Elitist Non-dominated Sorting Genetic Algorithm: NSGA-II

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Multi-objective optimization problem

- Problems with more than one objectives – typically conflicting objectives
- Cars: Luxury vs. Price
- Mathematical formulation

Minimize $\mathbf{F}(\mathbf{x})$,

where $\mathbf{F}(\mathbf{x}) = \{f_i: \forall i = 1, M\}$,

$\mathbf{x} = \{x_j: \forall j = 1, N\}$

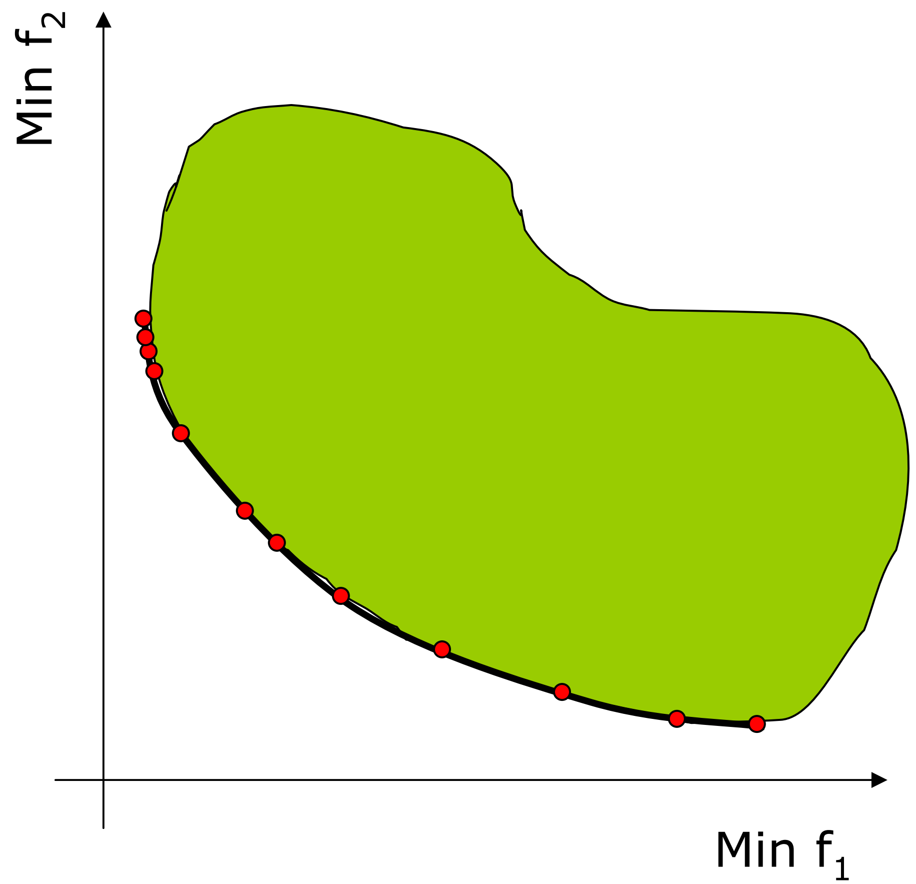
Subject to:

