Algorithm for Calculation of Earth Atmosphere Mass.

According to the I – law of thermodynamics heat change dq depends on the system's internal energy change du and object work change dw, thus total heat energy is $dq = du+dw$. For ideal gasses, I – law of thermodynamics heat change dq equation is:

$$\overbrace{pdv}^{dw}+\overbrace{vdp}^{du}=\overbrace{RdT}^{dq},\ [J/kg] \tag{1.0}$$

Here: R =287 $J/(kg\times K)$ – universal gas constant, p – gas pressure, [Pa], v=1/ρ – gas specific volume, [m^3/kg], ρ – gas density, [kg/m^3].

Isothermal – dq=0, dT=*const.* – slow gas expansion – compression then afterward gas temperature stays unchanged. Earth pressure density change from height is based on **isothermal** gas expansion from height change. Both are similar units: $\rho(h)\sim\rho(h)$ based on the ideal gas equation, for more details see (1.05) equation.

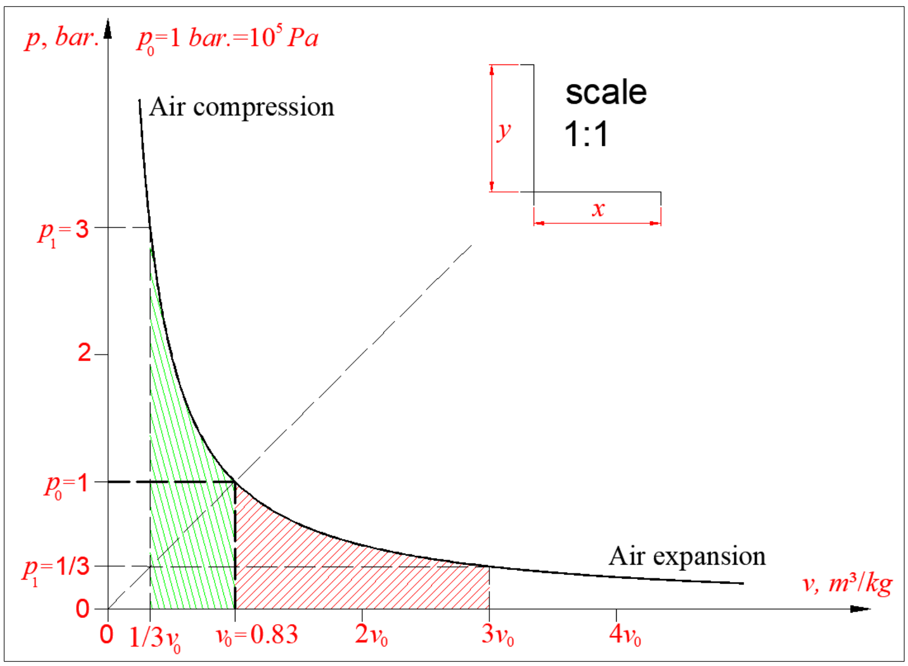

Fig. 1 Isothermal process p=*const.*/v of air compression – expansion.

Equation (1.04) is a square (isosceles) hyperbola math function and it defines a thermodynamic isothermal process of air compression – expansion.

The enthalpy equation of an ideal gas is:

$$pv = RT,\ [J/kg]\ \Rightarrow\ p/\rho = RT. \tag{1.01}$$

First, from ideal gas (1.01) eq. express gas (air) density term and plug this term into a differential equation for compressible gas:

$$\rho = \frac{p}{RT} \Rightarrow\ dp = \rho g dh\ \Rightarrow\ dp = \frac{p}{RT} g dh. \tag{1.02}$$

Multiply the (1.02) equation by 1/p and later integrate the left side from dp in a range from p_0 to p_1 and the right side from dh in a range from h_0=0 to h:

$$\int_{p_0}^{p}\frac{dp}{p} = \frac{g}{RT}\int_0^h dh\ \Rightarrow\ -\ln\left(\frac{p}{p_0}\right) = \frac{gh}{RT}\ \Rightarrow\ p = p_0 e^{\frac{-gh}{RT}}. \tag{1.03}$$

Equation (1.03) shows isothermal air pressure decreases from height altitude change $p(h)$. The density of air change $\rho(h)$ from height (altitude) change is:

$$\rho = \rho_0 e^{\frac{-gh}{RT}} \tag{1.04}$$

We can prove that pressure and density are similar units $p\sim\rho$ or $p\sim 1/v$:

$$p = p_0 e^{\frac{-gh}{RT}};\;\; v = v_0 e^{\frac{gh}{RT}};\;\; p_0 e^{\frac{-gh}{RT}} \cdot v_0 e^{\frac{gh}{RT}} = RT \;\;\Rightarrow\;\; pv = RT = const. \tag{1.05}$$

Calculation of average air density based on intensive property of density. Average air temp. is T=273+20°C =293K, at h_0=0, thus $\rho(h_0)$=1.205, to $h=10^6$ meters, thus $h\to\infty$, $\rho(h)\to 0$, a sum of air density performed by an increment of 1 meter $\delta=h/N=1$ till $h=N\delta$:

$$\bar{\rho} = \frac{\rho_0}{N}\sum_{n=0}^{N=10^6} e^{\frac{-g\delta}{RT}n} \tag{1.06}$$

