Inertia effect explanation based on Euler's formula for complex numbers

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1. Introduction

Mathematical investigation of the body with mass m inertia force in ideal conditions without friction air drag or any other factors that can change the total energy value of the investigative system. By applying the discrete calculation method, we try to generalize calculation by applying Euler's formula for complex numbers, which will define time, total energy value, and individually kinetic and potential terms energies values with one beautiful mathematical expression.

Keywords: Hooke's law, object kinetic and elastic spring potential energy, L-C electric circuit, Earth's gravity force.

2. Hooke's law and ideal harmonic oscillator

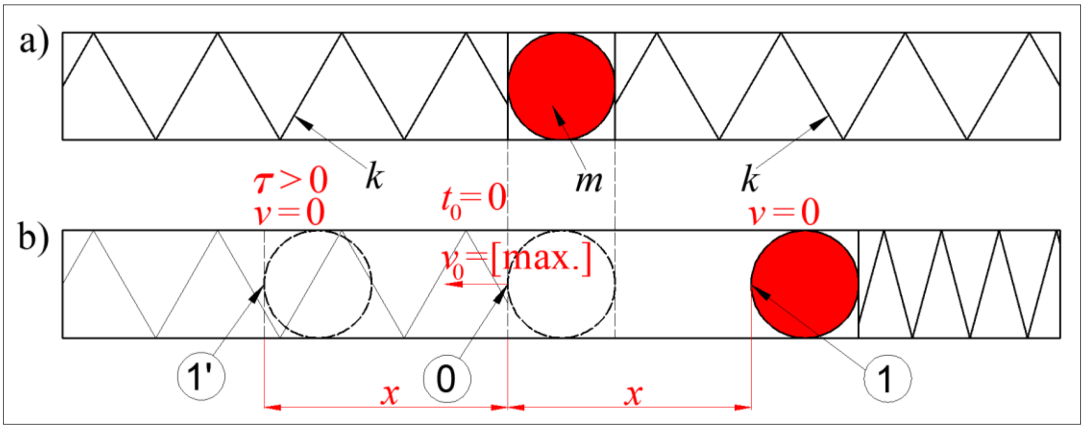

Fig. 1 a) Harmonic oscillator in a still position.
b) Harmonic oscillator with maximums spring potential energy.

In (Fig. 1 a)) an object with mass m=3.5 kg is at rest in 0 positions. In (Fig. 1 b)) the spring is loaded at a maximum distance of x=0.3 m, and at that moment spring has maximum potential energy which was derived from Hooke's force law for elastic materials. Here k=1500 N/m – spring's elasticity coefficient. At point 0 all elastic spring potential energy – $U=kx^2/2$ is converted into object kinetic energy $K=mv^2/2$ at maximum velocity value v_0=[max.], from moment t_0=0 at point 0 object contract spring again by exact distance x, and at point 1' object's velocity is again v=0, spring contraction at that time is equal to τ. In ideal conditions, only two energies are present, spring potential U and objects kinetic K energies.

Energy quantity conservation condition:

$$E_0 = U = K \tag{1.0}$$

$$E_0 = \frac{kx^2}{2} = \frac{mv_0^2}{2} \quad \Rightarrow \quad v_0 = x\sqrt{\frac{k}{m}} = 6.21059 \quad m/s \tag{1.1}$$

The algorithm sequence [6] to solve time τ is:
m=3, k=1500, $N=2\times10^4$, $\delta=x/N=1.5\times10^{-5}$, n=0, 1 … N, for n=0: t_0=0, v_0=6.21059.

$$U_n = n\frac{k\delta^2}{2} \Rightarrow K_{n+1} = \frac{m(v_n)^2}{2} - U_{n+1} \Rightarrow E_n = K_n - U_n \Rightarrow$$
$$\Rightarrow \sqrt{\frac{2E_n}{m}} = \overbrace{v_n}^{\Uparrow} \Rightarrow t_n = \frac{\delta}{v_n} \Rightarrow \tau = \delta\sum_{n=1}^{N}\frac{1}{v_n}. \quad (1.2)$$

