1 Singular 1

Função custo:

$$J = x_3(t_f) \tag{1}$$

Dinâmica:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= u(t) \\ \dot{x}_3(t) &= x_1^2(t) \end{aligned} \tag{2}$$

Estados e controles:

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T \tag{3}$$

Estado inicial:

$$\begin{aligned} x_1(0) &= 0 \\ x_2(0) &= 1 \\ x_3(0) &= 0 \end{aligned} \tag{4}$$

Restrições de caminho:

$$-1 \leq u(t) \leq 1 \tag{5}$$

2 Singular 2

Função custo:

$$J = x_3(t_f) \tag{6}$$

Dinâmica:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= u(t) \\ \dot{x}_3(t) &= x_1^2(t) + x_2^2(t) \end{aligned} \tag{7}$$

Estados e controles:

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T \tag{8}$$

Estado inicial:

$$\begin{aligned} x_1(0) &= 0 \\ x_2(0) &= 1 \\ x_3(0) &= 0 \end{aligned} \tag{9}$$

Restrições de caminho:

$$-1 \leq u(t) \leq 1 \tag{10}$$

3 Pendulo Invertido

Função custo:

$$J = \int_0^{t_f} F^2(t)dt \tag{11}$$

Dinâmica:

$$
\begin{aligned}
\dot{d}(t) &= v(t) \\
\dot{\theta}(t) &= \omega(t) \\
\dot{v}(t) &= \frac{l\, m_2\, \omega^2(t)\, sen\big(\theta(t)\big) + F(t) + m_2\, g\, cos\big(\theta(t)\big)\, sen\big(\theta(t)\big)}{m_1 + m_2\big[1 - cos^2\big(\theta(t)\big)\big]} \\
\dot{\omega}(t) &= -\frac{l\, m_2\, \omega^2(t)\, sen\big(\theta(t)\big)\, cos\big(\theta(t)\big) + F(t)\, cos\big(\theta(t)\big) + (m_1 + m_2) g\, sen\big(\theta(t)\big)}{l\, m_1 + l\, m_2\big[1 - cos^2\big(\theta(t)\big)\big]}
\end{aligned} \tag{12}
$$

Estados e controles:

$$x(t) = \begin{bmatrix} d(t) & \theta(t) & v(t) & \omega(t) \end{bmatrix}^T \tag{13}$$

Estado inicial:

$$
\begin{aligned}
d(0) &= 0\,\mathrm{m} \\
\theta(0) &= 0\,\mathrm{rad} \\
v(0) &= 0\,\mathrm{m/s} \\
\omega(0) &= 0\,\mathrm{rad/s}
\end{aligned} \tag{14}
$$

Estado final:

$$
\begin{aligned}
d(t_f) &= d_f \\
\theta(t_f) &= \theta_f \\
v(t_f) &= 0\,\mathrm{m/s} \\
\omega(t_f) &= 0\,\mathrm{rad/s}
\end{aligned} \tag{15}
$$

Restrições laterais:

$$
\begin{aligned}
-d_{max} \leq d(t) \leq d_{max} \\
-F_{max} \leq F(t) \leq F_{max}
\end{aligned} \tag{16}
$$

4 UAV

Função custo:

$$J = \int_0^{t_f} I(t) dt \tag{17}$$

$$J = K_i\, m\, g \int_0^{t_f} \frac{1}{cos\, \phi(t)} dt \tag{18}$$

Dinâmica:

$$
\begin{aligned}
\dot{d}_x(t) &= v_x(t) \\
\dot{d}_y(t) &= v_x(t) \\
\dot{v}_x(t) &= g\, tan\, \phi(t)\, cos\, \theta(t) - \frac{c_d}{m}\Big[v_x(t) - c_x\big(d_x(t), d_y(t)\big)\Big] \\
\dot{v}_y(t) &= g\, tan\, \phi(t)\, sen\, \theta(t) - \frac{c_d}{m}\Big[v_y(t) - c_y\big(d_x(t), d_y(t)\big)\Big]
\end{aligned} \tag{19}
$$

Estados e controles:

$$x(t) = \begin{bmatrix} d_x(t) & d_y(t) & v_x(t) & v_y(t) \end{bmatrix}^T \tag{20}$$

$$u(t) = \begin{bmatrix} \phi(t) & \theta(t) \end{bmatrix}^T \tag{21}$$

Estado inicial:

$$\begin{aligned} d_x(0) &= 1\,\mathrm{m} \\ d_y(0) &= 5\,\mathrm{m} \\ v_x(0) &= 0\,\mathrm{m/s} \\ v_x(0) &= 0\,\mathrm{m/s} \end{aligned} \tag{22}$$

