

## Singular Value Decomposition (SVD)

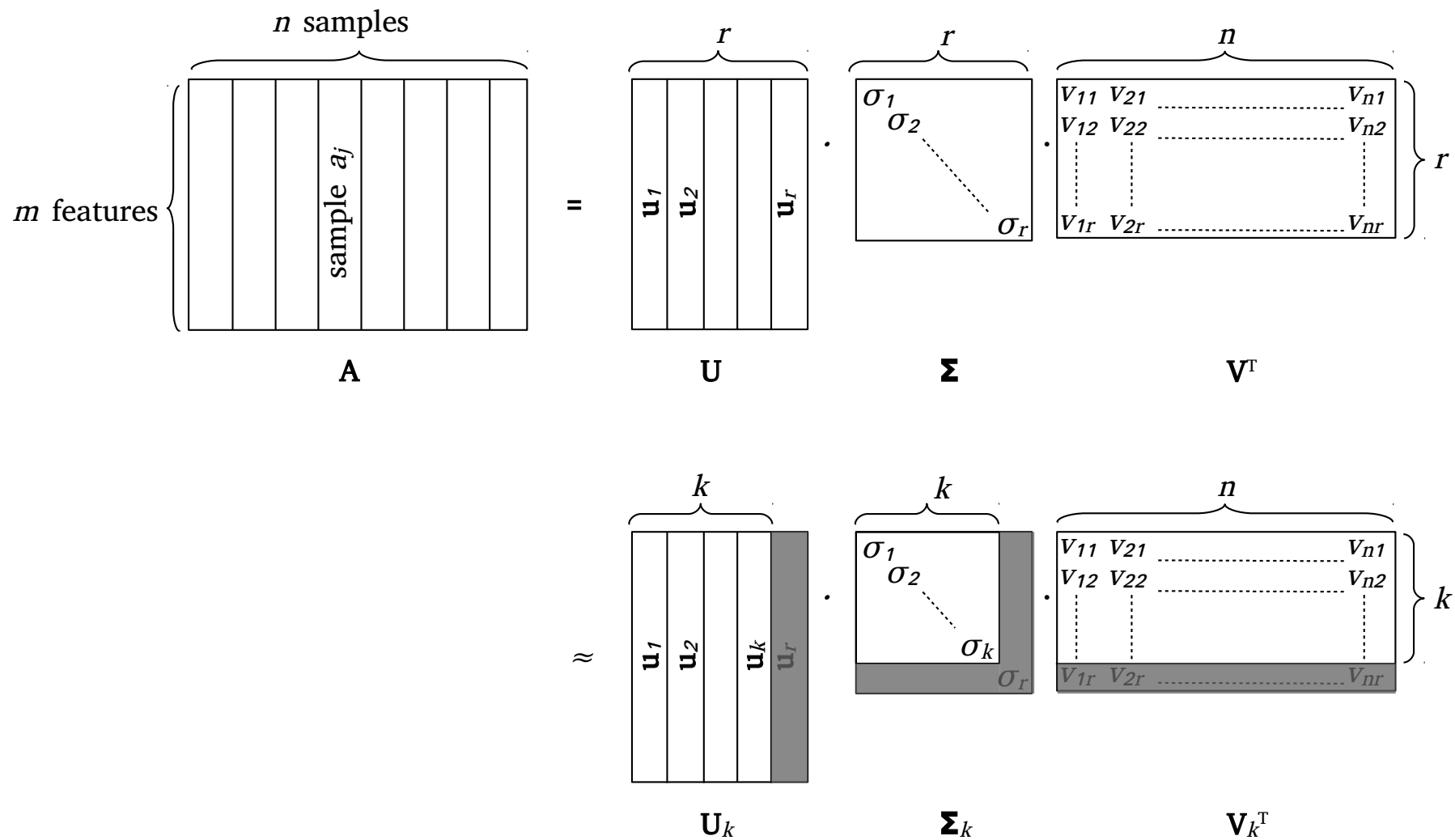
For any matrix  $\mathbf{A}$ , there exist  $\mathbf{U}$  and  $\mathbf{V}$  such that

$$\begin{array}{ccccc}
 \begin{array}{c} n \\ \hline \begin{array}{|c|} \hline \mathbf{A} \\ \hline \end{array} \\ m \end{array} & = & \begin{array}{c} r \\ \hline \begin{array}{|c|c|c|} \hline \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_r \\ \hline \end{array} \\ \mathbf{U} \end{array} \cdot \begin{array}{c} r \\ \hline \begin{array}{|c|} \hline \begin{array}{c} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_r \end{array} \\ \hline \end{array} \\ \mathbf{\Sigma} \end{array} \cdot \begin{array}{c} n \\ \hline \begin{array}{|c|c|c|} \hline \begin{array}{ccc} V_{11} & V_{21} & \dots & V_{n1} \\ V_{12} & V_{22} & \dots & V_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{1r} & V_{2r} & \dots & V_{nr} \end{array} \\ \hline \end{array} \\ \mathbf{V}^T \end{array} \Bigg\} r
 \end{array}$$
  

