Lecture 2: The Nature of Costs

ACCT 3210

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We are interested in how costs respond to business decisions? Why?

- Costs are resources. Business decisions that have costs require resources so we need to make sure we have the resources when required.
- Cost, Volume, Profit analysis. Which we will talk about in the following lectures, is based on the relationship between a key decision—the volume decision (how much to produce)—and costs.
- We have to know where the resources are and where they need to them to be in order to understand the resource bargins that need to be negotiated.

Why do we care about costs?

Profit = Revenue - Total Cost

Total Cost = Fixed Cost + $\sum$ (Cost Per Unit $\times$ # Units)

Revenue = $\sum$ (Price Per Unit $\times$ # Units)

$\sum$*: Summed across all products.*

- Apple chooses how many devices to produce based projections of how costs and revenues will respond to this decision.
- Today we will focus on modelling the cost portion of this equation.

Notate Bene:

- Some of you will think that I use too many “U.S.” examples.
- Please note that most of my examples reference Apple’s *production*.
- If you think that this is a U.S. example, I have some very surprising news for you. :]

Why do we care about profit?

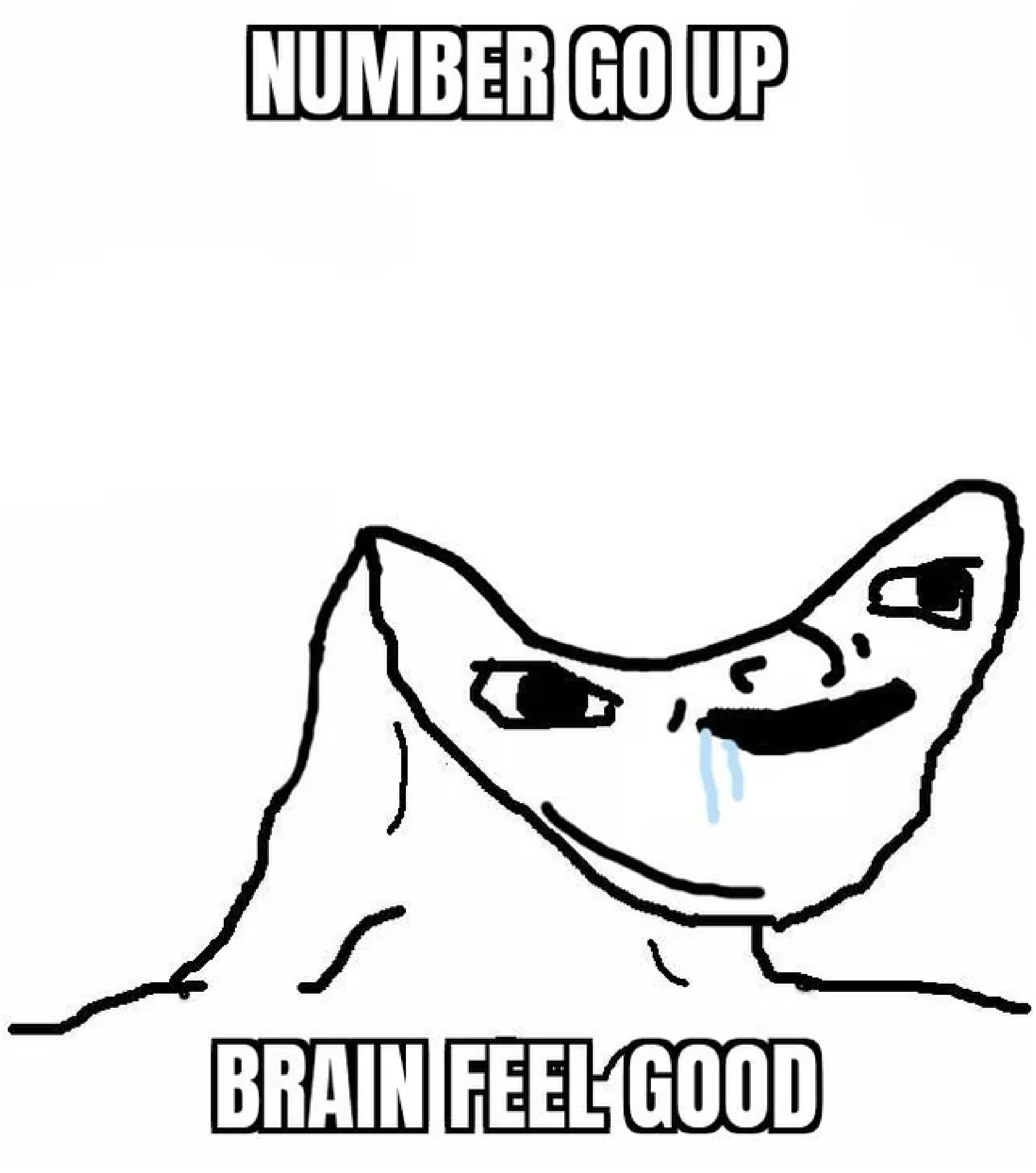

Figure 1: Zimmerman and Friedman's vision of the firm

Cost Functions

Models of the firm that we use to predict how cost will respond to various actions, which we express as variables in the model.