Estado final:

$$\big(d_x(t_f) - r_x\big)^2 + \big(p_y(t_f) - r_y\big)^2 \leq r \tag{23}$$

Restrições laterais:

$$\begin{aligned} 0 \text{ m} \leq d_x(t) \leq D_x \\ 0 \text{ m} \leq d_y(t) \leq D_y \\ 0 \text{ rad} \leq \phi(t) \leq \phi_{max} \\ -\pi \text{ rad} \leq \theta(t) \leq \pi \text{ rad} \end{aligned} \tag{24}$$

5 Foguete

Dinâmica: (OK)

$$\begin{aligned} \dot{\mathbf{r}}(t) &= \mathbf{v}(t) \\ \dot{\mathbf{v}}(t) &= -\frac{\mu}{|\mathbf{r}(t)|^3}\mathbf{r}(t) + \frac{T}{m(t)}\mathbf{u}(t) + \frac{\mathbf{D}(t)}{m(t)} \\ \dot{m} &= dm \end{aligned} \tag{25}$$

Vetores utilizados: (OK)

$$\begin{aligned} \mathbf{r}(t) &= \begin{bmatrix} r_x(t) & r_y(t) & r_z(t) \end{bmatrix}^T \\ \mathbf{v}(t) &= \begin{bmatrix} v_x(t) & v_y(t) & v_z(t) \end{bmatrix}^T \\ \mathbf{D}(t) &= \begin{bmatrix} D_x(t) & D_y(t) & D_z(t) \end{bmatrix}^T \\ \mathbf{u}(t) &= \begin{bmatrix} u_x(t) & u_y(t) & u_z(t) \end{bmatrix}^T \end{aligned} \tag{26}$$

Estados e controles: (OK)

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} r_x(t) & r_y(t) & r_z(t) & v_x(t) & v_y(t) & v_z(t) & m(t) \end{bmatrix}^T \\ \mathbf{u}(t) &= \begin{bmatrix} u_x(t) & u_y(t) & u_z(t) \end{bmatrix}^T \end{aligned} \tag{27}$$

Arrasto: (OK)

$$\mathbf{D}(t) = -\frac{1}{2}\, c_d\, A_{ref}\, \rho\, |\mathbf{v_{ref}}(t)|\, \mathbf{v_{ref}}(t) \tag{28}$$

$$\mathbf{v_{ref}}(t) = \mathbf{v}(t) + \boldsymbol{\omega} \times \mathbf{r}(t) \tag{29}$$

$$\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & \omega_e \end{bmatrix}^T \tag{30}$$

$$\rho(t) = \rho_0\, e^{-h(t)/h_0} \tag{31}$$

$$h(t) = |\mathbf{r}(t)| - R_e \tag{32}$$

Condições iniciais (OK):

$$\begin{aligned} \mathbf{r}(0) &= \mathbf{r_0} \\ \mathbf{v}(0) &= \mathbf{v_0} \\ m(0) &= m_0 \end{aligned} \tag{33}$$

$$\mathbf{r}_0 = \begin{bmatrix} r_{0x} \\ r_{0y} \\ r_{0z} \end{bmatrix} = \begin{bmatrix} R_e \; cos\, l_{cc} \\ 0 \\ R_e \; sen\, l_{cc} \end{bmatrix} \tag{34}$$

$$\mathbf{v}_0 = \boldsymbol{\omega} \times \mathbf{r}_0 \tag{35}$$

$$\mathbf{v}_0 = \begin{bmatrix} v_{0x} \\ v_{0y} \\ v_{0z} \end{bmatrix} = \begin{bmatrix} -r_{0y}\, w_e \\ r_{0x}\, w_e \\ 0 \end{bmatrix} \tag{36}$$