The sum of (1.06) eq. express via infinite geometric series, then $N=10^6\to\infty$, and $x<1$:

$$S = \sum_{n=0}^{N\to\infty} x^n = 1 + x + x^2 + x^3 + ... = \frac{1}{1-x},\;\; e^{-x} = e^{\frac{-g\delta}{RT}} = 0.999883 < 1 \tag{1.07}$$

$$\bar{\rho} = \frac{\rho_0}{N}\sum_{n=0}^{N} e^{-xn} = \frac{\rho_0}{N}\sum_{n=0}^{N} e^{\frac{-g\delta}{RT}n} = \frac{\rho_0}{N}\cdot\frac{1}{1-e^{-g\delta/RT}} = 0.0103298\ kg/m^3 \tag{1.08}$$

$$e^{-x} = \sum_{n=0}^{N\to\infty}(-1)^n\frac{x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ... \tag{1.09}$$

For $x=g\delta/RT=1.183\cdot 10^{-4}\approx 0$ we approximate (1.09) eq. to such expression:

$$e^{-x} \approx 1 - x \;\;\Rightarrow\;\; e^{\frac{-g\delta}{RT}} \approx 1 - g\delta/RT. \tag{1.10}$$

Plug (1.10) eq. into (1.08) eq., we get:

$$\bar{\rho} = \frac{\rho_0}{N}\cdot\frac{1}{1-e^{-g\delta/RT}} = \frac{\rho_0}{N}\cdot\frac{1}{1-(1-g\delta/RT)} = \frac{\rho_0 RT}{gN\delta} = \frac{p_0}{gh} \tag{1.11}$$

The result of (1.11) eq. is known, see (1.08) eq., thus atmospheric pressure is:

$$p_0 = \bar{\rho}gh = 1.01335\cdot 10^5 \cong 10^5\ Pa \tag{1.12}$$

Atmospheric pressure result in (1.12) eq. is accurate to the actual measured value – $p_0=1.01325\times 10^5\,Pa$ [1]. Mass of air is calculated by summing air density from range $h_0=0$ till $h=10^6\ m$, $N=10^6$, over $\delta=h/N=1$ meter increments, and multiplying by sphere volumes in the shape of "onion rings" over $\delta=h/N=1$ meter radius increments starting from a radius of Earth – $R_{eth}=3178000\,m$, for more details see (Fig .2):

$$M_{air} = \rho_0\sum_{n=0}^{N} e^{\frac{-g\delta}{RT}n}\cdot\frac{4}{3}\pi\sum_{n=0}^{N}\left[\left(R_{eth}+\delta(n+1)\right)^3 - (R_{eth}+\delta n)^3\right] = 1.318\times 10^{18}\ kg \tag{1.13}$$

The convergence condition to the (1.13) equation is:

$$M_{air} = \lim_{n\to\infty}\left(\sum\rho_n\cdot\sum V_n\right)\to \text{finite} \tag{1.14}$$

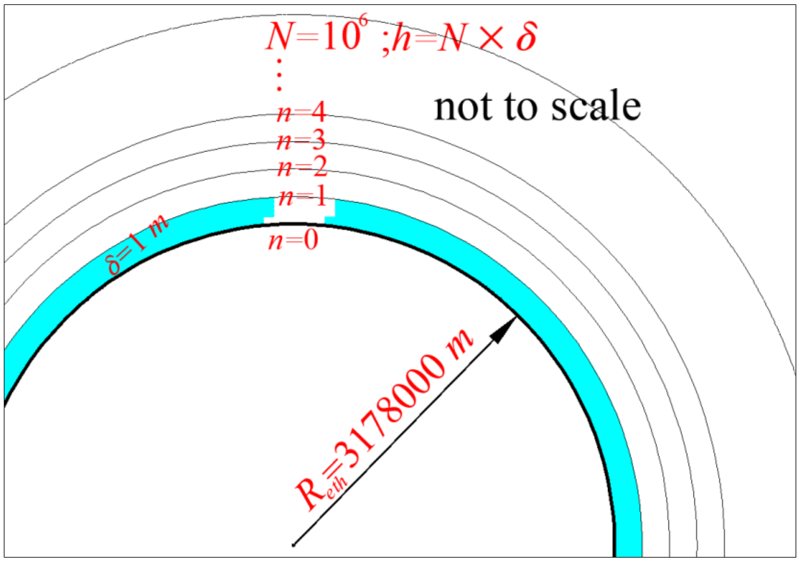

Fig. 1 Sketch for Earth air mass calculation over 1-meter increment.

Error calculation; two facts increase the total mass of the Earth's atmosphere and one factor decreases. Temperature increases starting from h=0, $T(h)$=0.0065 K/m till 100km height – Karman line also has name Thermosphere, from that point temperature starts again decreasing. And free fall acceleration constant dependable from height:

$$g(h) = \frac{GM_{earth}}{(R_{erth} + h)^2}, \left[\frac{m}{s^2}\right] \tag{1.15}$$

Most of Earth's atmosphere is close to Earth's ground thus 2 km where atmosphere is dense. A negative factor is a landmass, which is any object that is above sea level, mountains peaks displace Earth's atmosphere mostly. By all consideration, Earth's mass is 10^{15} kg bigger than calculated by the (1.13) equation:

$$M_{air} = 1.318 \times 10^{18} kg + 10^{15}\ kg = 1.319 \times 10^{18} kg \tag{1.16}$$

Errors are 1/1000th or 0.1% bigger than calculated.

According to Google search, Earth's mass is M_{air}=5.15×10^{18} kg, the ratio is ≅4 times bigger than calculated here. Chat-gpt inquiry is very close to the same result of M_{air}=5.14×10^{18} kg.

GitHub Python code: https://github.com/ArthurKarbocius/Earth-s_air_mass