MAE 280A

Linear system theory

Final Report

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CONTENTS

STATE-ESTIMATE FEEDBACK CONTROLL DESIGN

By Zhu's master thesis, we have state-space:

$$\begin{bmatrix} \dot{\theta(t)} \\ \dot{\phi(t)} \\ \theta(t) \end{bmatrix} = \begin{bmatrix} -13.528 & 13.528 & 175.457 \\ 17.735 & -17.735 & -115.561 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta(t)} \\ \dot{\emptyset(t)} \\ \theta(t) \end{bmatrix} + \begin{bmatrix} -90.456 \\ 118.585 \\ 0 \end{bmatrix} \cdot u(t)$$

$$\begin{bmatrix} \dot{\theta(t)} \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta(t)} \\ \dot{\phi}(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot u(t)$$

Where

$$A = \begin{bmatrix} -13.528 & 13.528 & 175.457 \\ 17.735 & -17.735 & -115.561 \\ 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -90.456 \\ 118.585 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Since our task is to design discrete-time linear state-estimate feedback controller, I use MATLAB code c2d to build new state-space for discrete-time.
Discrete-time state space is:

$$\begin{bmatrix} \dot{\theta(t+1)} \\ \dot{\emptyset(t+1)} \\ \theta(t+1) \end{bmatrix} = \begin{bmatrix} 0.8917 & 0.1165 & 1.581 \\ 0.1475 & 0.8475 & 0.9253 \\ 0.009416 & 0.00612 & 1.008 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta(t)} \\ \dot{\emptyset(t)} \\ \theta(t) \end{bmatrix} + \begin{bmatrix} -0.7791 \\ 1.02 \\ -0.004092 \end{bmatrix} \cdot u(t)$$

$$\begin{bmatrix} \dot{\theta(t+1)} \\ \dot{\emptyset(t+1)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta(t)} \\ \dot{\emptyset(t)} \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot u(t)$$

Where

$$AD = \begin{bmatrix} 0.8917 & 0.1165 & 1.581 \\ 0.1475 & 0.8475 & 0.9253 \\ 0.009416 & 0.00612 & 1.008 \end{bmatrix} \quad BD = \begin{bmatrix} -0.7791 \\ 1.02 \\ -0.004092 \end{bmatrix}$$

$$CD = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad DD = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Observer

In this discrete-time state-space, eigenvalues of AD are 0.7046, 1.0918, 0.9509, i.e., matrix AD is not stable. Although AD is not stable, [AD CD] is observable, which means I can place eigenvalues arbitrary. I assign three new poles (0.95, 0.94, 0.93) to get L (called Lmine in the following) by using MATLAB code `place`.

$$\dot{\hat{x}} = (\text{AD-Lmine}\cdot\text{CD})\hat{x} + \text{B}\cdot\text{u} + \text{L}\cdot\text{y}$$

Where

$$\text{Lmine}=\begin{bmatrix} -0.2548 & -0.3520 \\ -0.2686 & 0.1821 \\ 0.0010 & -0.0186 \end{bmatrix}$$

Feedback Control

I also find [A B] is reachable, thus I can again place eigenvalues arbitrary. Assigning (0.95, 0.94, 0.93) to find K (called Kmine in the following) by using MATLAB code `place`.

$$\dot{\hat{x}} = (\text{AD-Lmine}\cdot\text{CD-BD}\cdot\text{Kmine})\hat{x} + \text{L}\cdot\text{y}$$
$$\text{u=-Kmine}\cdot\hat{x}$$

Where

$$\text{Kmine}=\begin{bmatrix} -0.1156 & -0.1731 & -3.3744 \end{bmatrix}$$

To plot the figure of states & estimates, I add all $\hat{x}$ into the output. Now I have my state-estimate feedback controller to be

$$\begin{bmatrix} \widehat{\theta(t\dot{+}1)} \\ \emptyset(t\dot{+}1) \\ \theta(t+1) \end{bmatrix} = \begin{bmatrix} 1.0564 & 0.3336 & -1.0478 \\ -0.0032 & 0.8419 & 2.5160 \\ 0.0079 & 0.0186 & 0.9944 \end{bmatrix} \cdot \begin{bmatrix} \widehat{\theta(\dot{t})} \\ \emptyset(\dot{t}) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} -0.2548 & -0.3520 \\ 0.2686 & 0.1821 \\ 0.0010 & -0.0186 \end{bmatrix} \cdot \begin{bmatrix} \theta(\dot{t}) \\ \phi(\dot{t}) \end{bmatrix}$$

$$\begin{bmatrix} \widehat{\dot{\theta}(t)} \\ \widehat{\dot{\emptyset}(t)} \\ \widehat{\theta(t)} \\ V(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.1156 & -0.1731 & -3.3744 \end{bmatrix} \cdot \begin{bmatrix} \widehat{\theta(\dot{t})} \\ \dot{\phi}(t) \\ \theta(t) \end{bmatrix}$$

Where V(t)=u(t).