3. Square or isosceles hyperbola

From the algorithm (1.2) sequence [6], the last expression τ is a parametric mathematical square or isosceles hyperbola expression:

$$y = \frac{const.}{x} \Rightarrow y\cdot x = const. \quad \begin{cases} x\in(+\infty,\ 0) \\ y\in(0,\ +\infty) \end{cases} \quad (2.0)$$

$$t_n = \frac{\delta}{v_n} \Rightarrow t_n\cdot v_n = \delta = const. \quad \begin{cases} v_n\in[v_1,\ v_N] \\ t_n\in[t_1,\ t_N] \end{cases} \quad (2.1)$$

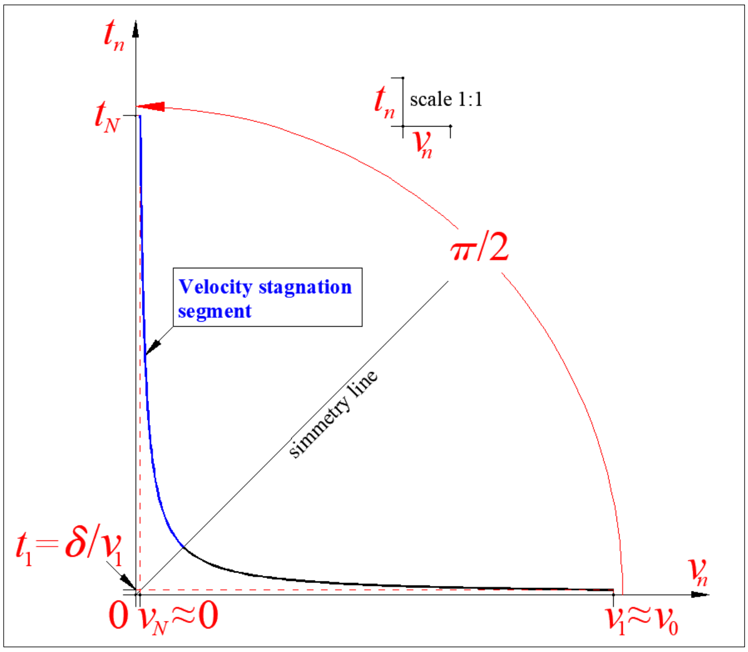

Fig. 2 Square hyperbola of function $t_n = \delta / v_n$.

In the chart (Fig. 2) at a scale of the x-axis and y-axis spring compression time t_n from velocity change v_n has a square hyperbola progression. In a chart, the symmetry line at 45° degree angle divides the hyperbola function curve into two equal-length curve segments. Velocity change from maximum v_1 to minimum v_N has a maximum 90°=π/2 angle change. Alternative total time in seconds of spring contraction calculated in such way:

$$\tau = \frac{\pi}{2}\sqrt{\frac{m}{k}} = 0.07587\ldots\ s \Rightarrow \tau = \sum_{n=1}^{N\to\infty} t_n \Rightarrow \tau = \delta\sum_{n=1}^{N\to\infty}\frac{1}{v_n} \quad (3.0)$$

$$\tau = \sum_{n=1}^{N=19999} t_n = \delta\sum_{n=1}^{19999}\frac{1}{v_n} = 0.07552\ldots\ s \quad (3.1)$$

Interestingly, our calculation algorithm is done without any periodic trigonometric f-ions help, all results are obtained through arithmetic operations. For a simple harmonic oscillator [1], where oscillation change $x(t)$ sinusoidal f-ion is given fact based on observation, not mathematical derivation [6]:

$$x(t) = A\cos(\omega t + \varphi),\ \ \omega = \sqrt{k/m},\ [s^{-1}],\ \ A = 0.3\ m,\ \ \varphi = -\pi/2 \tag{3.2}$$

Proof that in spring contraction – expansion result comes from parametric energy conservation law condition (1.0) eq., where at any given time the closed energetic systems total value of potential and kinetic energies in absolute value terms equal to unit E_0=1:

$$\begin{aligned} &E_0 = \frac{kx^2}{2} + \frac{mv_0^2}{2} = 1 \ \Rightarrow\ \frac{x^2}{2} + \frac{y^2}{2} = 1 \ \Rightarrow\ x^2 + y^2 = 2 \ \Rightarrow \\ &\Rightarrow\ \text{Area} = \int_0^R \sqrt{R^2 - x^2}\,dx = \frac{\pi}{2};\ R = \sqrt{2} \end{aligned} \tag{3.3}$$

