Proper cost allocation of support departments.

Step-down method: Does the order matter?

Does the order matter?

- The costs allocated to Cars and Trucks differ by only $27,000 depending on whether telecommunications or IT is chosen first.
- The difference of $27,000 is less than 1 percent of the total costs allocated.

Does the order matter?

- However, very different incentives result depending on which method is used.
- Allocated costs are taxes, and taxes effect behavior.
- And these can lead to the **Death Spiral** if the tax is too high!

Illustration:

- To illustrate lets expand the telecommunications and IT example.
- Suppose the allocation base in telecommunications is the number of telephones in each department, and
- in IT the allocation base is the number of gigabytes of disk space used.

Illustration:

- Transfer prices are to be established for telephones and gigabytes.
- Allocated costs will be used to compute the transfer prices.

The allocation bases:

	Allocation base
Telecomm	3,000 Telephones
IT	12 million gigabytes

Cost allocated per phone

Number of phones

	Direct	Step, Telecomm first	Step, IT first
Telecoms	–	–	–
IT	–	20% × 3,000 = 600	–
Cars	40% × 3,000 = 1,200	40% × 3,000 = 1,200	40% × 3,000 = 1,200
Trucks	30% × 3,000 = 900	30% × 3,000 = 900	30% × 3,000 = 900
Phones	2,100	2,700	2,100

- *Note: that telecom is always '–' here because we are considering how to allocate it's costs. The in the 'IT first' column the telecom costs already include IT costs.*

Cost allocated per phone

	Direct	Step, Telecomm first	Step, IT first
Cost per phone	\$2M/2,100 = \$ 952	\$2M/2,700 = \$ 741	\$3.765M/2,100 = \$1,793
Number of phones: Cars	1,200	1,200	1,200
Telecoms charged to Cars	\$1.143	\$0.889	\$ 2.151

Does the order matter?

The order can lead to large changes in the 'tax' on the allocation base!

Cost allocated per Gigabyte of Storage

Number of Gigabytes of Storage

	Direct	Step, Telecomm first	Step, IT first
Telecoms	–	–	25% × 12 = 3.0
IT	–	–	–
Cars	35% × 12 = 4.2	35% × 12 = 4.2	35% × 12 = 4.2
Trucks	25% × 12 = 3.0	25% × 12 = 3.0	25% × 12 = 3.0
Gigs	7.2	7.2	10.2

- *Note: that IT is always '–' here because we are considering how to allocate it's costs. The in the 'Telecom first' column the IT costs already include Telecom costs.*

Cost allocated per Gigabyte of Storage

	Direct	Step, Telecomm first	Step, IT first
Cost per gig	\$6/7.2 = \$0.833	\$6.44/7.2 = \$0.895	\$6/10.2 = \$0.588
Number of gigs in Cars	4.2	4.2	4.2
IT charged to Cars	\$3.5	\$3.759	\$2.470

Cost allocated per Giga of storage (Millions except cost per Gb)

Consider the impact on behavior:

- The sequence of service departments in the step-down method changes the costs of each service.
- Because the cost per phone (which represents the transfer price) varies depending whether or not it includes IT costs,
- the cost allocation scheme affects the decision of each department to add or drop phones.
- The same conclusions hold for the information technology department.

Does the order matter?

- Note the wide variation in cost per gigabyte.
- The cost varies from \$0.588 per gigabyte under the step-down method with IT chosen first
- to \$0.895 under the step-down method with telecommunications chosen first.
- **The step-down method is an example of a sub-optimal status quo.**

The central issues with the step-down method:

- The sequence used is arbitrary and large differences can result in the cost per unit of service using different sequences.
- Also, the step-down method ignores the fact that although departments earlier in the sequence use service departments later in the sequence, earlier departments are not allocated these costs.
- This creates an artificially low tax on the first department and an artificially high tax on the second department.
- Get this wrong and risk the death spiral.