$\mathbf{C}(\mathbf{x}) \leq 0$, where $\mathbf{C} = \{C_k: \forall\ k = 1, P\}$

$\mathbf{H}(\mathbf{x}) = 0$, where $\mathbf{H} = \{H_l: \forall\ l = 1, Q\}$

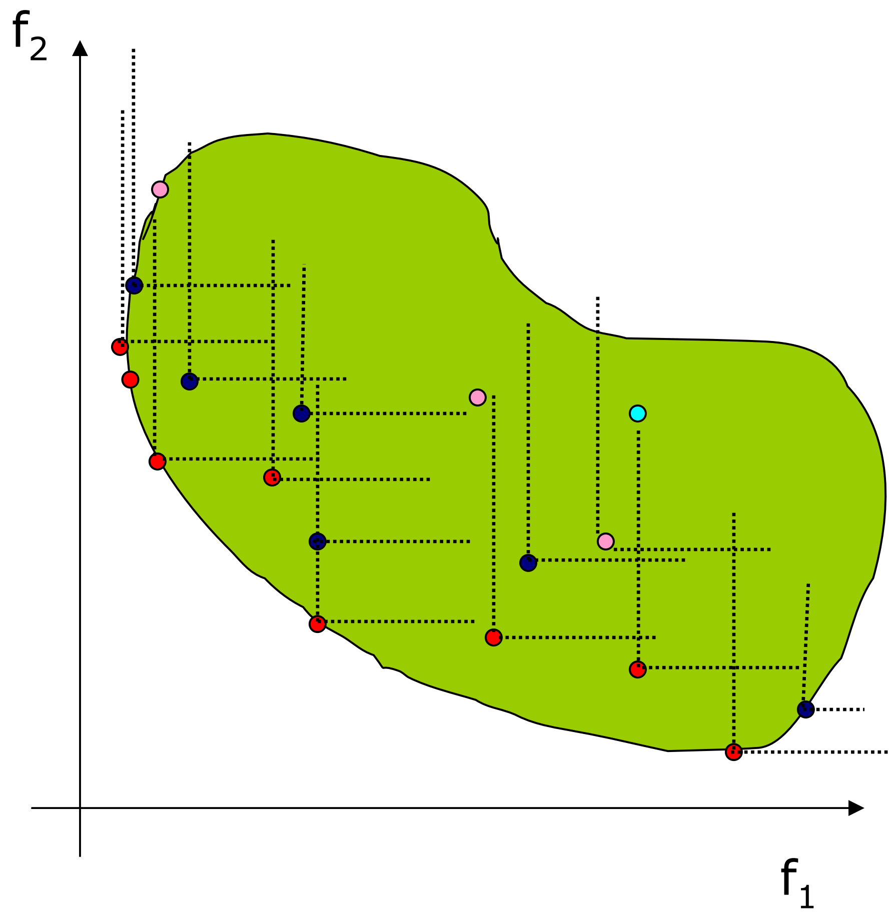
Pareto optimal front

- Many optimal solutions
- Usual approaches: weighted sum strategy, ε-constraint modeling, Multi-objective GA
- Algorithm requirements:
  - Convergence
  - Spread