4. Life is driven by e^x – motions light is driven by i – motions

This chapter has no context, but we know that light is an electromagnetic wave (EMW) and dual electric wave change induces magnetic wave change discovered by J. C. Maxwell (1831–1879). A decrease in the magnetic field induces an electric field and vice versa, that is how light as an electromagnetic wave sustains a dual formation. Same in our case total energy E_0=1, if kinetic energy K=1, thus spring potential energy U=0. My thesis is that function of change is based on trigonometric functions. I claim that in general laws of Nature are written based on transcendental math contestants, $i = \sqrt{-1}$, e^x, π, $\sin(2\pi n)$, $\cos(2\pi n)$. And God or Random Nature based on a belief system plugs fundamental physics constants into trigonometric or exponential functions. This approach in theoretical physics theory is called mathematical formalism, instead of claiming the cliché notion that Nature speaks in mathematical language my claim is that Nature is built based on mathematical language. From this notion, we can discard Egg and Chicken conundrum who was first. Because transcendental math constants and God is alpha and omega e, π, $i = \sqrt{-1}$ together with and trig. functions existed forever and Intelligence design or Random Nature creates our reality based on these functions e^x, $\sin(x)$, $\cos(x)$. Matter, in my opinion, is based on the result Fourier series (superposition principle), and a result of the Fourier series in a 2D plane gives a holographic projection of 3D reality. < *In fact the special whole in every part of a hologram is one of the by-products that occurs when an image or pattern is translated into the Fourier language of wave forms.* [2] >. Our example is the most simple, straight linear motion, to prove the point. In (4.0) eq. we discarded $i = \sqrt{-1}$ value and only care about the time real number value part.

Object's velocity and spring contraction equation from time variable expressed as Euler's formula for complex numbers:

$$E_0 e^{i\Theta} = v_0\cos(\Theta) + ix\sin(\Theta);\ \ \Theta \in [0,\ \pi/2];\ \ R = E_0 = 1 \tag{4.0}$$

In (Fig. 3), object velocity and spring contraction distance expressing via angle change:

$$t = a\tau = a\frac{\pi}{2}\sqrt{\frac{m}{k}},\ \ \Theta = \frac{\pi}{2}a;\ a \in \mathbb{R} \qquad \begin{cases} a \in [0,\ 1] \\ \Theta \in [0,\ \pi/2] \\ t \in [0,\ \tau] \end{cases} \tag{4.1}$$

$$E_0 e^{i\frac{2}{\pi}\sqrt{\frac{k}{m}}t} = v_0 \cos\left(\frac{2}{\pi}\sqrt{\frac{k}{m}}t\right) + ix\sin\left(\frac{2}{\pi}\sqrt{\frac{k}{m}}t\right),\ t\in[0,\ \tau] \tag{4.2}$$

$$E_0 e^{\frac{i\pi}{2}a} = v_0 \cos\left(\frac{\pi}{2}a\right) + ix\sin\left(\frac{\pi}{2}a\right),\ a\in[0,\ 1],\ a\in\mathbb{R} \tag{4.3}$$

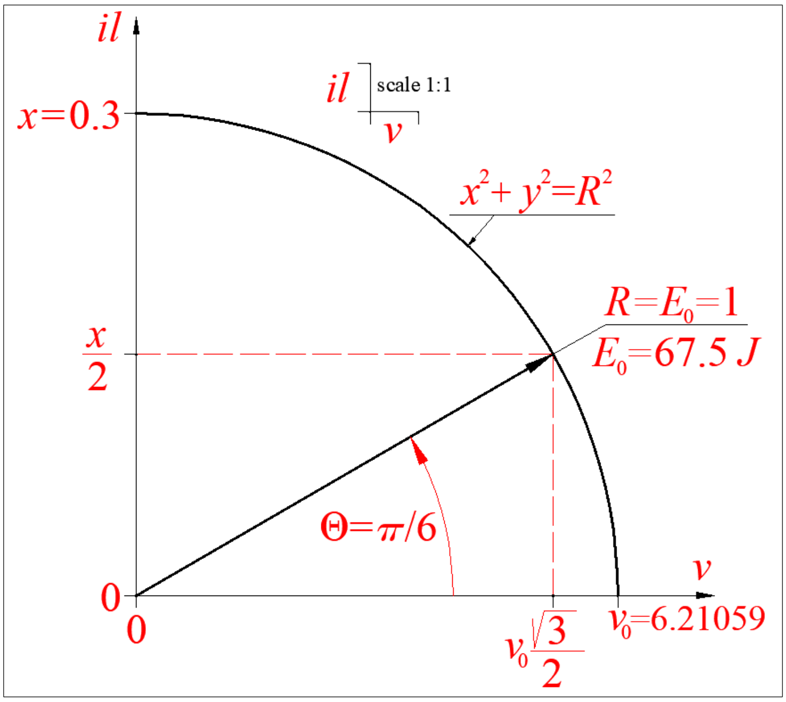

Fig . 3 Object velocity and spring contraction values from a time-angle $\Theta=\pi/6$.