A simple example where everything is linear:

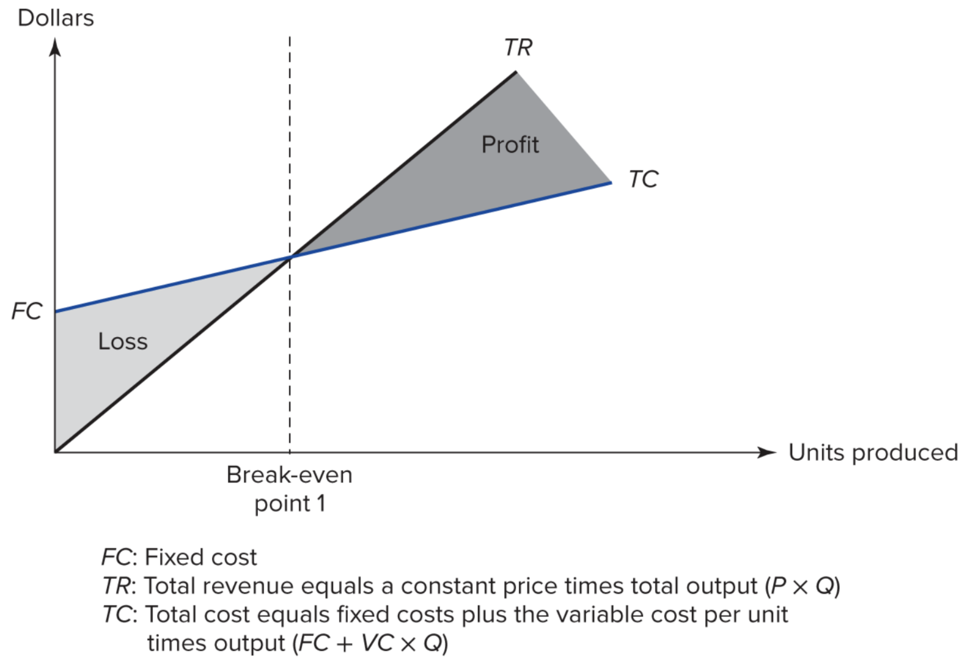

Figure 2: Linear Cost and Revenue Functions

Let's look at some cost terms in the context of a single-product firm.

- **Fixed cost (FC):** Cost at zero output. Also used to refer to costs that do not vary with output (or some other driver).
- **Variable cost (VC):** Cost that vary with output (or some other driver).
- **Marginal cost (MC):** The cost per unit at the margin (i.e. the point of interest). This is the rate of change of cost at the margin.
- **Incremental cost (IC):** The cost of producing the next unit. Often MC and IC have the same value, but they are slightly different things!

- **Average cost (ATC):** Total Cost of producing the output over the number of units of output. This is a simple average for single product firms. It is not simple at all for multi-product firms.

iPRS here.

- **Cost object:** An activity or item for which we want to measure cost.
- **Cost driver:** Any factor or activity whose change leads to a change in costs.

A simple example where everything is linear:

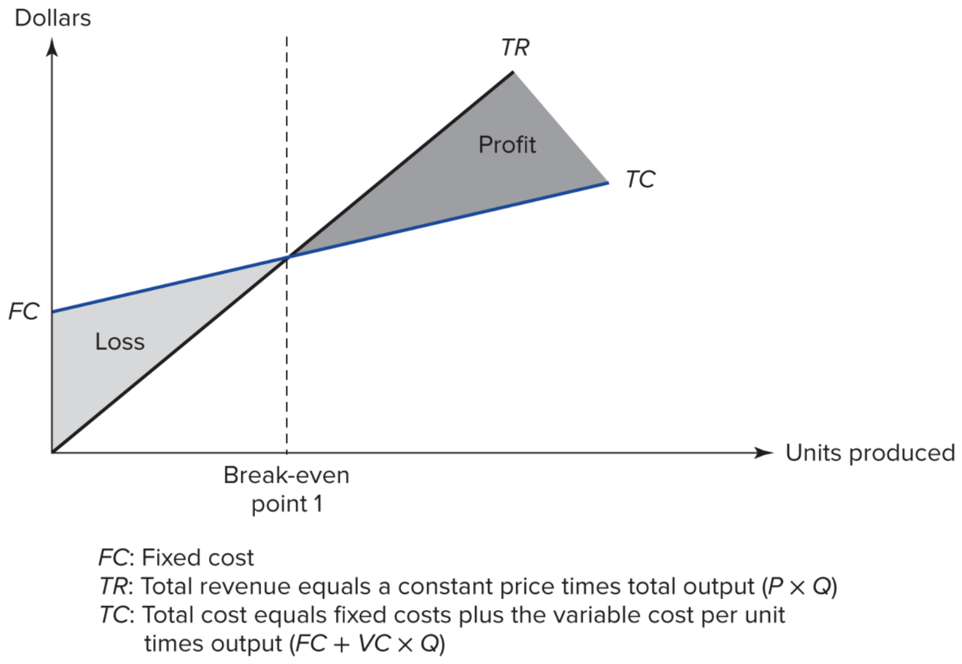

Figure 3: Linear Cost and Revenue Functions

Can you see anything unrealistic in this graph?