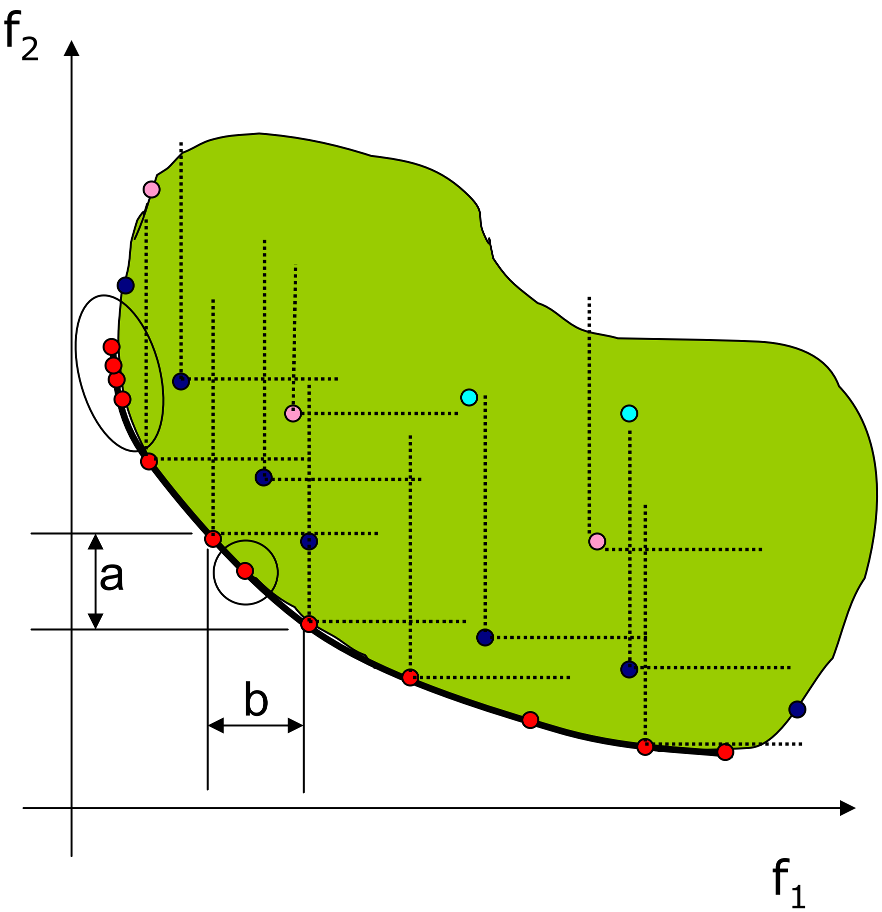
Terminology

- Non-domination criterion
- Ranking



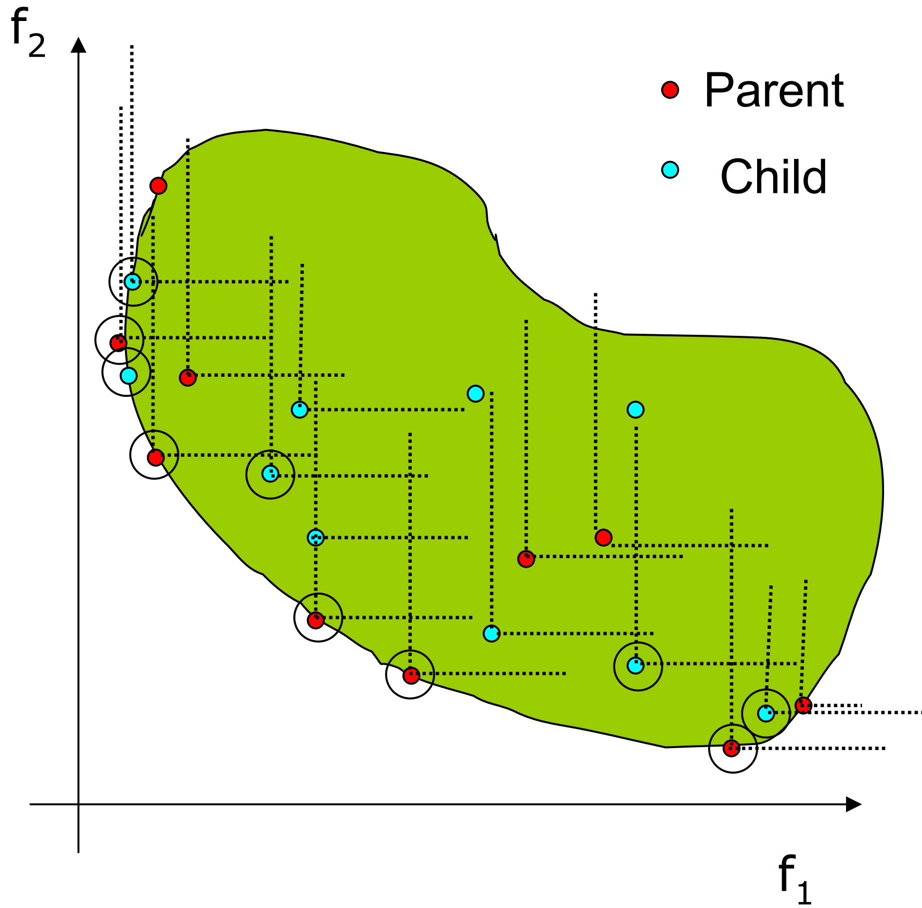
Terminology

- Niching – parametric
- Crowding distance
  - c = a + b
  - Ends have infinite crowding distance