$$E_0 e^{i\pi/6} = v_0 \cos\left(\frac{\pi}{6}\right) + ix\sin\left(\frac{\pi}{6}\right) = v_0\frac{\sqrt{3}}{2} + x\frac{1}{2} \tag{4.4}$$

In (4.4) eq. after one-third a=1/3 of the total time is passed $t=\tau/3$, half $x/2$ of the total spring length is already contracted, and velocity still holds $0.86v_0$ of its maximum value.

Equation (4.2) solves a quarter of the period, for full period $T=4\tau$ and periodic process of spring contraction – expansion $t\in[0,\ \infty)$, where ω, $[s^{-1}]$ – angular frequency, we get:

$$E_0 e^{i\omega t} = v_0 \cos(\omega t) + ix\sin(\omega t);\quad \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \tag{4.5}$$

Euler's formula for complex numbers of unit energy value E_0=1 encompasses all parameters: energy, velocity, distance, and time–angle. It is also related to Pythagoras' theorem in trigonometric form for the right triangle $\cos(\Theta)^2+\sin(\Theta)^2=1$, in our case constant radius unit circle:

$$E_0 = K\cos(\Theta)^2 + U\sin(\Theta)^2 = 1 \ \Rightarrow$$
$$\Rightarrow\ E_0 = K\cos\left(\frac{\pi}{6}\right)^2 + U\sin\left(\frac{\pi}{6}\right)^2 = K\left(\frac{\sqrt{3}}{2}\right)^2 + U\left(\frac{1}{2}\right)^2 = 1 \tag{4.6}$$

The left side of the (4.6) eq. described the total energy value E_0=1 which is also the constant value of the circle radius, which is also an intrinsic property of the circle. The right side of the equation side gives a result of velocity and distance at any given time t moment, except in the final result we discard the imaginary number for length result in $l=x\sin(\Theta)$.

5. <u>The universality of square hyperbola function. Earth's gravity energy.</u>

In this chapter, we show an example where the Eulers equation not holds, but square hyperbola as a function $t_n = \delta / v_n$ is always true. In this example, a projectile with mass m=3.5 kg is shoot into the air with an initial Θ_0=90° angle with the ground and with initial velocity v_0=7.67202 m/s. Total energy transformation holds Pythagoreos theorem, but this solution is trivial because all projectile goes perpendicular to the ground in a straight line and all kinetic energy is converted and point h into potential gravity energy:

$$E_0 = K\cos(\Theta_0)^2 + U\sin(\Theta_0)^2 = K\cos(90^\circ)^2 + U\sin(90^\circ)^2 = U = 1$$

According to classical mechanics, an object dropped from height h with initial velocity v=0 will hit the ground with velocity v_0, or in our case opposite example perpendicular to the ground shoot object with initial velocity v_0=7.67202 m/s will reach maximum height h equal to:

$$h = \frac{v_0^2}{2g} = 3\ m \tag{6.0}$$

The algorithm sequence [6] to solve projectile time τ value to reach height h is:
m=3.5, N= 2×10^4, δ=h/N=1.5×10^{-5}, n=0, 1 ... N, for n=0: t_0=0, v_0=7.672, K_0=103.

$$\begin{gathered} U_n = mg\cdot n\delta \;\Rightarrow\; K_n = \overbrace{\frac{mv_0^2}{2}}^{K_0=const.} - U_n \;\Rightarrow\; v_n = \sqrt{\frac{2K_n}{m}} \;\Rightarrow \\ \Rightarrow\; t_n = \frac{\delta}{v_n} \;\Rightarrow\; \tau = \delta\sum_{n=1}^{N}\frac{1}{v_n}. \end{gathered} \tag{6.1}$$