Most firms' costs are non-linear

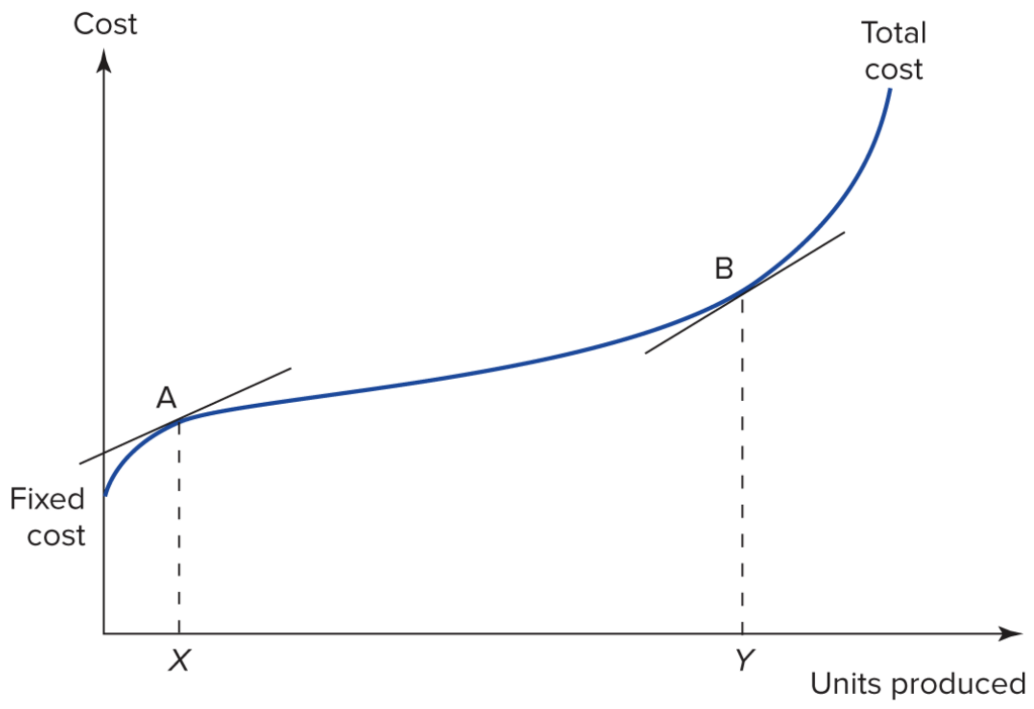

Figure 4: Non-linear Cost and Revenue Functions

Most firms' costs are non-linear

Questions about the previous slides:

1. What is the economic significance of the area to the left of the line from X to A?
2. What is the economic significance of the area between X->A and Y->B?
3. What is the economic significance of the area to the right of Y->B?

Note: I am not providing the answers here because I want you to ponder these questions.

Cost Types

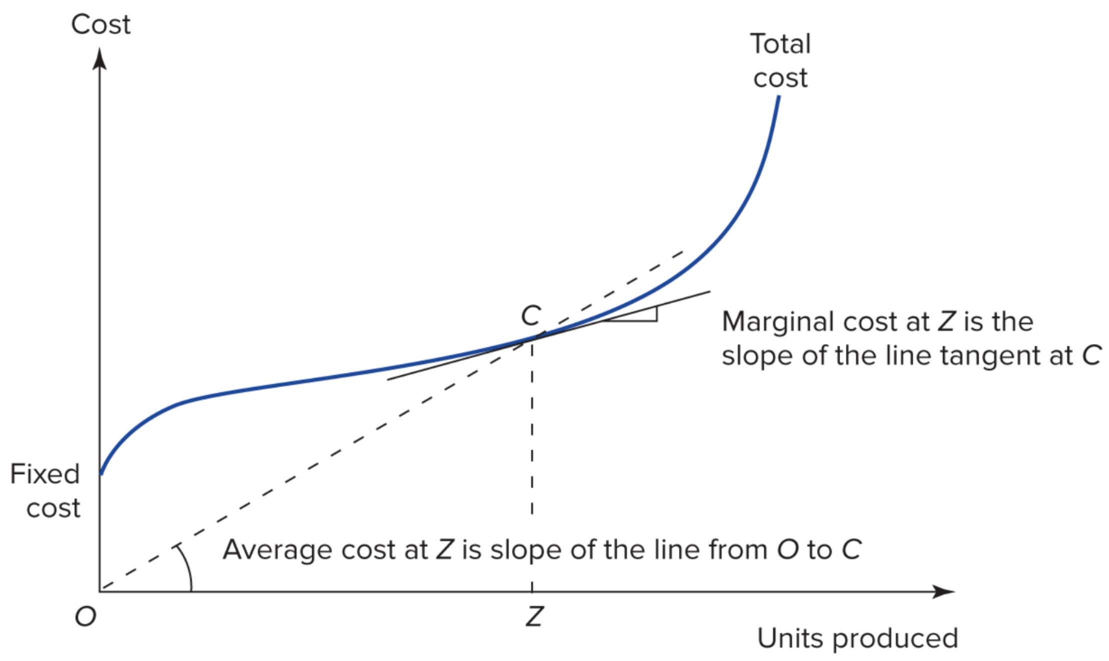

Figure 5: Marginal and Incremental Costs

1. If marginal cost is the slope of the tangent line and incremental cost is the cost of one unit, then when are they the same on this graph?
2. When are they different?

