 7x + y \end{pmatrix} = k \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

$$\begin{aligned} (x - kx) + 7y &= 0 \\ 7x + (y - ky) &= 0 \end{aligned}$$

$$\begin{aligned} (1 - k) + 7 &= 0 \\ 7 + (1 - k) &= 0 \end{aligned}$$

$$k^2 - 2k - 48 = 0$$

$$k^1 = -6$$

$$k^2 = 8$$

Seto E

2

$$3x + 4y - z = 0$$

$$2x - 2y + 3z = 0$$

$$x + y = 0$$

$$\begin{array}{ccc|c} 3 & 4 & -1 & 0 \\ 2 & -1 & 3 & 0 \\ 1 & 1 & 0 & 0 \end{array} \quad \rightarrow \quad \begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & 4 & -1 & 0 \end{array} \quad \rightarrow \quad \begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 2 & -1 & 0 \end{array}$$

$$0 + 0 = 0$$



$\frac{0}{0} = \text{infinites}$
 $\frac{0}{0} = \text{soluções}$

Sistema D

③

$$x + y + z = 0$$

$$kx + 3y + 4z = 0$$

$$x + ky + 3z = 0$$

$$\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ k & 3 & 4 & k & 3 \\ 1 & k & 3 & 1 & k \end{array}$$

~~Handwritten scribbles~~

~~Handwritten scribbles~~

$$9 + 4 + x^2 - (3 + 4x + 3x)$$

$$10 - 7x + x^2$$

$$x^2 - 7x + 10 = 0$$

$$x^1 = 5$$

$$x^2 = 2$$

$$x^1 + x^2 = 7$$

Setra D

(4)

$$x + kz = 0$$

$$kx + y = 0$$

$$x + ky = 0$$

$$\begin{array}{cccc} \textcircled{1} & 0 & k & = 0 \\ -k & k & 1 & 0 = 0 \\ & 1 & k & 0 = 0 \end{array}$$

$$k+1 \neq 0$$

$$-k - k^2 \neq 0$$

$$-k^2 + 1 = 0$$

$$\frac{0}{-k^2 + 1} = 0$$



$$k^1 = 1$$

$$k^2 = -1$$

Letra A

⑤

$$-x + 2y - 3 = 0$$

$$3x - y + 3 = 0$$

$$2x - 4y + 6 = 0$$

$$-x + 2y = 3$$

$$3x - y = -3$$

$$2x - 4y = -6$$

$$R \begin{array}{cc|c} -1 & 2 & 3 \end{array}$$

$$\begin{array}{cc|c} 3 & -1 & -3 \end{array}$$

$$\begin{array}{cc|c} 2 & -4 & -6 \end{array}$$

$$-3x + 6y = 9$$

$$3x - 1y = -3$$

$$5y = 6$$

$$y = \frac{6}{5}$$

Setra B