With the help of calculus, the time equation τ is:

$$\tau = \sqrt{2}\sqrt{\frac{h}{g}} = 0.7820618..\ s \;\Rightarrow\; \tau = \delta\sum_{n=1}^{N\to\infty}\frac{1}{v_n} \tag{6.2}$$

$$\tau = \delta\sum_{n=1}^{19999}\frac{1}{v_n} = 0.778014..\ .\ s \tag{6.3}$$

Kinetic energy conversion in Earth gravity is very “easy” compared to spring or electric circuit, “easy” means straightforward kinetic energy converted into Earth gravity potential energy increases at every n-step $U_n = mg\cdot n\delta$. The final result is $U_N = 1,\ K_N = 0$. The spring experiment requires doing a recursive calculation of kinetic energy K_{n+1} for that reason intermediate (“imaginary”) energy E_n = K_n - U_n term is required, and the final results are $U_N = 0,\ K_N = 0,\ E_N = 0$. The explanation of recursive calculation is the counter-force of spring loading, not present in the Earth gravity potential field.

Algorithm sequence (6.1) has interesting mathematical relation of time t, velocity v, and height y change, from non-dimensional coefficient $a \in \mathbb{R}$ variable:

$$v_0 \;\Rightarrow\; h = \frac{v_0^2}{2g} \;\Rightarrow\; \tau = \sqrt{\frac{2h}{g}};\quad a = \begin{cases} t = \tau a \\ v = v_0(1-a) \\ y = ha(2-a) \end{cases};\quad \begin{cases} a \in [0,\ 1] \\ t \in [0,\ \tau] \\ v \in [v_0,\ 0] \\ y \in [0,\ h] \end{cases} \tag{7.0}$$

6. Square root $\sqrt{2}$ constant in case object is in Earth gravity force field

In spring and LC electric circuits we have π and e math constant which is also an irrational number. In Earth's gravity, we lack these two very important math and physics constants but instead have also irrational numbers $\sqrt{2}=1.414...$. That does not make it less important, we live on Earth not in an air atmosphere with normal atmospheric pressure but under Earth's gravity force, which to these days origin is not known to scientists, but in the priority of things, it is more important because the air atmosphere in which we are surrounded, including water, which boiling pressure point is directly related to atmosphere pressure depends from Earth gravity force magnitude, expressed as free fall gravity physics constant g=9.81 m/s^2. Earth's environment is submerged inside normal air pressure $p_0 \approx 10^5$ Pa atmosphere which is a consequence of Earth gravity force compression. Based only on the reason that we live in Earth's gravity field, is good to know the mathematical relation of a projectile shooting - falling time, velocity, and height change from independent variable.

From (7.0) eq. a system, time, velocity, and height change at $a=1/3$ are:

$$a=\frac{1}{3}=\begin{cases} t=a\tau=\dfrac{\tau}{3} \\ v=v_0(1-a)=v_0\dfrac{2}{3} \\ y=ha(2-a)=h\dfrac{5}{9} \end{cases} \qquad \begin{cases} a\in[0,\ 1] \\ t\in[0,\ \tau] \\ v\in[v_0,\ 0] \\ y\in[0,\ h] \end{cases} \tag{7.1}$$

At one-third of total flight time $t=\tau/3$, the projectile still has 2/3 of the initial maximum velocity $v=2/3v_0$, and the projectile already reached above halfway $y=5/9h>h/2$ of maximum height. If the projectile dropped from height h, in (7.1) eq. system dependency flips the order and the time countdown starts from the max. value to zero $a\in[1,\ 0]$, velocity and height change interval order too $y\in[h,\ 0]$, $v\in[0,\ v_0]$.

At any given moment in (7.1) eq. total energy at an absolute value of kinetic K=1 and potential U=1 energies quantity is preserved:

$$E_0=K+U=\frac{mv^2}{2}+mgy=K\left(\frac{2}{3}\right)^2+U\frac{5}{9}=1 \tag{8.0}$$

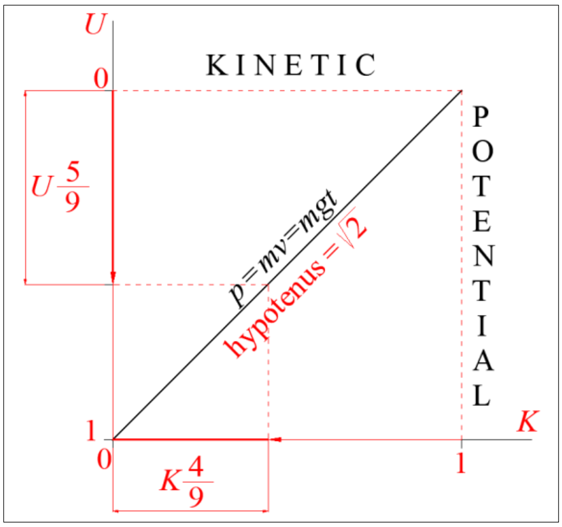

Fig . 5 Absolute value of unit energy term E_0=1 between kinetic (velocity) and Earth's gravity potential (height) energies.