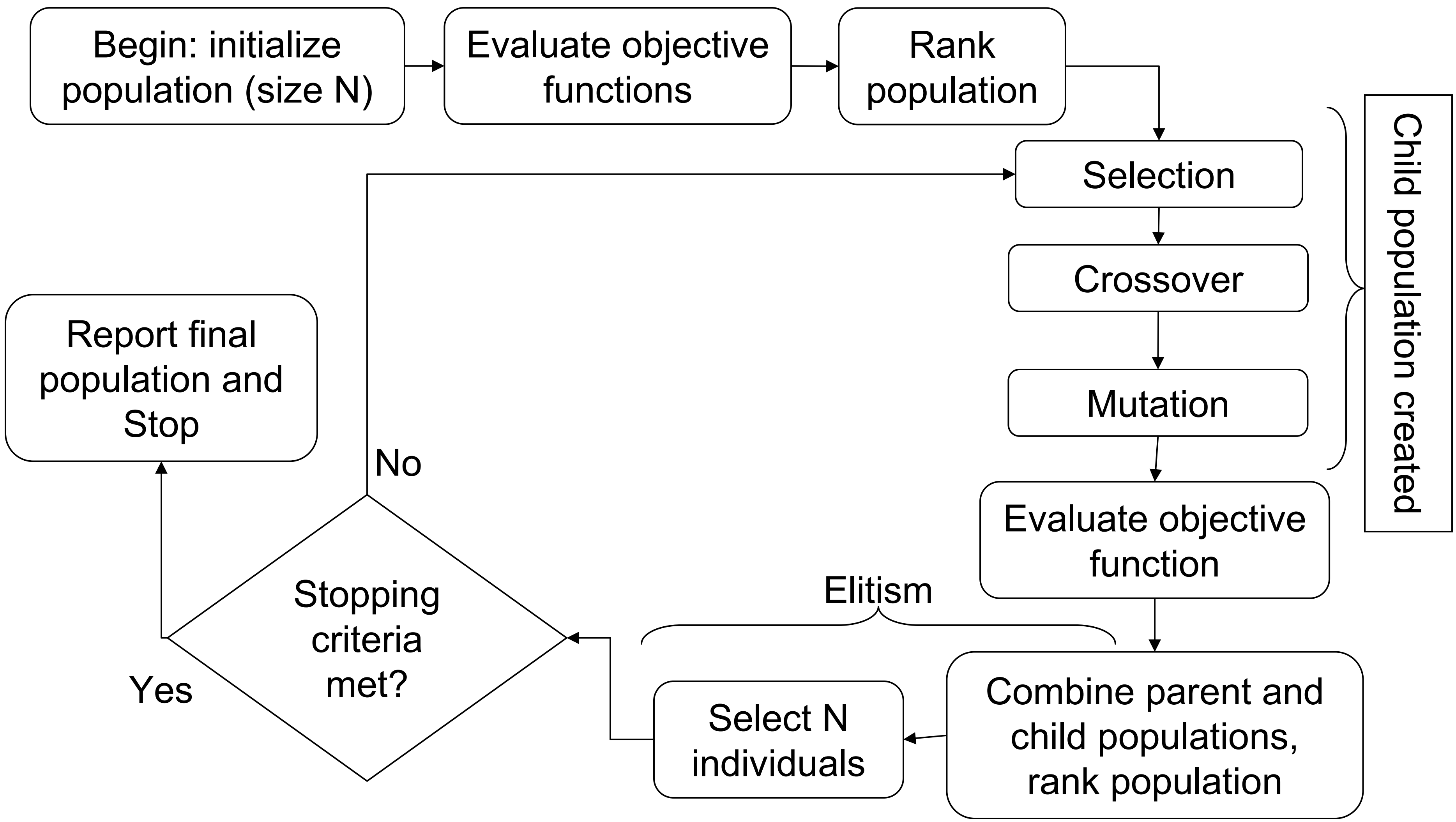


Elitism

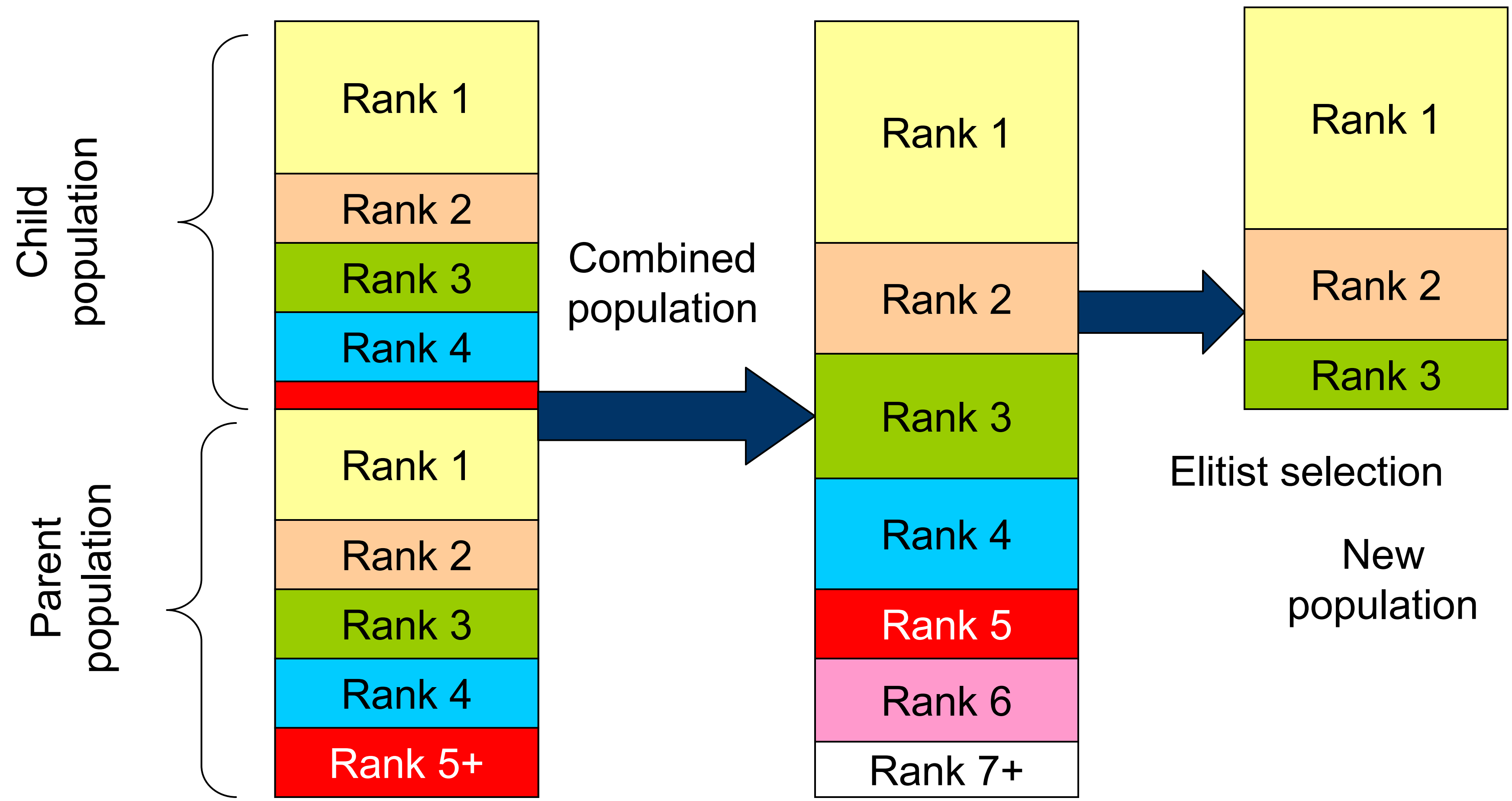
- Elitism: Keep the best individuals from the parent and child population