Continuidade das posições e velocidades (OK):

$$\begin{aligned} \mathbf{r}^{(2)}\left(t_0^{(2)}\right) &= \mathbf{r}^{(1)}\left(t_f^{(1)}\right) \\ \mathbf{r}^{(3)}\left(t_0^{(3)}\right) &= \mathbf{r}^{(2)}\left(t_f^{(2)}\right) \\ \mathbf{r}^{(4)}\left(t_0^{(4)}\right) &= \mathbf{r}^{(3)}\left(t_f^{(3)}\right) \end{aligned} \tag{37}$$

$$\begin{aligned} \mathbf{v}^{(2)}\left(t_0^{(2)}\right) &= \mathbf{v}^{(1)}\left(t_f^{(1)}\right) \\ \mathbf{v}^{(3)}\left(t_0^{(3)}\right) &= \mathbf{v}^{(2)}\left(t_f^{(2)}\right) \\ \mathbf{v}^{(4)}\left(t_0^{(4)}\right) &= \mathbf{v}^{(3)}\left(t_f^{(3)}\right) \end{aligned} \tag{38}$$

Conexão entre fases: massa (OK)

$$\begin{aligned} m^{(2)}\left(t_0^{(2)}\right) &= m^{(1)}\left(t_f^{(1)}\right) - \Delta m^{(1)} \\ m^{(3)}\left(t_0^{(3)}\right) &= m^{(2)}\left(t_f^{(2)}\right) - \Delta m^{(2)} \\ m^{(4)}\left(t_0^{(4)}\right) &= m^{(3)}\left(t_f^{(3)}\right) - \Delta m^{(3)} \end{aligned} \tag{39}$$

Restrições terminais (OK)

$$\begin{aligned} a(t_4) &= a_f \\ e(t_4) &= e_f \\ i(t_4) &= i_f \\ \Omega(t_4) &= \Omega_f \\ \omega(t_4) &= \omega_f \end{aligned} \tag{40}$$

Palpites iniciais para posição e velocidade das últimas fases:

$$\mathbf{r}_f = \begin{bmatrix} r_{fx} \\ r_{fy} \\ r_{fz} \end{bmatrix} = \begin{bmatrix} 4397,29 \\ 4243,77 \\ 2379,47 \end{bmatrix} \text{ km} \tag{41}$$

$$\mathbf{v}_f = \begin{bmatrix} v_{fx} \\ v_{fy} \\ v_{fz} \end{bmatrix} = \begin{bmatrix} -5826,69 \\ 7819,62 \\ -3178,43 \end{bmatrix} \text{ m/s} \tag{42}$$

Conversão de posição e velocidade para coordenadas orbitais:

$$\begin{gathered}
\mathbf{h}(t) = \mathbf{r}(t) \times \mathbf{v}(t) \\
\mathbf{e}'(t) = \frac{\mathbf{v}(t) \times \mathbf{h}(t)}{\mu} - \frac{\mathbf{r}(t)}{|\mathbf{r}(t)|} \\
\mathbf{e}'(t) = \begin{bmatrix} e'_x(t) & e'_y(t) & e'_z(t) \end{bmatrix}^T \\
\mathbf{h}(t) = \begin{bmatrix} h_x(t) & h_y(t) & h_z(t) \end{bmatrix}^T \\
\mathbf{n}(t) = \begin{bmatrix} -h_y(t) & h_x(t) & 0 \end{bmatrix}^T \\
i(t) = cos^{-1} \frac{h_z(t)}{|\mathbf{h}(t)|} \\
e(t) = |\mathbf{e}'(t)| \\
a(t) = \frac{1}{\dfrac{2}{|\mathbf{r}(t)|} - \dfrac{|\mathbf{v}(t)|^2}{\mu}} \\
\Omega(t) = \begin{cases} cos^{-1} \dfrac{n_x(t)}{|\mathbf{n}(t)|} & \text{se } n_y(t) \geq 0 \\ 2\pi - cos^{-1} \dfrac{n_x(t)}{|\mathbf{n}(t)|} & \text{se } n_y(t) < 0 \end{cases} \\
\omega(t) = \begin{cases} cos^{-1} \dfrac{\mathbf{n}(t) \bullet \mathbf{e}'(t)}{|\mathbf{n}(t)||\mathbf{e}'(t)|} & \text{se } e'_z(t) \geq 0 \\ 2\pi - cos^{-1} \dfrac{\mathbf{n}(t) \bullet \mathbf{e}'(t)}{|\mathbf{n}(t)||\mathbf{e}'(t)|} & \text{se } e'_z(t) < 0 \end{cases}
\end{gathered} \tag{43}$$

$$\Omega(t) = U_s\big(n_y(t)\big)\, cos^{-1} \frac{n_x(t)}{|\mathbf{n}(t)|} + U_s\big(-n_y(t)\big) \left(2\pi - cos^{-1} \frac{n_x(t)}{|\mathbf{n}(t)|}\right) \tag{44}$$