At any scale there is a range within which production is efficient

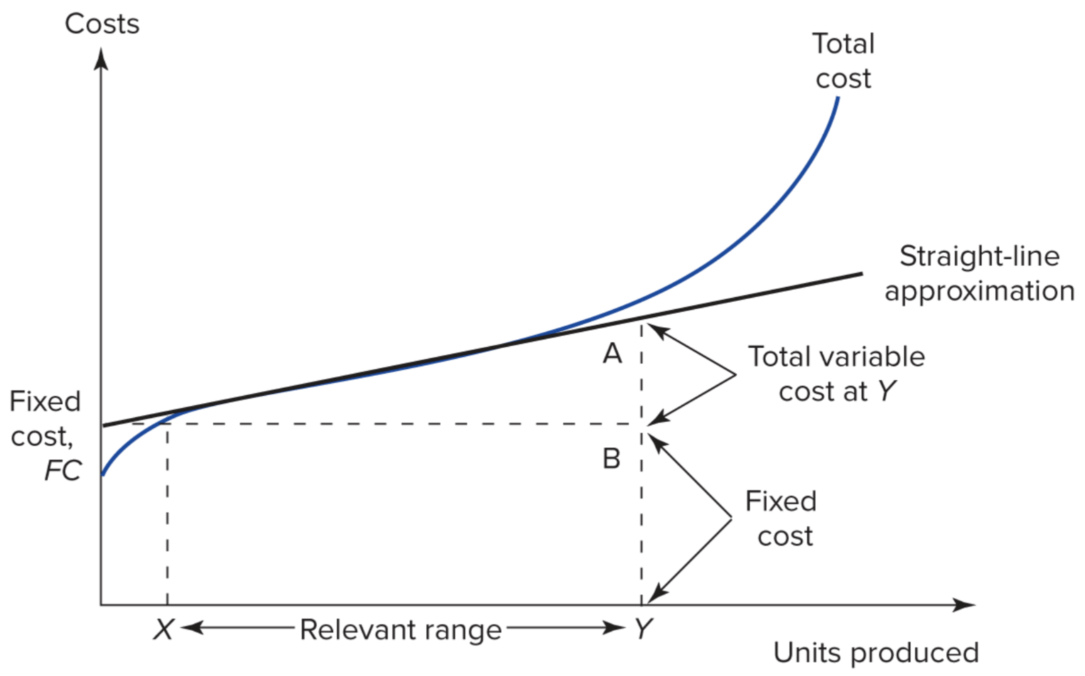

Figure 6: Producing outside of this range is less efficient, unless we change the scale of the firm.

Costs are not always smooth

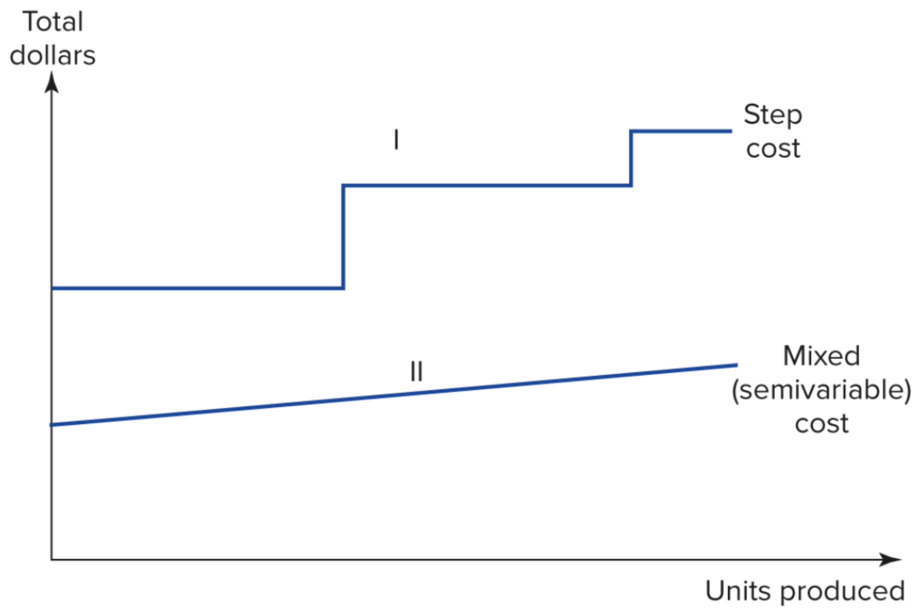

Figure 7: Most firms have a mix of these attributes. Steps occur when the scale of the firm changes (e.g. we add a new factory).

Let's talk about the homework assignment!

- Download the Excel File here
- **Please note that Python, Excel, or any other set of tools can be used on the homework!**

Cost in a Multiproduct Firm:

Consider three firms that produce two products with quantities denoted q_1 and q_2. The three distinct cost functions are:

- $C_1(q_1, q_2) = 10q_1 + 5q_2$
- $C_2(q_1, q_2) = 6q_1 + q_1^2 + 8q_2 + q_2^2$
- $C_3(q_1, q_2) = 7q_1 + 9q_2 + q_1 q_2$

Cost in a Multiproduct Firm:

1. Fill in the following table for each of the cost functions. (Incremental cost refers to the incremental cost of one additional unit of output.)

Output	Total Cost	Average Cost	Marginal Cost	Incremental Cost
q_1, q_2		q_1, q_2	q_1, q_2	q_1, q_2
100, 50				
60, 50				
40, 50				
30, 10				
30, 50				
30, 70				

Total cost:

- Plug the output data into each cost function!