Flowchart of NSGA-II



Elitism Process



Example: Bicycle Frame Design

- Objectives
  - Minimize area
  - Minimize max. deflection
- Constraints
  - Component should be a valid geometry
  - Maximum stress < Yield stress

$$\left(\sigma_{\max} < \sigma_{allowed}\right)$$

  - Maximum deflection < Allowed deflection

$$\left(\delta_{\max} < \delta_{allowed}\right)$$

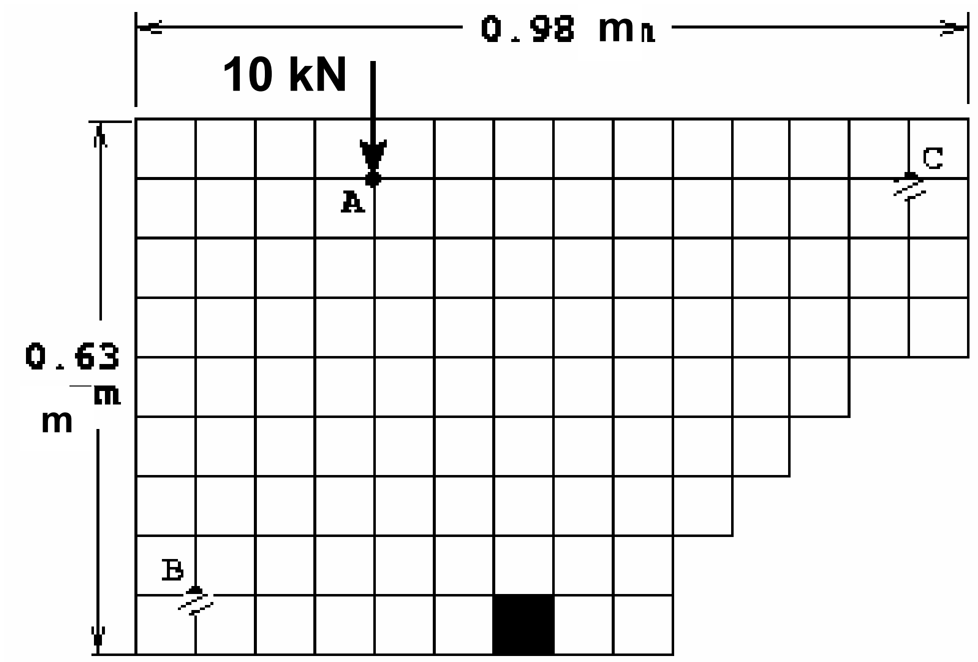


Plate thickness = 20 mm

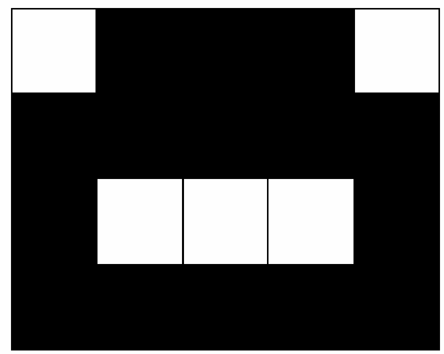
Problem Modeling

- Shapes are represented by binary strings, where ‘0’ represents void region and ‘1’ represents material region
- **Example :** A typical binary string is

  01110 11111 10001 11111

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Left to Right Representation