Kinetic and potential energies are derived from the momentum p equation:

$$p = \frac{dU}{dv} = F_g t = mgt \;\Rightarrow\; \int dU = mgt \int dv \;\Rightarrow\; U = mgtv = mgy. \qquad (9.0)$$

$$p = \frac{dK}{dv} = mv \;\Rightarrow\; \int dK = m \int v dv \;\Rightarrow\; K = \frac{mv^2}{2} \;\Rightarrow$$
$$\Rightarrow\; \sqrt{2} = \sqrt{\frac{mv^2}{K}} \;\Rightarrow\; \sqrt{2} = \sqrt{\frac{mv^2}{U}} \;\Rightarrow\; \sqrt{2} = v\sqrt{\frac{1}{gy}} = const. \qquad (9.1)$$

The right isosceles triangle property is in scale $x=y$, $K=U=1$, see (Fig. 5), velocity and time change is independent of their mass m, mass cancels out $m/m=1$, see (9.1) equation. Earth's gravity field pushes all objects to the ground by constant acceleration value g and the pushing force is independent of their mass. The Hypotenuse of unit one of the right isosceles triangle is the hypotenuse of constant value $\sqrt{2}$.

7. Inertia, halfway, and Kurt Gödel's Incompleteness Theorem in Physics

In all chapters, we derived objects' velocity, distance, time; energy based on Euler's formula for complex numbers but not mentioned, inertia's physical effect. The inertia effect is proportional to object mass. All objects with mass m require time $t > 0$ to slow down, accelerate, and change direction of motion or move from a still position. The inertia effect is behind the explanation of the paradoxical situation.

The best example of the inertia effect is at an angle – $\Theta=45^o=\pi/4$:

$$E_0 = K\cos\left(\frac{\pi}{4}\right)^2 + U\sin\left(\frac{\pi}{4}\right)^2 = 1 \;\Rightarrow\; K\left(\frac{1}{\sqrt{2}}\right)^2 = U\left(\frac{1}{\sqrt{2}}\right)^2 \qquad (10.0)$$

In (10.0) eq. at an angle–time point $\Theta=\pi/4$, kinetic energy at this point reached equilibrium with spring potential energy, from the perspective of the force balance, motion forwards is impossible. Inertia plays the main role as an imaginary force that moves forwards even if the counter-force of the potential energy term has a bigger value than the kinetic energy term. Inertia effect based on physics a postulate. The postulate is a fact without rigid proof. An analogy of a postulate is an axiom in mathematics, for example, Euclidian axioms state that parallel lines never intersect, experimentally it is impossible to prove; even an axiom statement is obvious. Same to the inertia force postulate, it is obvious from the energy quantity conservation condition perspective.

Hyperbola explains the "velocity stagnation segment", for more details see chart (Fig. 3), more spring is contracted more contra-force it exerts back to object forward motion with velocity v. It is itself in the paradoxical situation, more spring contracted, less velocity, less velocity and higher spring contra-force more time it will take to contract spring, stagnation segment curve are very steep $\approx\pi/2$. Reversible process, after $\pi/2$ contraction K=0, we have an opposite effect of spring expansion when spring potential energy is again converted back into the object's kinetic energy.

In electronics exsist similar processes, current leads by $\pi/2$ voltage or voltage lags behind current by $\pi/2$ and vice versa.

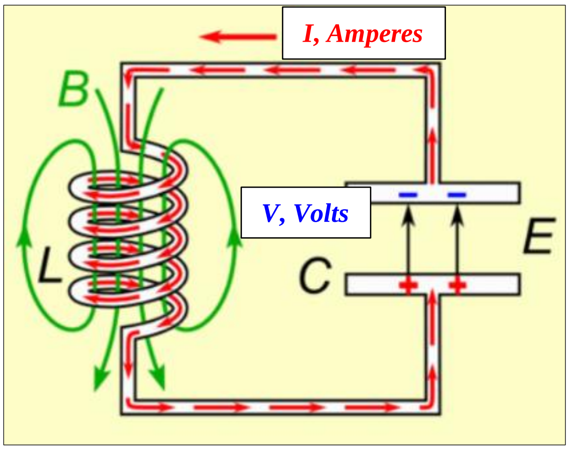

Fig. 4 Diagram showing the operation of a tuned circuit (LC circuit) [3].