Let's fill this out using Python (**don't panic**), Excel (also **don't panic**).

We'll start with Excel

Reference items:

- **Marginal cost (MC):** The cost per unit at the margin (i.e. the point of interest). This is the rate of change of cost at the margin.
- **Incremental cost (IC):** The cost of producing the next unit. Often MC and IC have the same value, but they are slightly different things!
- **Average cost (ATC):** Total Cost of producing the output over the number of units of output. This is a simple average for single product firms. It is not simple at all for multi-product firms.
- $C_1(q_1, q_2) = 10q_1 + 5q_2$
- $C_2(q_1, q_2) = 6q_1 + q_1^2 + 8q_2 + q_2^2$
- $C_3(q_1, q_2) = 7q_1 + 9q_2 + q_1 q_2$

Now Python

You can follow along in colab.

Set up: load libraries and data

First we need to load some data science libraries:

```python
# import pandas so we can put everything into a nice friendly data frame
import pandas as pd
import numpy as np

# lets put what we know into a dict (python dicts are POWERFUL use them when in doubt)
outputs = {
    "q1" : [100, 60, 40, 30, 30, 30],
    "q2" : [50, 50, 50, 10, 50, 70],
}
outputs
```

```
{'q1': [100, 60, 40, 30, 30, 30], 'q2': [50, 50, 50, 10, 50, 70]}
```

Table for Firm 1

```python
# use pandas to make that into a dataframe
cost_frame_1 = pd.DataFrame(outputs)
cost_frame_2 = pd.DataFrame(outputs)
cost_frame_3 = pd.DataFrame(outputs)
cost_frame_1
```

	q1	q2
0	100	50
1	60	50
2	40	50
3	30	10
4	30	50
5	30	70

Next write down our cost functions as... well... functions

```python
# write down our cost functions
## total cost
def cost_1(q1,q2):
  return 10 * q1 + 5 * q2
def cost_2(q1,q2):
  # note that we have to use ** in place of ^ here
  return 6 * q1 + q1**2 + 8 * q2 + q2**2
def cost_3(q1,q2):
  return 7 * q1 + 9 * q2 + q1 * q2

cost_1(1,2)
```

```
20
```

Notice how close this is to how you might type this on your phone!

Then we can use those functions to calculate average cost

- We can just pass q1,q2 as arguments to the cost functions

```python
cost_1(100,50)
```

```
1250
```

```python
cost_2(100,50)
```

```
13500
```

```python
cost_3(100,50)
```

```
6150
```

Now we need to do this to all the data in the data frames

slow simple way:

```
TotalCost1 = []
for q1,q2 in zip(outputs['q1'],outputs['q2']):
    TotalCost1.append(cost_1(q1,q2))
print(TotalCost1)
outputs
```

```
[1250, 850, 650, 350, 550, 650]
```

```
{'q1': [100, 60, 40, 30, 30, 30], 'q2': [50, 50, 50, 10, 50, 70]}
```

```
cost_1(100,50)
```

```
1250
```

less simple but faster way

```
TotalCost1 = [cost_1(q1,q2) for q1,q2 in zip(outputs['q1'],outputs['q2'])]
print(TotalCost1)
```

```
[1250, 850, 650, 350, 550, 650]
```

super fast way that scales to large datasets

```
cost_frame_1["Total Cost"] = np.vectorize(cost_1)(cost_frame_1['q1'],cost_frame_1['q2'])
cost_frame_1
```

	q1	q2	Total Cost
0	100	50	1250
1	60	50	850
2	40	50	650
3	30	10	350
4	30	50	550
5	30	70	650

```
# we can do this for the other to firms:
cost_frame_2["Total Cost"] = np.vectorize(cost_2)(cost_frame_2['q1'],cost_frame_2['q2'])
cost_frame_3["Total Cost"] = np.vectorize(cost_3)(cost_frame_3['q1'],cost_frame_3['q2'])
cost_frame_2
```

	q1	q2	Total Cost
0	100	50	13500
1	60	50	6860
2	40	50	4740
3	30	10	1260
4	30	50	3980
5	30	70	6540

Average cost

The average cost of a each product is the total cost for producing **that product alone** divided by the number of units produced.

For firm 1 & 2 this is straightforward, each firm has an AC for each product where we plug in zero for the other product:

- $AC_1(q_1) = (10q_1 + 0)/q_1$
- $AC_1(q_2) = (0 + 5q_2)/q_2$
- $AC_2(q_1) = (6q_1 + q_1^2 + 0 + 0)/q_1$
- $AC_2(q_2) = (0 + 0 + 8q_2 + q_2^2)/q_2$

What about firm 3?

$$C_3(q_1, q_2) = 7q_1 + 9q_2 + q_1 q_2$$

What does $q_1 \times q_2$ mean?

- when two products are multiplied like this we often refer to it as an “interaction”
- Plugging in zero no longer separates the costs.
- Calculating the average cost for each product requires us to separate the costs of the products.

- When there are interactions between products their costs are **inseparable**!!
- So "average cost" is no longer a meaningful number!

One way to think of this is that average cost requires us to pretend that the firm only produces one product. When we can separate costs then this pretend firm tells us something about the real firm. When we cannot separate costs, this pretend firm **does not tell us anything about the real firm**!!