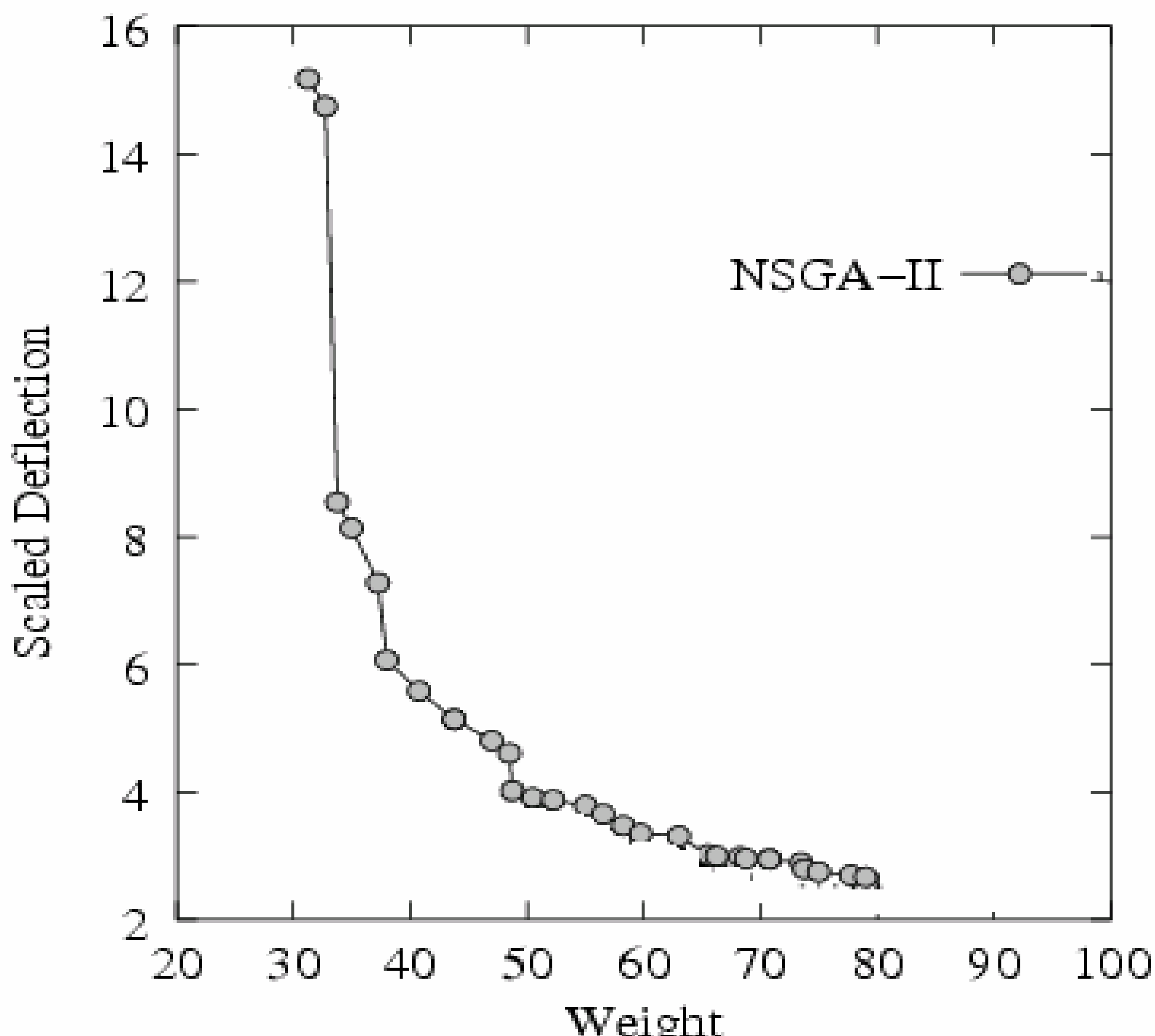
Shape Corresponding to the Binary String

Material Properties and GA Parameters

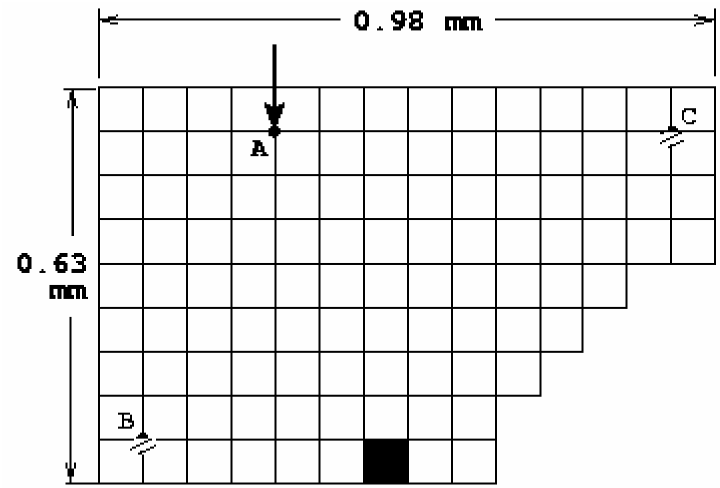
- Material Properties
  - Yield Stress $\left(\sigma_{allowed}\right)$ 140 MPa
  - Max Deflection $\left(\delta_{allowed}\right)$ 5 mm
  - Young's Modulus (E) 80GPa
  - Poisson's Ratio $\left(\nu\right)$ 0.25
- GA Parameters
  - Binary String Size (*L*) 14x9
  - Population Size 30
  - Crossover Probability 0.95
  - Mutation Probability 1/*L*
  - # of Generations 150

