**Algorithm for Calculation of Earth Atmosphere Mass.**

According to the I – law of thermodynamics heat change $dq$ depends on the system's internal energy change $du$ and object work change $dw$, thus total heat energy is $dq = du + dw$. For ideal gasses, I – law of thermodynamics heat change $dq$ equation is:

$$\overbrace{pdv}^{dw} + \overbrace{vdp}^{du} = \overbrace{RdT}^{dq}, \ [J/kg] \qquad (1.0)$$

Here: $R$ =287 $J/(kg\times K)$ – universal gas constant, $p$ – gas pressure, [$Pa$], $v=1/\rho$ – gas specific volume, [$m^3/kg$], $\rho$ – gas density, [$kg/m^3$].

**Isothermal** – $dq=0$, $dT=const.$ – slow gas expansion – compression then afterward gas temperature stays unchanged. Earth pressure density change from height is based on **isothermal** gas expansion from height change. Both are similar units: $\rho(h)\sim\rho(h)$ based on the ideal gas equation, for more details see (1.05) equation.

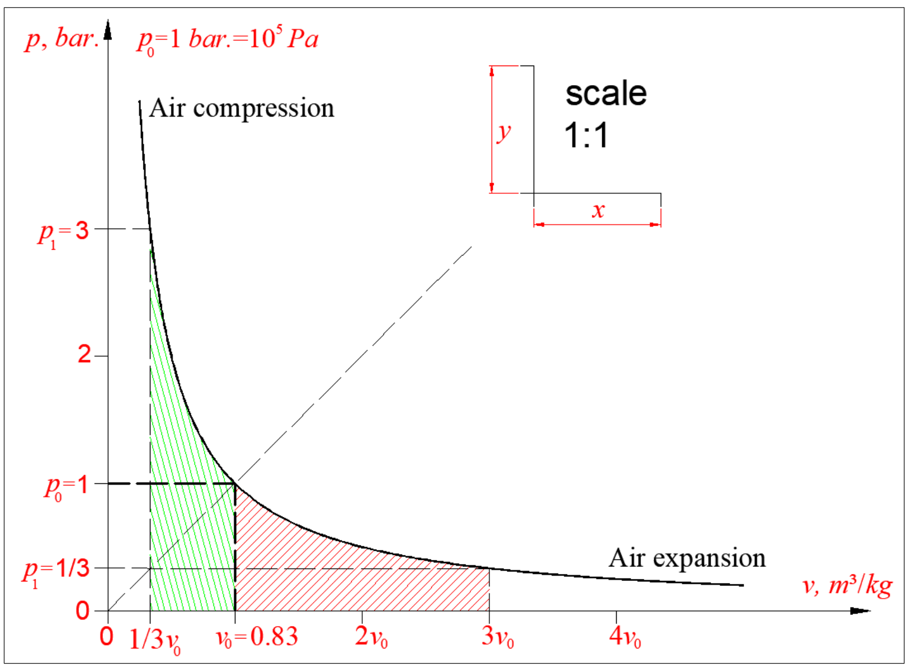


Fig. 1 Isothermal process $p=const./v$ of air compression – expansion.

Equation (1.04) is a square (isosceles) hyperbola math function and it defines a thermodynamic isothermal process of air compression – expansion.

The enthalpy equation of an ideal gas is:

$$pv = RT, \ [J/kg] \ \Rightarrow \ p/\rho = RT. \qquad (1.01)$$

First, from ideal gas (1.01) eq. express gas (air) density term and plug this term into a differential equation for compressible gas:

$$\rho = \frac{p}{RT} \Rightarrow \ dp = \rho g dh \ \Rightarrow \ dp = \frac{p}{RT} g dh. \qquad (1.02)$$

Multiply the (1.02) equation by $1/p$ and later integrate the left side from $dp$ in a range from $p_0$ to $p_1$ and the right side from $dh$ in a range from $h_0=0$ to $h$:

$$\int_{p_0}^{p} \frac{dp}{p} = \frac{g}{RT}\int_0^h dh \ \Rightarrow \ -\ln\left(\frac{p}{p_0}\right) = \frac{gh}{RT} \ \Rightarrow \ p = p_0 e^{\frac{-gh}{RT}}. \qquad (1.03)$$

Equation (1.03) shows isothermal air pressure decreases from height altitude change $p(h)$. The density of air change $\rho(h)$ from height (altitude) change is:

$$\rho = \rho_0 e^{\frac{-gh}{RT}} \tag{1.04}$$

We can prove that pressure and density are similar units $p \sim \rho$ or $p \sim 1/v$:

$$p = p_0 e^{\frac{-gh}{RT}}; \;\; v = v_0 e^{\frac{gh}{RT}}; \;\; p_0 e^{\frac{-gh}{RT}} \cdot v_0 e^{\frac{gh}{RT}} = RT \;\; \Rightarrow \;\; pv = RT = const. \tag{1.05}$$

Calculation of average air density based on intensive property of density. Average air temp. is $T$=273+20°$C$ =293$K$, at $h_0$=0, thus $\rho(h_0)$=1.205, to $h$=$10^6$ meters, thus $h \to \infty$, $\rho(h) \to 0$, a sum of air density performed by an increment of 1 meter $\delta = h/N = 1$ till $h = N\delta$:

$$\bar{\rho} = \frac{\rho_0}{N} \sum_{n=0}^{N=10^6} e^{\frac{-g\delta}{RT}n} \tag{1.06}$$

