

Algorithm for Calculation of Earth Atmosphere Mass.

According to the I – law of thermodynamics heat change dq depends on the system's internal energy change du and object work change dw , thus total heat energy is $dq = du + dw$. For ideal gasses, I – law of thermodynamics heat change dq equation is:

$$\overbrace{pdv}^{dw} + \overbrace{vdp}^{du} = \overbrace{RdT}^{dq}, [J / kg] \quad (1.0)$$

Here: $R = 287 J/(kg \times K)$ – universal gas constant, p – gas pressure, $[Pa]$, $v = 1/\rho$ – gas specific volume, $[m^3/kg]$, ρ – gas density, $[kg/m^3]$.

Isothermal – $dq=0$, $dT=0$. – slow gas expansion – compression then afterward gas temperature stays unchanged. Earth pressure density change from height is based on **isothermal** gas expansion from height change. The Atmosphere pressure and density are similar units: $\rho(h) \sim \rho(h)$ based on the ideal gas equation condition, for more details see (1.05) equation.

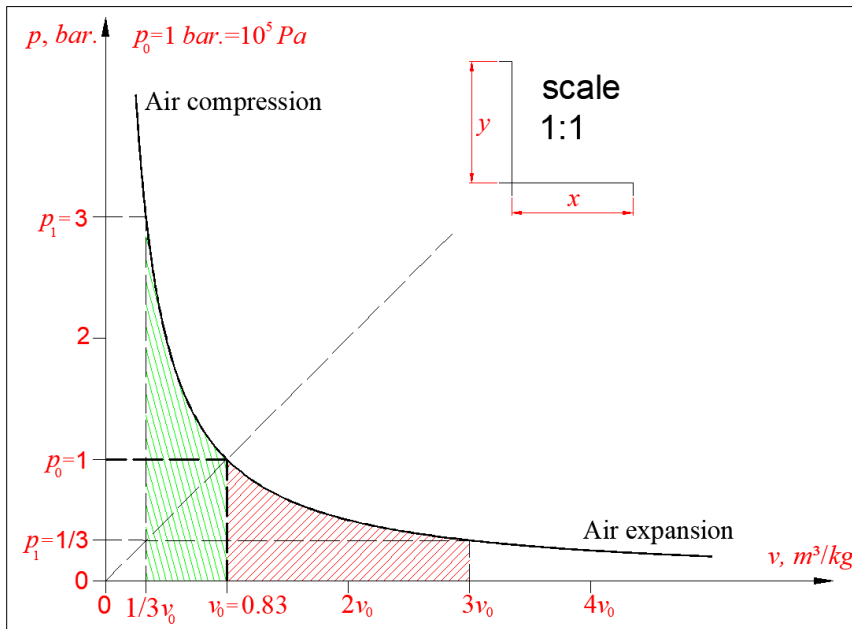


Fig. 1 Isothermal process $p = \text{const.}/v$ of air compression – expansion.

Equation (1.04) is a square (isosceles) hyperbola math function and it defines a thermodynamic isothermal process of air compression – expansion.

The enthalpy equation of an ideal gas is:

$$pv = RT, [J / kg] \Rightarrow p / \rho = RT. \quad (1.01)$$

First, from ideal gas (1.01) eq. express gas (air) density term and plug this term into a differential equation for compressible gas:

$$\rho = \frac{p}{RT} \Rightarrow dp = \rho g dh \Rightarrow dp = \frac{p}{RT} g dh. \quad (1.02)$$

Multiply the (1.02) equation by $1/p$ and later integrate the left side from dp in a range from p_0 to p_1 and the right side from dh in a range from $h_0=0$ to h :

$$\int_{p_0}^{p_1} \frac{dp}{p} = \frac{g}{RT} \int_0^h dh \Rightarrow -\ln\left(\frac{p}{p_0}\right) = \frac{gh}{RT} \Rightarrow p = p_0 e^{\frac{-gh}{RT}}. \quad (1.03)$$

Equation (1.03) shows isothermal air pressure decreases from height altitude change $p(h)$. The density of air change $\rho(h)$ from height (altitude) change is:

$$\rho = \rho_0 e^{\frac{-gh}{RT}} \quad (1.04)$$

We can prove that pressure and density are similar units $p \sim \rho$ or $p \sim 1/v$:

$$p = p_0 e^{\frac{-gh}{RT}}; \quad v = v_0 e^{\frac{gh}{RT}}; \quad p_0 e^{\frac{-gh}{RT}} \cdot v_0 e^{\frac{gh}{RT}} = RT \Rightarrow pv = RT = \text{const.} \quad (1.05)$$

Calculation of average air density based on intensive property of density. Average air temp. is $T=273+20^\circ\text{C}=293\text{ K}$, at $h_0=0$, thus $\rho(h_0)=1.205$, to $h=10^6$ meters, thus $h \rightarrow \infty$, $\rho(h) \rightarrow 0$, a sum of air density performed by an increment of 1 meter $\delta=h/N=1$ till $h=N\delta$:

$$\bar{\rho} = \frac{\rho_0}{N} \sum_{n=0}^{N=10^6} e^{\frac{-g\delta}{RT}n} \quad (1.06)$$