Pareto Optimal Front

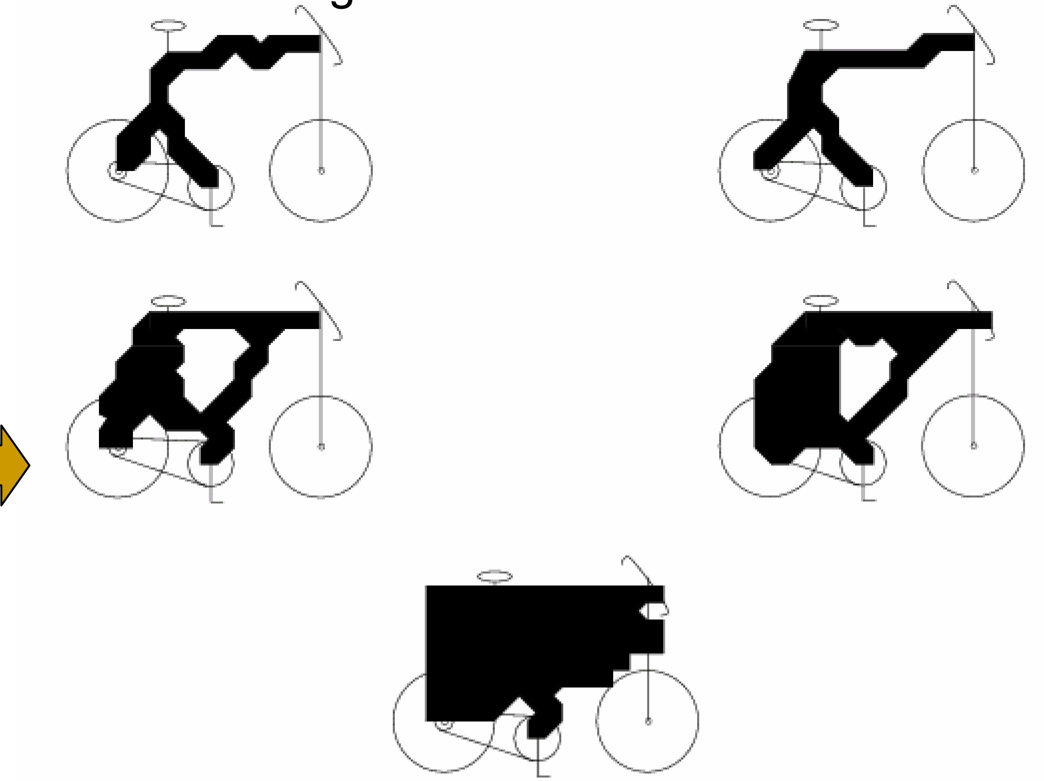
- Small increase in weight leads to large drop in deflection
- Similarly small change in deflection allows significant reduction of the weight



Optimal shapes



Least weight



Least deflection

Different conceptual designs

Some More Engineering Applications

- Structural designs of mechanical components
- Design of turbo-machinery components
- Bioinformatics – protein unfolding
- VLSI circuit designs
- Packaging
- …

Other related topics of interest

- Real-coded genetic algorithms
- Other multi-objective evolutionary algorithms
  - Pareto archived evolutionary strategies (PAES)
  - Strength Pareto evolutionary algorithm (SPEA)
  - ε – multi-objective evolutionary algorithm (ε-MOEA)
- Hybrid GAs
- Particle swarm algorithms
- Ant colony optimization