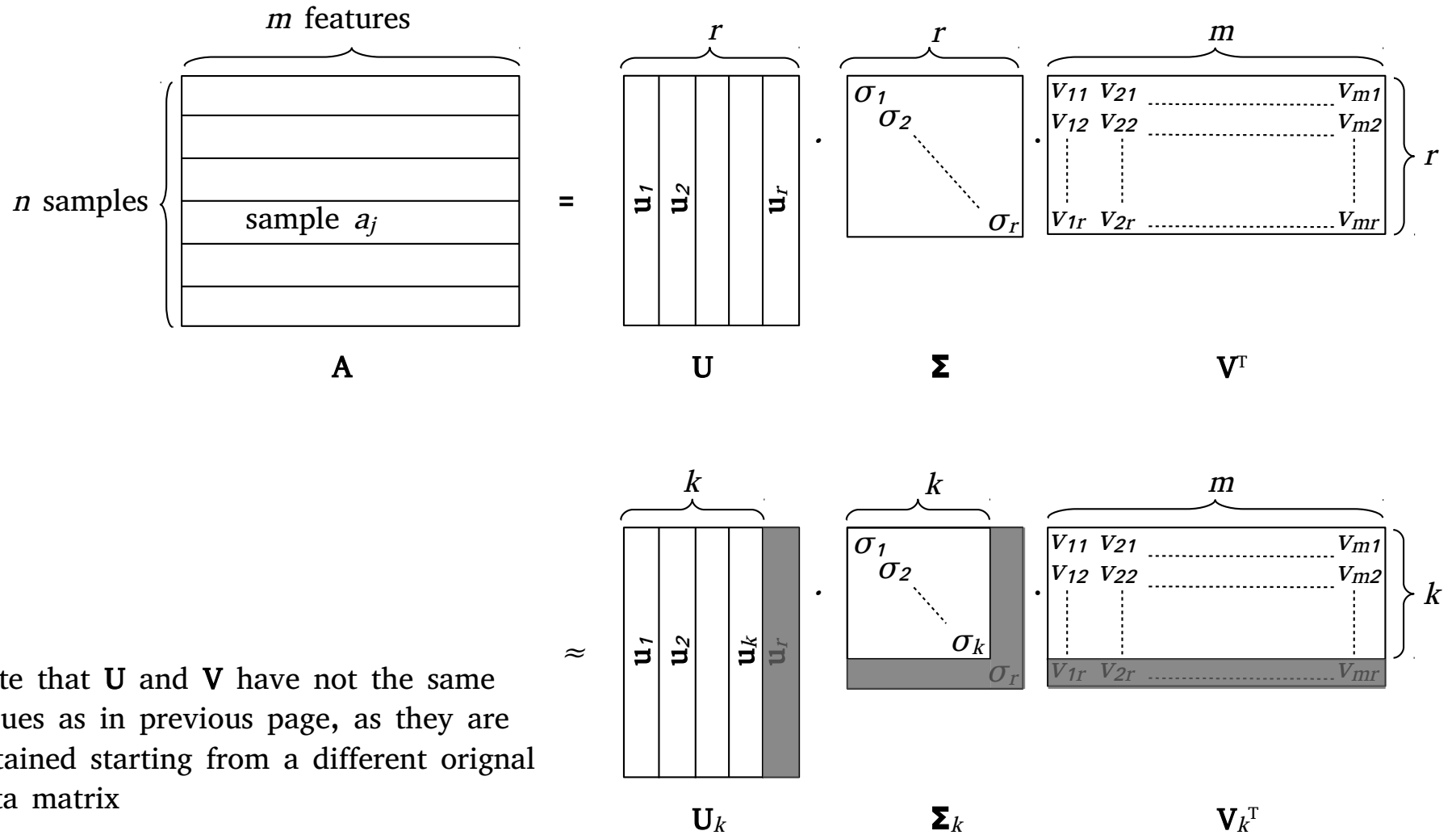
$$\begin{array}{ccccc}
 & \approx & \begin{array}{c} k \\ \hline \begin{array}{|c|c|c|} \hline \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_k & \mathbf{u}_r \\ \hline \end{array} \\ \mathbf{U}_k \end{array} \cdot \begin{array}{c} k \\ \hline \begin{array}{|c|} \hline \begin{array}{c} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_k \end{array} \\ \hline \end{array} \\ \mathbf{\Sigma}_k \end{array} \cdot \begin{array}{c} n \\ \hline \begin{array}{|c|c|c|} \hline \begin{array}{ccc} V_{11} & V_{21} & \dots & V_{n1} \\ V_{12} & V_{22} & \dots & V_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{1r} & V_{2r} & \dots & V_{nr} \end{array} \\ \hline \end{array} \\ \mathbf{V}_k^T \end{array} \Bigg\} k
 \end{array}$$

In data science, we can apply singular value decomposition in two ways.

Application 1 (as in the slides):



Application 2 (as in the code or as in this [Wikipedia](#) page):



Note that  $U$  and  $V$  have not the same values as in previous page, as they are obtained starting from a different original data matrix



Source: <https://www.alimentipedia.it/come-si-fa-il-formaggio.html>