TASK 5

Equilibrium

With $\theta(0) = \dot{\theta}(0) = \dot{\emptyset}(0)$ =0, and noise of $\dot{\theta}$, $\dot{\emptyset}$ are also zero, the system is in equilibrium position (shown in Figure 1, Figure 2, Figure 3)

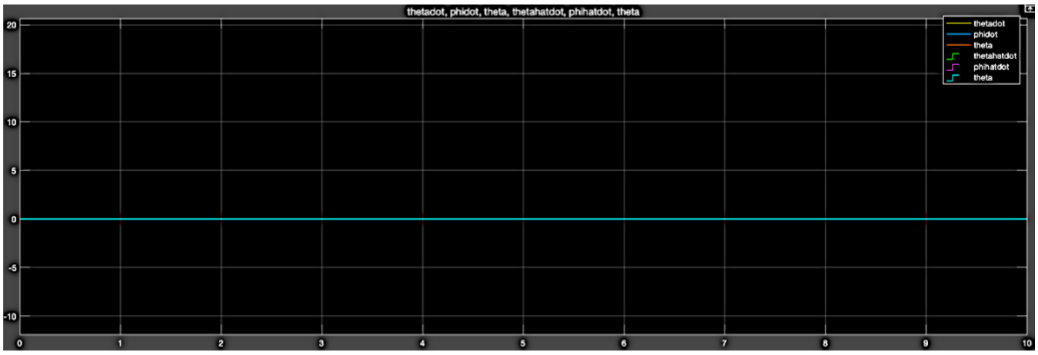

Figure 1 State-estimate when equilibrium

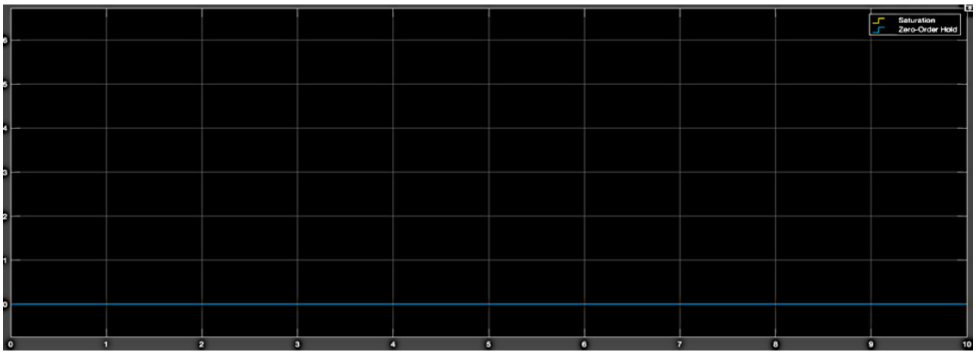

Figure 2 Applied Control (V(t)) when equilibrium

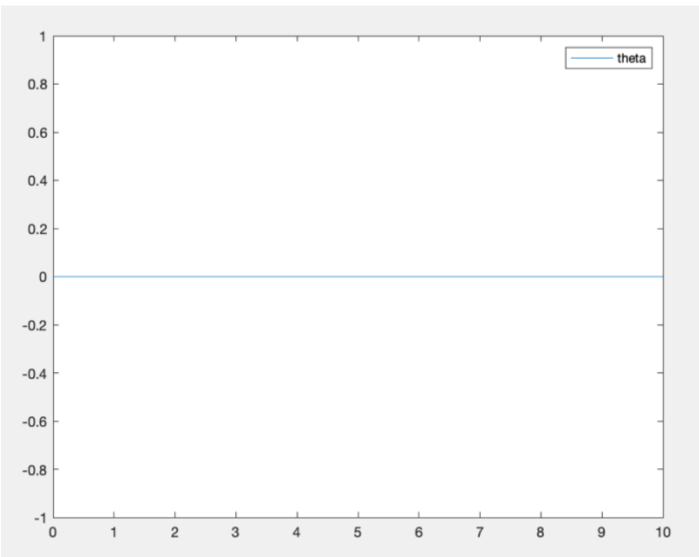

Figure 3 Theta when equilibrium

Nonlinear system stability

After trying several initial conditions of theta, I find nonlinear system cannot reach stable when thetaic=0.37 radian. (shown in Figure 4, Figure 5)