In (Fig. 4), the coil (L) represents the kinetic energy term $K_{el}=LI^2/2$ of electric current flow–inductance, and capacitor (C) plates shows potential electric energy storage capacitance $U_{el}=CV^2/2$. These two expressions are quadratic $x^2+y^2=2$ and similar to our rigid body mechanics physics problem without total energy loss – $E_0=K_{el}=U_{el}$.

LC circuit charge-discharge time τ is equal to:

$$\tau=\frac{\pi}{2}\sqrt{LC},\ [s] \tag{11.0}$$

Here: L – coil inductance coefficient, [H], C – plates capacitance coefficient, [F], I – the electric current of the circuit, [*Amperes*], V – voltage of a circuit, [*Volts*].

The equation above is not 100% the same as our expression of time τ, (3.0) equation, even though we consider that Nature always plays by the same rules without any exceptions from the rule. What happened?

In our example k=1500 N/m – spring's elasticity coefficient. The inverse coefficient of elasticity is spring's rigidity coefficient $C_k=1/k=6.67\times10^{-5}\ m/N$. Spring rigidity coefficient $C_k=1/k$ shows spring contraction – expansion length per one Newton force applied to rigid material (spring). Another example of an inverse physics constant (coefficient) is specific volume $v=1/\rho$, [m^3/kg], specific volume is an inverse measurement unit of density. Specific volume measurement unit shows a volume per one kilogram of material mass. The usefulness of such inverse coefficients is to make physics expressions neat.

With the new modified coefficient k, the time τ (3.0) equation is equal to:

$$\tau=\frac{\pi}{2}\sqrt{\frac{m}{k}}\ \Rightarrow\ \tau=\frac{\pi}{2}\sqrt{mC_k},\ [s] \tag{11.1}$$

The equation (11.1) eq. in rigid body mechanics is the exact expression of the electric LC circuit time τ solution, (11.0) equation. The Fractal Nature of physics is mesmerizing it always holds a pattern. To be a physicist memory as such thing not required☺, need to know and memorize one specific equation from huge amounts of formulas, and apply again with a self-similar fractal pattern in mind to another similar physics behavior, for example, from Newtonian rigid body mechanics to electricity:

$$E_0e^{i\Theta}=I\cos(\Theta)+iV\sin(\Theta) \tag{12.0}$$

Another strange fact is time-angle and related to imaginary numbers. Angle is a convenient representation of time, that duration per seconds' units, see (4.2) equation. Euler's equation shows rotation in our case arrow of unit energy arrow on a circle. That gives some thought that about time measurement unit itself < *Where geometry allowed the exploration of space, Hamilton believed, algebra allowed the investigation of "pure time", a rather esoteric concept he had derived from Immanuel Kant that was meant to be a kind of Platonic ideal of time, distinct from the real-time we humans experience*. [5]> It is based on very stubborn Euclidian geometry math teacher Lewis Carroll (1832-1898) who mocked new math including complex numbers, in his very well-written children's book "Alice in Wonderland". He criticized radical new ideas in math. Including imaginary numbers together with the quaternions concept, which was discovered by mathematician Rowan Hamilton (1805 -1865) who tried to explain rotation outside the rigid body Newtonian mechanical world. In my view, an attempt is worth the effort. We concluded that in spring and in Earth's gravity case time change is based on isosceles hyperbola progression $t=dx/dv$. In all classical mechanics, dt is linear $v=dx/dt$, in our case space (distance) is linear as a product of time and velocity. Maybe "pure time" is not related to the rotation but directly with angle change, and not with the circle but with isosceles hyperbola, it is a more general tendency because for Earth gravity field circle does not hold a pattern, but isosceles hyperbola do. Rowan Hamilton's effort was to investigate time beyond physics measurement units. One second is artificial units for convenience with agreements around the world as a standardized unit. A most primitive description of time is an event, and an event is a measure (observation) of change. In physics, the event is the duration between two arbitrary points $t_0=0$ and $t > 0$. And only possible candidate for a non-dimensional unit of the time change angle measurement unit measured in "π – pies" ☺. Rowan Hamilton cares about the depiction of rotation without the physics angular velocity concept, but spring loading-unloading is a linear physics problem, with no rotation involved. In (LC circuit) is an electric circuit loop so we expect circular periodic progression, anyway in both cases π and e constant are present. And in my view time can be easily linear $0 \to \pi/2$. We do not investigate cases when the projectile is shot with an angle with the x-axis $\Theta<\pi/2$. In that case, all energy – information is stored inside trigonometric functions. It is out of the scope of our investigative system. It is an obvious fact that projectile trajectory in the air under the influence of the Earth's gravity field has nothing to do with harmonic oscillation, a trajectory path directly related to sine and cosine trigonometric function. When we think of physics and trig function, we always relate to, periodicity, rotation, frequency, and angular velocity, but it can be a very conservative (linear) estimate of total energy based on Pythagoras theorem in trig. form for a right triangle where all sides of a triangle are straight lines. Lewis Carroll was a very conservative Victorian-age mathematician, but in particular cases Euclidian geometric and their axioms can explain things directly (linearly) without imaginary number help or involving rotation.