The sum of (1.06) eq. express via infinite geometric series, then $N=10^6 \rightarrow \infty$, and $x < 1$:

$$S = \sum_{n=0}^{N \rightarrow \infty} x^n = 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}, \quad e^{-x} = e^{\frac{-g\delta}{RT}} = 0.999883 < 1 \quad (1.07)$$

$$\bar{\rho} = \frac{\rho_0}{N} \sum_{n=0}^N e^{-xn} = \frac{\rho_0}{N} \sum_{n=0}^N e^{\frac{-g\delta}{RT}n} = \frac{\rho_0}{N} \cdot \frac{1}{1 - e^{-g\delta/RT}} = 0.0103298 \text{ kg/m}^3 \quad (1.08)$$

$$e^{-x} = \sum_{n=0}^{N \rightarrow \infty} (-1)^n \frac{x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots \quad (1.09)$$

For $x=g\delta/RT=1.183 \cdot 10^{-4} \approx 0$ we approximate (1.09) eq. to such expression:

$$e^{-x} \approx 1 - x \Rightarrow e^{\frac{-g\delta}{RT}} \approx 1 - g\delta/RT. \quad (1.10)$$

Plug (1.10) eq. into (1.08) eq., we get:

$$\bar{\rho} = \frac{\rho_0}{N} \cdot \frac{1}{1 - e^{-g\delta/RT}} = \frac{\rho_0}{N} \cdot \frac{1}{1 - (1 - g\delta/RT)} = \frac{\rho_0 RT}{gN\delta} = \frac{p_0}{gh} \quad (1.11)$$

The result of (1.11) eq. is known, see (1.08) eq., thus atmospheric pressure is:

$$p_0 = \bar{\rho}gh = 1.01335 \cdot 10^5 \approx 10^5 \text{ Pa} \quad (1.12)$$

Atmospheric pressure result in (1.12) eq. is accurate to the actual measured value – $p_0=1.01325 \times 10^5 \text{ Pa}$ [1]. Mass of air is calculated by summing air density from range $h_0=0$ till $h=10^6 \text{ m}$, $N=10^6$, over $\delta=h/N=1$ meter increments, and multiplying by sphere volumes in the shape of “onion rings” over $\delta=h/N=1$ meter radius increments starting from a radius of Earth – $R_{eth}=3178000 \text{ m}$, for more details see (Fig .2):

$$M_{air} = \rho_0 \sum_{n=0}^N e^{\frac{-g\delta}{RT}n} \cdot \frac{4}{3} \pi \sum_{n=0}^N [(R_{eth} + \delta(n+1))^3 - (R_{eth} + \delta n)^3] = 1.318 \times 10^{18} \text{ kg} \quad (1.13)$$

The convergence condition to the (1.13) equation is:

$$M_{air} = \lim_{n \rightarrow \infty} \sum \rho_n \cdot \sum (V_{n+1} - V_n) \rightarrow \text{finite} \quad (1.14)$$

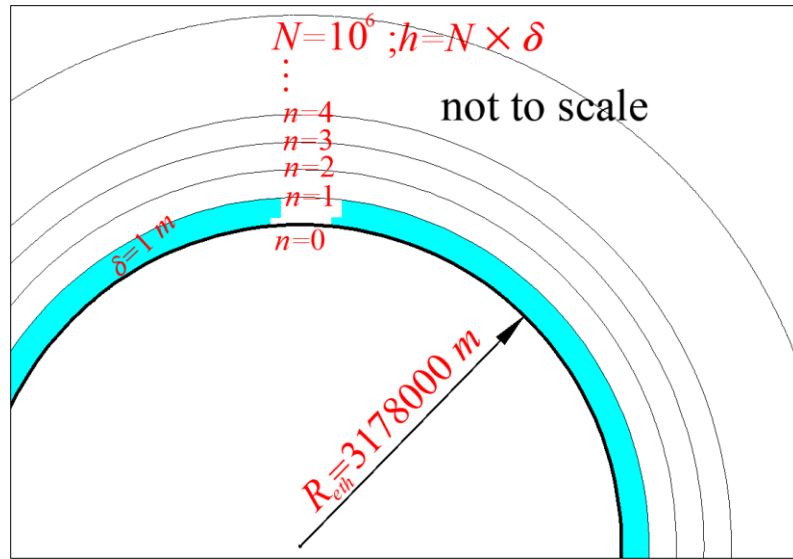


Fig. 1 Sketch for Earth air mass calculation over 1-meter increment.

Error calculation; two facts increase the total mass of the Earth's atmosphere and one factor decreases. Temperature increases starting from $h=0$, $T(\delta)=0.0065 \text{ K/m}$ till 100 km height – Karman line also has name Thermosphere, from that point temperature starts again decreasing. And free fall acceleration constant dependable from height:

$$g(\delta) = \frac{GM_{\text{earth}}}{(R_{\text{earth}} + \delta)^2}, \left[\frac{m}{s^2} \right] \quad (1.15)$$

Most of Earth's atmosphere is close to Earth's ground thus 2 km where atmosphere is dense. A negative factor is a landmass, which is any object that is above sea level [1], mostly high mountains peaks displace Earth's atmosphere. The Earth's mass is 10^{15} kg bigger than calculated by the (1.13) equation:

$$M_{\text{air}} = 1.318 \times 10^{18} \text{ kg} + 10^{15} \text{ kg} = 1.319 \times 10^{18} \text{ kg} \quad (1.16)$$

Errors are 1/1000th or 0.1% bigger than calculated.

According to Google search, Earth's mass is $M_{\text{air}}=5.15 \times 10^{18} \text{ kg}$, the ratio is $\cong 4$ times bigger than calculated here. Chat-gpt inquiry is close to the same result of $M_{\text{air}}=5.14 \times 10^{18} \text{ kg}$.

GitHub Python code: https://github.com/ArthurKarbocius/Earth-s_air_mass