One way to do this in python is to write a function

```python
# avg cost by product
def avg_cost(cost_function,q1=0,q2=0):
    """
    cost_function: the cost function you are averaging
    pass either q1 or q2 but not both to tell which product to use
    """
    if q1!=0 & q2!=0:
        print("only pass one nonzero argument")
        return None
    else:
        return cost_function(q1,q2) / (q1+q2)
```

Firm 1

```python
# average cost the fast way
cost_frame_1["AC q1"] = np.vectorize(avg_cost)(cost_1,q1=cost_frame_1['q1'])
cost_frame_1["AC q2"] = np.vectorize(avg_cost)(cost_1,q2=cost_frame_1['q2'])
cost_frame_1
```

	q1	q2	Total Cost	AC q1	AC q2
0	100	50	1250	10.0	5.0
1	60	50	850	10.0	5.0
2	40	50	650	10.0	5.0
3	30	10	350	10.0	5.0
4	30	50	550	10.0	5.0
5	30	70	650	10.0	5.0

Firm 2

```
# average cost the fast way
cost_frame_2["AC q1"] = np.vectorize(avg_cost)(cost_2,q1=cost_frame_2['q1'])
cost_frame_2["AC q2"] = np.vectorize(avg_cost)(cost_2,q2=cost_frame_2['q2'])
cost_frame_2
```

	q1	q2	Total Cost	AC q1	AC q2
0	100	50	13500	106.0	58.0
1	60	50	6860	66.0	58.0
2	40	50	4740	46.0	58.0
3	30	10	1260	36.0	18.0
4	30	50	3980	36.0	58.0
5	30	70	6540	36.0	78.0

Marginal Cost

The marginal cost is the derivative of the cost function wrt. the product.

$$C_1(q_1, q_2) = 10q_1 + 5q_2$$
$$MC_1(q_1) = 10$$
$$MC_1(q_2) = 5$$

$$C_2(q_1, q_2) = 6q_1 + q_1^2 + 8q_2 + q_2^2$$
$$MC_2(q_1) = 6 + 2q_1$$
$$MC_2(q_2) = 8 + 2q_2$$

$$C_3(q_1, q_2) = 7q_1 + 9q_2 + q_1 q_2$$
$$MC_3(q_1) = 7 + q_2$$
$$MC_3(q_2) = 9 + q_1$$

- This might help with the intuition for the average cost in this case!

Hate Calculus?

let's make python do the work

```
# we'll use symbolic python
import sympy as sp
# we need to tell it which symbols to use
q1,q2 = sp.symbols('q1 q2')
q1
```

q_1

```
# sympy can take the derivative for us
c1 = "10 * q1 + 5 * q2"
s_mcost_1_q1 = sp.diff(c1 , q1)
s_mcost_1_q2 = sp.diff(c1 , q2)
print(s_mcost_1_q1,s_mcost_1_q2)
```

```
10 5
```

```
# and we can convert that to a function
mcost_1_q1 = sp.lambdify(q1,s_mcost_1_q1)
mcost_1_q2 = sp.lambdify(q2,s_mcost_1_q2)
mcost_1_q1(100),mcost_1_q2(100)
```

```
(10, 5)
```

Firm 1

```
# marginal cost
cost_frame_1["MC q1"] = np.vectorize(mcost_1_q1)(cost_frame_1['q1'])
cost_frame_1["MC q2"] = np.vectorize(mcost_1_q2)(cost_frame_1['q2'])
cost_frame_1
```

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2
0	100	50	1250	10.0	5.0	10	5
1	60	50	850	10.0	5.0	10	5

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2
2	40	50	650	10.0	5.0	10	5
3	30	10	350	10.0	5.0	10	5
4	30	50	550	10.0	5.0	10	5
5	30	70	650	10.0	5.0	10	5

Firm 2

```
q1,q2 = sp.symbols('q1 q2')
# sympy can take the derivative for us
c2 = "6 * q1 + q1**2 + 8 * q2 + q2**2"
s_mcost_2_q1 = sp.diff(c2 , q1)
s_mcost_2_q2 = sp.diff(c2 , q2)
print(s_mcost_2_q1,s_mcost_2_q2)
# and we can convert that to a function
mcost_2_q1 = sp.lambdify(q1,s_mcost_2_q1)
mcost_2_q2 = sp.lambdify(q2,s_mcost_2_q2)
mcost_2_q1(100),mcost_2_q2(50)
```

```
2*q1 + 6 2*q2 + 8
```

```
(206, 108)
```

Firm 2 Table

```
# marginal cost
cost_frame_2["MC q1"] = np.vectorize(mcost_2_q1)(cost_frame_2['q1'])
cost_frame_2["MC q2"] = np.vectorize(mcost_2_q2)(cost_frame_2['q2'])
cost_frame_2
```

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2
0	100	50	13500	106.0	58.0	206	108
1	60	50	6860	66.0	58.0	126	108
2	40	50	4740	46.0	58.0	86	108
3	30	10	1260	36.0	18.0	66	28
4	30	50	3980	36.0	58.0	66	108

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2
5	30	70	6540	36.0	78.0	66	148