Mathematician Kurt Gödel's (1906 -1978) "Incompleteness theorem" states that in any reasonable mathematical system, there will always be true statements that cannot be proved. In our, it was an adaptation of Euler's formula for complex numbers for our physics problem on the basis that it fits our discrete algorithm sequence results. Nothing wrong with that, intuition and science go hand in hand with the intuition (heuristic) approach to a problem. However, it is not a systematically proven theorem. Kurt Gödel's incompleteness theorem interpretation for physics is that you can not create one postulate (axiom) or a specific set of rules that can enclose all physics problems. Keep in mind, an adaptation of the Eulers formula for our system is not complete - smooth step-by-step proof, and we discard imaginary numbers because it is not required for our final results. From the perspective of math, an imaginary number is necessary, without an imaginary number Euler's formula for complex numbers has no sense. To know your limitations, for example, Fourier series image generation pattern speed [4]. The same image is done by computer and by light itself (naturally). Nature doing math $2/(3\times10^{-9})\approx10^{9}$ times faster, taking into count that exists superfast computer programs with high-frequency (CPU), still no denying [4] that Nature doing calculations much faster compared to computer-generated digital images. Discrete methods especially based on a limit approaching infinity $N\rightarrow\infty$, have a limit itself because computers can not do infinitely many operations, and sacrifice accuracy over time of computation. Maybe Nature has another method that we do not completely understands or uses a different counting system, different numerals, and different method to deal with the infinity limit. Maybe, in this case, imaginary numbers are the main reason for the fast solution by Nature. In my view, the Eulers equation for complex numbers is the most universal formula of Pythagoras's theorem, but it requires imaginary numbers that in the end we discard as unnecessary calculation "leftover". Physics in general only cares about real value part results; energy, time, velocity, length, etc.

8. Conclusions

A spring contraction (Fig. 1) time τ result is based on a square hyperbola, unit radius R=1 circle mathematical model. The unifying formula of spring parameters is based on Euler's formula for complex numbers:

$$E_0 e^{\frac{i\pi}{2}a} = v_0\cos\left(\frac{\pi}{2}a\right) + ix\sin\left(\frac{\pi}{2}a\right), \quad \begin{cases} a\in[0,\ 1] \\ t\in[0,\ \tau] \end{cases}$$

Where E_0=R=1 is the absolute value of the energy of the system equal to the unit. That is Pythagoras's theorem for a circle, where the radius of the circle at any given moment–angle is a constant value. The analogy of spring contraction–expansion is LC electric circuit, where in ideal conditions an electric field with voltage potential energy U_{el} in the capacitor (C) converts its maximum energy into magnetic field electric current flow energy K_{el} in inductance coil (L). Symbolically formula of LC circuit charge–discharge time τ (11.0) eq. is identical to the spring compression - expansion formula time τ (11.1) equation. For Earth's gravity potential energy, time τ solution formula, instead of imaginary numbers and math constants π and e we have a constant that is a result of the integrand of momentum expression in a derivative form which is hypotenuse length of $\sqrt{2}$. The constant $\sqrt{2}$ is only applicable for Earth's gravity field where velocity, time height change is independent of body mass.

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