The reciprocal method:

- **Solves the problem by making the allocation simultaneously**

Start by setting up the equations

Costs before allocation:

Consumer:	Telecoms	IT	Cars	Trucks	Total
Provider:					
Telecoms	10%	20%	40%	30%	100%
IT	25%	15%	35%	25%	100%
Cost incurred	\$2M	\$6M			8M
Total to allocate:	T	I			

I and T are unknown because they include unallocated costs. We need to set up a system of equations and solve it to get these numbers.

Telecoms equation:

- T = Telecom Cost incurred, plus the portion of those costs that Telecom incurred, and the portion of IT that Telecom incurred.
- The equation is:

$$T = \$2M + 0.10 \times T + 0.25 \times I$$

- Notice that the $0.10 \times T$ term is decreasing the amount of T to allocate, and $0.25 \times I\$$ is increasing it.
- The equation simplifies to:

$$0.9 \times T = \$2M + 0.25 \times I$$

$$T = \$2M/.9 + 0.25/.9 \times I$$

IT equation:

- I = IT cost incurred, plus the portion of those costs that IT itself incurred, and the portion of Telecom that IT incurred.
- The equation is:

$$I = \$6.0 + .20 \times T + .15 \times I$$

$$.85I = \$6.0 + .20 \times T$$

- Notice that the $.15 \times I$ term is decreasing the amount of I to allocate.

Now algebra :)

- Now we have two equations and two unknowns and we can solve by hand.
- As a proof of concept now we will use Google's Colab platform to solve this

Pass the following to the colab notebook:

```
# load symbolic python
import sympy as sp
# initialize I and T
I, T = sp.symbols('I, T')
```

Now define the equations

```
# - use the comma for '='
# - and simplify as little as you like
tel_eq = sp.Eq(
    2 + .25 * I , .9 * T
)
it_eq = sp.Eq(
    6 + .2 * T , .85 * I
)
```

Now ask for a solution

```
solution = sp.solve((tel_eq, it_eq),(I,T))
```

yields:

```
{I: 8.11188811188811, T: 4.47552447552448}
```

- This approach scales until google starts charging you! And after that until you run out of cash :)
- If we really wanted to have fun we could load weights and costs from a spreadsheet and do the calculation with matrix notation for hundreds of departments.

Service department cost allocation:

Consumer:	Telecoms	IT	Cars	Trucks	Total
Provider:					
Costs before allocation	\$2M	\$6M			\$8M
Telecoms tot. to alloc.	\$(4.475)				\$(4.475)
Amount allocated from Telecoms:	\$4.475 × .10 = \$.448	\$4.475 × .20 = \$.895	\$4.475 × .40 = \$1.790	\$4.475 × .30 = \$1.34.	\$4.475
IT tot. to alloc		\$(8.112)			\$(8.112)

Consumer:	Telecoms	IT	Cars	Trucks	Total
Amount allocated from IT:	\$8.112 × .25 = \$2.028	\$8.112 × .15 = \$1.217	\$8.112 × .35 = \$2.839	\$8.112 × .25 = \$2.028	\$8.112
Total overhead allocated	0.000	0.000	\$4.629	\$3.371	\$8.000

Cost per phone:

	Telecoms	IT	Cars	Trucks	Total
Allocated Telecoms costs (M)	\$ 0.448	\$ 0.448	\$1.790	\$1.343	\$ 4.475
÷ Number of phones	300	300	1,200	900	3,000
Cost per phone (M)	**\$ 1,492**	**\$ 1,492**	**\$1,492**	**\$1,492**	**\$ 1.492**
Allocated IT costs	\$ 2.028	\$ 1.217	\$2.839	\$2.028	\$ 8.111
÷ Number of gigabytes (M)	3.0	1.8	4.2	3.0	12.0
Cost per gigabyte	**\$ 0.676**	**\$ 0.676**	**\$0.676**	**\$0.676**	**\$ 0.676**