Firm 3

```
q1,q2 = sp.symbols('q1 q2')
# sympy can take the derivative for us
c3 = "7*q1 + 9*q2 + q1*q2"
s_mcost_3_q1 = sp.diff(c3 , q1)
s_mcost_3_q2 = sp.diff(c3 , q2)
print(s_mcost_3_q1,s_mcost_3_q2)
# and we can convert that to a function
# note tht we flip the inputs to match the function
mcost_3_q1 = sp.lambdify(q2,s_mcost_3_q1)
mcost_3_q2 = sp.lambdify(q1,s_mcost_3_q2)
mcost_3_q1(50),mcost_3_q2(100)
```

```
q2 + 7 q1 + 9
```

```
(57, 109)
```

Firm 3 Table

```
# marginal cost
cost_frame_3["MC q1"] = np.vectorize(mcost_3_q1)(cost_frame_3['q2'])
cost_frame_3["MC q2"] = np.vectorize(mcost_3_q2)(cost_frame_3['q1'])
cost_frame_3
```

	q1	q2	Total Cost	MC q1	MC q2
0	100	50	6150	57	109
1	60	50	3870	57	69
2	40	50	2730	57	49
3	30	10	600	17	39
4	30	50	2160	57	39
5	30	70	2940	77	39

Incremental Cost

$$IC(q_1) = C(q_1 + 1, q_2) - C(q_1, q_2)$$
$$IC(q_2) = C(q_1, q_2 + 1) - C(q_1, q_2)$$

Which I find a little easier to write than to say :)

In python we'll just write a little function for this

```python
# incremental cost if the cost of making the next unit by product
def inc_cost(cost_function,q1=q1,q2=q2,increment=str):
    '''
    cost_function: total cost function that you'd like to increment (over q1,q2)
    q1: the quantity you'd like to pass to the cost func as q1, defaults q1
    q2: same, default q2
    increment: the quantity you'd like to increment
    '''
    C_0 = cost_function(q1,q2)
    if increment == "q1":
        q1=q1+1
    elif increment == "q2":
        q2=q2+1
    else:
        print("increment must be one of q1,q2")
        return None
    C_1 = cost_function(q1,q2)
    return C_1 - C_0
```

Firm 1

```python
# Incremental cost
cost_frame_1["IC q1"] = np.vectorize(inc_cost)(
                                    cost_1,
                                    cost_frame_1['q1'],
                                    cost_frame_1['q2'],
                                    increment="q1"
                                    )
cost_frame_1["IC q2"] = np.vectorize(inc_cost)(
                                    cost_1,
                                    cost_frame_1['q1'],
                                    cost_frame_1['q2'],
                                    increment="q2"
                                    )
cost_frame_1
```

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2	IC q1	IC q2
0	100	50	1250	10.0	5.0	10	5	10	5
1	60	50	850	10.0	5.0	10	5	10	5
2	40	50	650	10.0	5.0	10	5	10	5
3	30	10	350	10.0	5.0	10	5	10	5
4	30	50	550	10.0	5.0	10	5	10	5
5	30	70	650	10.0	5.0	10	5	10	5

Firm 2

```
# Incremental cost
cost_frame_2["IC q1"] = np.vectorize(inc_cost)(
                                    cost_2,
                                    cost_frame_2['q1'],
                                    cost_frame_2['q2'],
                                    increment="q1"
                                    )
cost_frame_2["IC q2"] = np.vectorize(inc_cost)(
                                    cost_2,
                                    cost_frame_2['q1'],
                                    cost_frame_2['q2'],
                                    increment="q2"
                                    )
cost_frame_2
```

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2	IC q1	IC q2
0	100	50	13500	106.0	58.0	206	108	207	109
1	60	50	6860	66.0	58.0	126	108	127	109
2	40	50	4740	46.0	58.0	86	108	87	109
3	30	10	1260	36.0	18.0	66	28	67	29
4	30	50	3980	36.0	58.0	66	108	67	109

	q1	q2	Total Cost	AC q1	AC q2	MC q1	MC q2	IC q1	IC q2
5	30	70	6540	36.0	78.0	66	148	67	149

Firm 3

```
# Incremental cost
cost_frame_3["IC q1"] = np.vectorize(inc_cost)(
                                    cost_3,
                                    cost_frame_3['q1'],
                                    cost_frame_3['q2'],
                                    increment="q1"
                                    )
cost_frame_3["IC q2"] = np.vectorize(inc_cost)(
                                    cost_3,
                                    cost_frame_3['q1'],
                                    cost_frame_3['q2'],
                                    increment="q2"
                                    )
cost_frame_3
```

	q1	q2	Total Cost	MC q1	MC q2	IC q1	IC q2
0	100	50	6150	57	109	57	109
1	60	50	3870	57	69	57	69
2	40	50	2730	57	49	57	49
3	30	10	600	17	39	17	39
4	30	50	2160	57	39	57	39
5	30	70	2940	77	39	77	39

Let's make a 3d graph in Python!!!