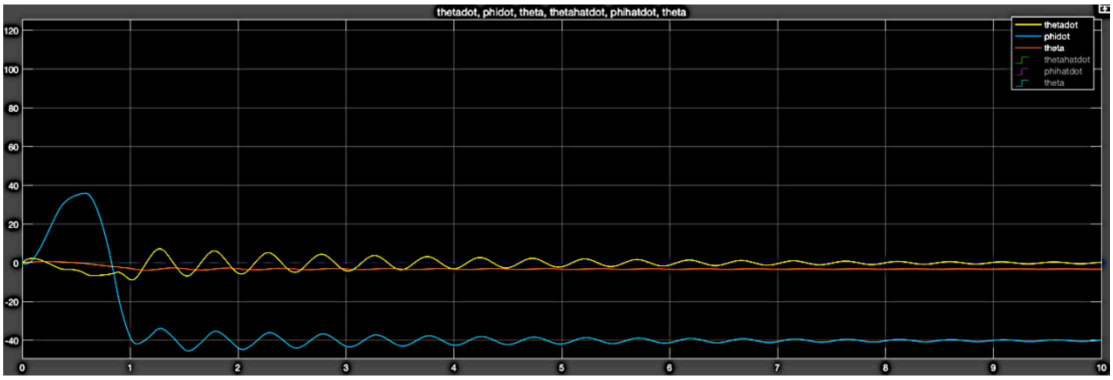

Figure 4 State of nonlinear system when thetaic = 0.37

The same when thetaic=-0.37

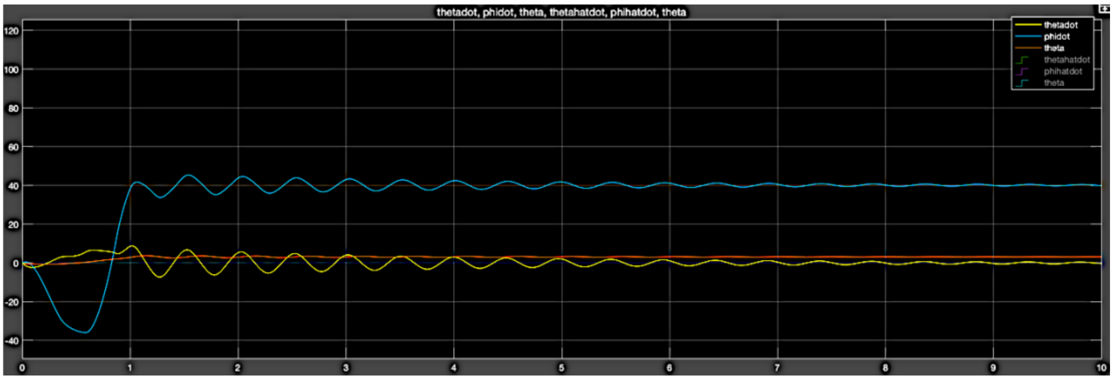

Figure 5 State of nonlinear system when thetaic =- 0.37

From Figure 4 and Figure 5, I conclude the range of allowable theta initial conditions for stability of the nonlinear system is -0.37< theta initial condition < 0.37.

Noise power

By selecting different noise power of thetadot, I plot several graphs of states & estimates. The following are some graphs that can show states and estimates goes unstable as the noise power grow.