The sum of (1.06) eq. express via infinite geometric series, then $N=10^6 \to \infty$, and $x<1$:

$$S = \sum_{n=0}^{N\to\infty} x^n = 1 + x + x^2 + x^3 + ... = \frac{1}{1-x}, \;\; e^{-x} = e^{\frac{-g\delta}{RT}} = 0.999883 < 1 \tag{1.07}$$

$$\bar{\rho} = \frac{\rho_0}{N} \sum_{n=0}^{N} e^{-xn} = \frac{\rho_0}{N} \sum_{n=0}^{N} e^{\frac{-g\delta}{RT}n} = \frac{\rho_0}{N} \cdot \frac{1}{1 - e^{-g\delta/RT}} = 0.0103298 \; kg/m^3 \tag{1.08}$$

$$e^{-x} = \sum_{n=0}^{N\to\infty} (-1)^n \frac{x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ... \tag{1.09}$$

For $x = g\delta/RT = 1.183 \cdot 10^{-4} \approx 0$ we approximate (1.09) eq. to such expression:

$$e^{-x} \approx 1 - x \;\; \Rightarrow \;\; e^{\frac{-g\delta}{RT}} \approx 1 - g\delta/RT. \tag{1.10}$$

Plug (1.10) eq. into (1.08) eq., we get:

$$\bar{\rho} = \frac{\rho_0}{N} \cdot \frac{1}{1 - e^{-g\delta/RT}} = \frac{\rho_0}{N} \cdot \frac{1}{1 - (1 - g\delta/RT)} = \frac{\rho_0 RT}{gN\delta} = \frac{p_0}{gh} \tag{1.11}$$

The result of (1.11) eq. is known, see (1.08) eq., thus atmospheric pressure is:

$$p_0 = \bar{\rho} gh = 1.01335 \cdot 10^5 \cong 10^5 \;\; Pa \tag{1.12}$$

Atmospheric pressure result in (1.12) eq. is accurate to the actual measured value – $p_0 = 1.01325 \times 10^5$ $Pa$ [1]. Mass of air is calculated by summing air density from range $h_0$=0 till $h$=$10^6$ $m$, $N$=$10^6$, over $\delta = h/N = 1$ meter increments, and multiplying by sphere volumes in the shape of "onion rings" over $\delta = h/N = 1$ meter radius increments starting from a radius of Earth – $R_{eth}$=3178000 $m$, for more details see (Fig .2):

$$M_{air} = \rho_0 \sum_{n=0}^{N} e^{\frac{-g\delta}{RT}n} \cdot \frac{4}{3}\pi \sum_{n=0}^{N} \left[ (R_{eth} + \delta(n+1))^3 - (R_{eth} + \delta n)^3 \right] = 1.318 \times 10^{18} \;\; kg \tag{1.13}$$

The convergence condition to the (1.13) equation is:

$$M_{air} = \lim_{n\to\infty} \left( \sum \rho_n \cdot \sum V_n \right) \to \text{finite} \tag{1.14}$$

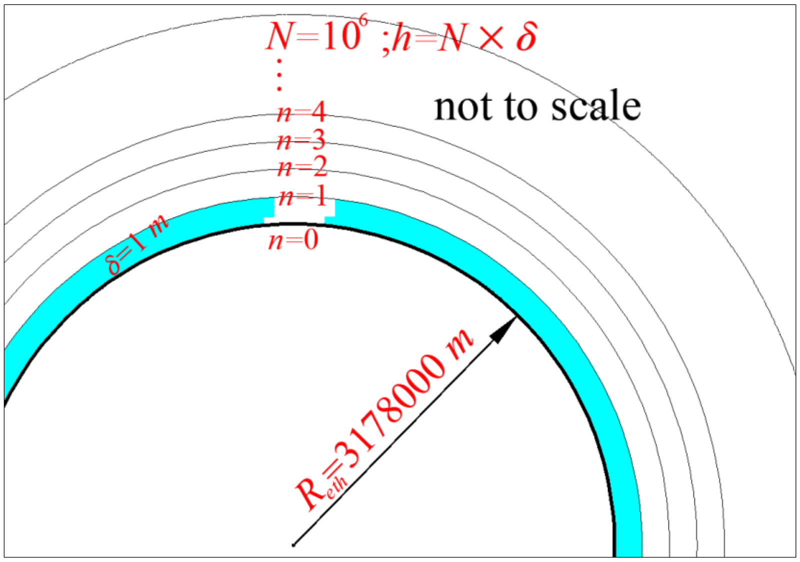


Fig. 1 Sketch for Earth air mass calculation over 1-meter increment.

Error calculation two facts increase the total mass of atmosphere and one factor decrease. Temperature increases starting form $h$=0 About $T(h)$=0.0065 $K/m$ till 100$km$ height – Karman line also has name Thermosphere from that point temperature starts again increasing. And free fall acceleration constant dependable from height:

$$g(h)=\frac{GM_{earth}}{(R_{erth}+h)^2},\ \left[\frac{m}{s^2}\right] \tag{1.15}$$

Most of Earth's atmosphere is close to Earth's ground thus 2 $km$ where atmosphere is dense. A negative factor is a landmass, which is any object that is above sea level, mountains peaks displace Earth's atmosphere mostly. By all consideration, Earth's mass is $10^{15}$ $kg$ bigger than calculated by the (1.13) equation:

$$M_{air}=1.318\times 10^{18}kg+10^{15}\ kg=1.319\times 10^{18}kg \tag{1.16}$$

Errors are 1/1000th or 0.1% bigger than calculated.

According to Google search, Earth's mass is $M_{air}$=5.15×$10^{18}$ $kg$, the ratio is ≅4 times bigger than calculated here. Chat-gpt inquiry is very close to the same result of $M_{air}$=5.14×$10^{18}$ $kg$.