First load libraries and make the data

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np # we already have np
```

```
# Create data for the plot
q1 = np.linspace(0, 1_000, 1_000)
q2 = np.linspace(0, 1_000, 1_000)
Q1, Q2 = np.meshgrid(q1, q2)
# calc costs
C1 = 10 * Q1 + 5 * Q2
C2 = 6 * Q1 + Q1**2 + 8 * Q2 + Q2**2
C3 = 7 * Q1 + 9 * Q2 + Q1 * Q2
```

Let’s look at what is in these variables:

```
print("q1")
print(q1)
print("q2")
print(q2)
print("Q1")
print(Q1)
print("Q2")
print(Q2)
```

```
q1
[   0.            1.001001      2.002002      3.003003      4.004004
    5.00500501    6.00600601    7.00700701    8.00800801    9.00900901
   10.01001001   11.01101101   12.01201201   13.01301301   14.01401401
   15.01501502   16.01601602   17.01701702   18.01801802   19.01901902
   20.02002002   21.02102102   22.02202202   23.02302302   24.02402402
   25.02502503   26.02602603   27.02702703   28.02802803   29.02902903
   30.03003003   31.03103103   32.03203203   33.03303303   34.03403403
   35.03503504   36.03603604   37.03703704   38.03803804   39.03903904
   40.04004004   41.04104104   42.04204204   43.04304304   44.04404404
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 ...
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  1000.       ]]
Q2
[[   0.           0.           0.        ...    0.           0.
     0.       ]
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   998.998999]
 [1000.        1000.        1000.        ... 1000.        1000.
  1000.       ]]
```
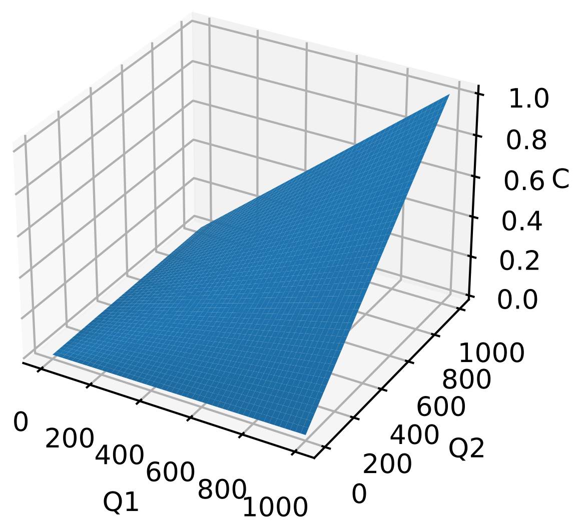
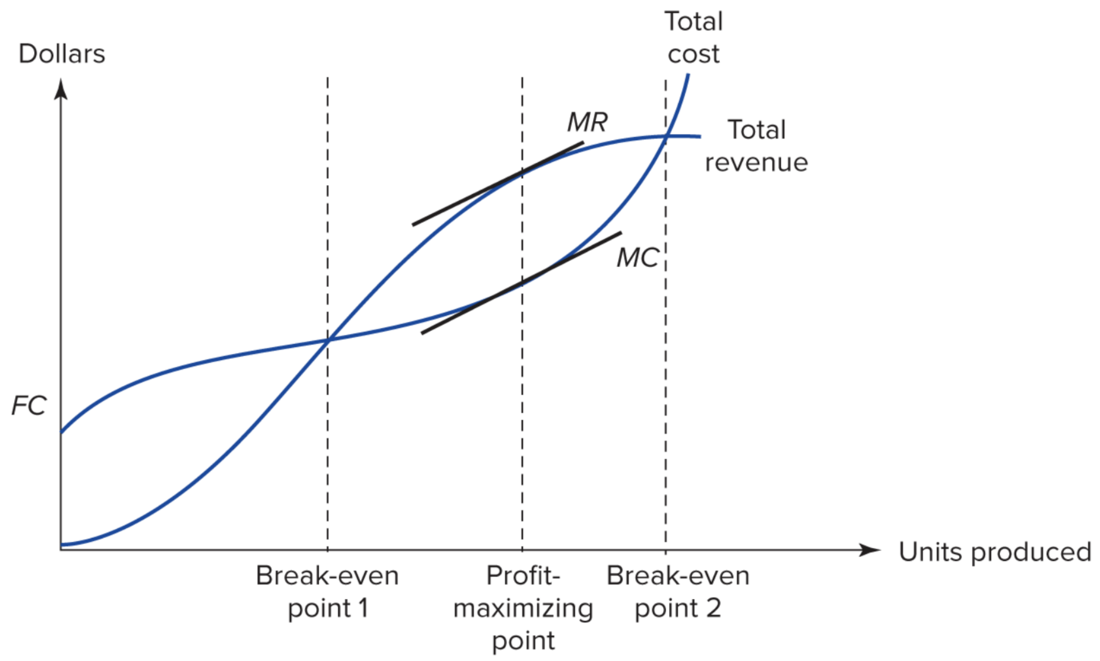

Make the plot:

```
# Create the figure and add a 3D axis
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot the data
# ax.plot_surface(Q1, Q2, C1)
# ax.plot_surface(Q1, Q2, C2)
ax.plot_surface(Q1, Q2, C3)

# Set axis labels and show the plot
ax.set_xlabel('Q1')
ax.set_ylabel('Q2')
ax.set_zlabel('Cost')
plt.show()
```



1.0
0.8
0.6
0.4
0.2
0.0
C
0
200
400
600
800
1000
Q1
0
200
400
600
800
1000
Q2

Turns out we are able to model non-linear functions pretty well!



MC: Marginal cost is the slope of the total cost curve.
MR: Marginal revenue is the slope of the total revenue curve.
MC and *MR* are equal at the profit-maximizing point.