$$\begin{multline}
\omega(t) = U_s\big(e'_z(t)\big)\, cos^{-1} \frac{\mathbf{n}(t) \bullet \mathbf{e}'(t)}{|\mathbf{n}(t)||\mathbf{e}'(t)|} + \\
U_s\big(-e'_z(t)\big) \left(2\pi - cos^{-1} \frac{\mathbf{n}(t) \bullet \mathbf{e}'(t)}{|\mathbf{n}(t)||\mathbf{e}'(t)|}\right)
\end{multline} \tag{45}$$

$$U'_s(x) = 0,5 \left(1 + tanh\left(\frac{x}{A}\right)\right) \tag{46}$$

Função objetivo (OK):

$$J = -m(t_4) \tag{47}$$

Restrições de caminho (OK):

$$|\mathbf{r}(t)| \geq R_e \tag{48}$$

$$|\mathbf{u}(t)| = 1 \tag{49}$$

Restrições laterais: (OK)

$$\begin{aligned} 2\,R_e \leq r_x(t) \leq 0 \\ 2\,R_e \leq r_y(t) \leq 0 \\ 2\,R_e \leq r_z(t) \leq 0 \end{aligned} \tag{50}$$

$$\begin{aligned} -20000 \leq v_x(t) \leq 20000 \\ -20000\,R_e \leq v_y(t) \leq 20000 \\ -20000\,R_e \leq v_z(t) \leq 20000 \end{aligned} \tag{51}$$

Constantes (OK):

$$\begin{aligned} v_{0x} &= 0 \text{ km} \\ v_{0y} &= 408,74 \text{ m/s} \\ r_{0x} &= 5605,2 \text{ km} \\ r_{0y} &= 3043,4 \text{ km} \\ m_0 &= 301454 \text{ kg} \\ \Delta m^{(1)} &= 13680 \text{ kg} \\ \Delta m^{(2)} &= 6840 \text{ kg} \\ \Delta m^{(3)} &= 8830 \text{ kg} \\ A_{ref} &= 4\pi \text{ m}^2 \\ c_d &= 0,5 \\ \rho_0 &= 1,225 \text{ kg/m}^3 \\ h_0 &= 7,2 \text{ km} \\ t_0 &= 0 \text{ s} \\ t_1 &= 75,2 \text{ s} \\ t_2 &= 150,4 \text{ s} \\ t_3 &= 261 \text{ s} \\ a_f &= 24361,14 \text{ km} \\ e_f &= 0,7308 \\ i_f &= 28,5^\circ \\ \Omega_f &= 269,8^\circ \\ \omega_f &= 130,5^\circ \end{aligned} \tag{52}$$

Cálculo de I (OK):

$$\begin{aligned} \delta m_{sb} &= \frac{m_{sb}}{t_{sb}} \\ I_{sb} &= \frac{T_{sb}}{g_0\,\delta m_{sb}} \end{aligned} \tag{53}$$

$$\begin{aligned} \delta m_{s1} &= \frac{m_{s1}}{t_{s1}} \\ I_{s1} &= \frac{T_{s1}}{g_0\,\delta m_{s1}} \end{aligned} \tag{54}$$

$$
\begin{aligned}
\delta m_{s2} &= \frac{m_{s2}}{t_{s2}} \\
I_{s2} &= \frac{T_{s2}}{g_0\,\delta m_{s2}}
\end{aligned} \tag{55}
$$

Cálculo de dm (OK):

$$
\begin{aligned}
dm_{sb} &= \frac{T_{sb}}{g_0\,I_{sb}} \\
dm_{s1} &= \frac{T_{s1}}{g_0\,I_{s1}} \\
dm_{s2} &= \frac{T_{s2}}{g_0\,I_{s2}}
\end{aligned} \tag{56}
$$

T e dm de cada fase (OK):

$$
\begin{aligned}
T^{(1)} &= 6\,T_{sb} + T_{s1} \\
dm^{(1)} &= -6\,dm_{sb} - dm_{s1}
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
T^{(2)} &= 3\,T_{sb} + T_{s1} \\
dm^{(2)} &= -3\,dm_{sb} - dm_{s1}
\end{aligned} \tag{58}
$$

$$
\begin{aligned}
T^{(3)} &= T_{s1} \\
dm^{(3)} &= -dm_{s1}
\end{aligned} \tag{59}
$$

$$
\begin{aligned}
T^{(4)} &= T_{s2} \\
dm^{(4)} &= -dm_{s2}
\end{aligned} \tag{60}
$$