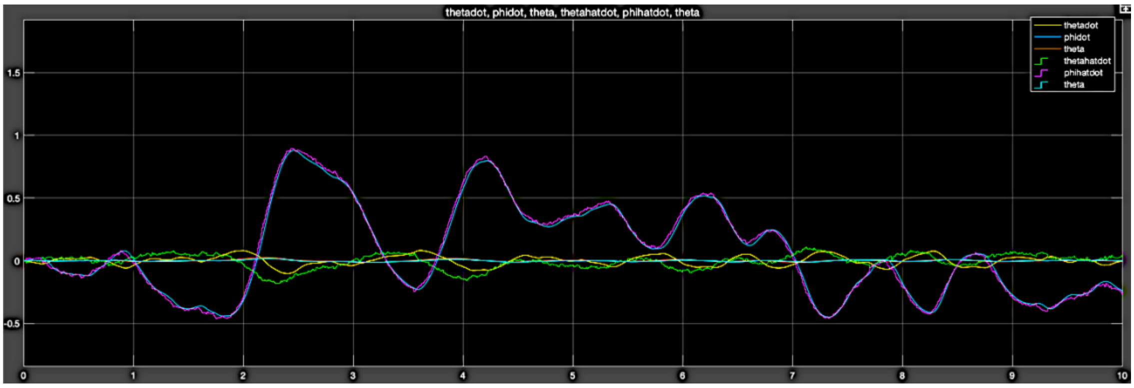

Figure 6 States and estimates when noise power of thetadot = 0.00001

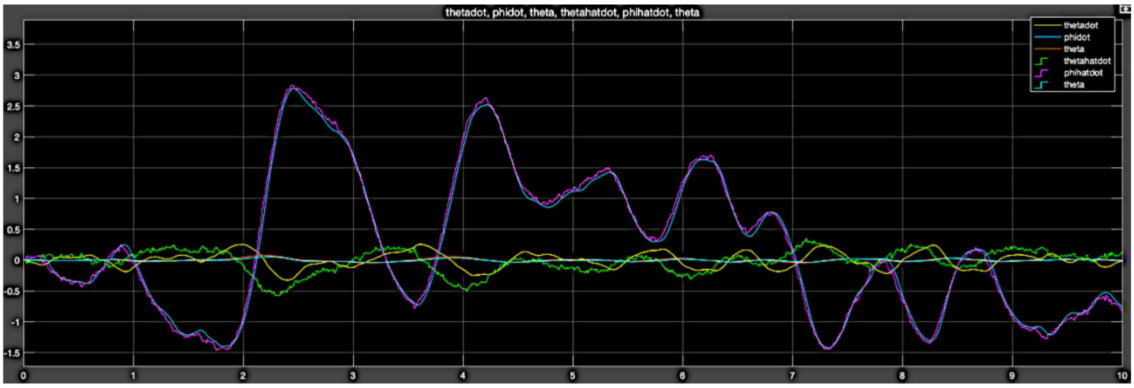

Figure 7 States and estimates when noise power of thetadot = 0.0001

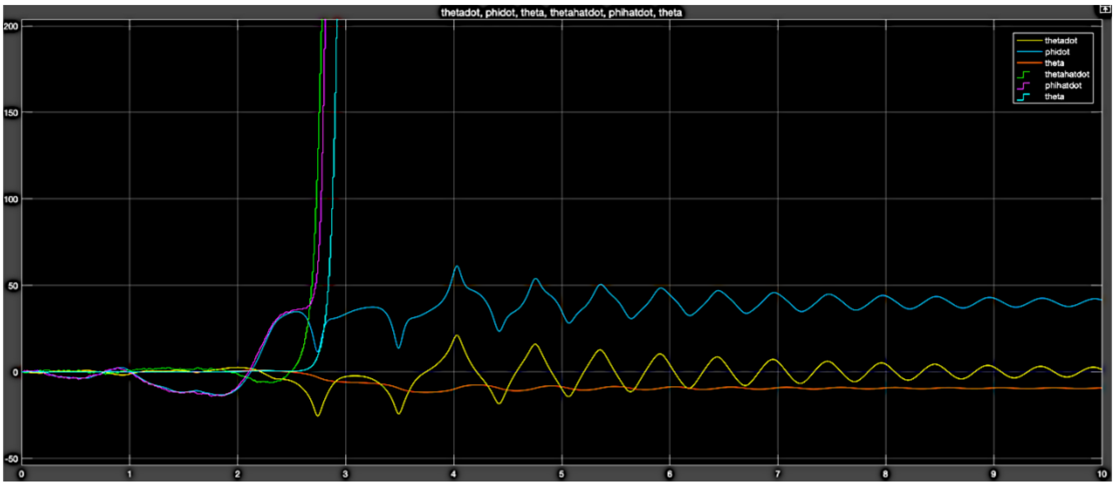

Figure 8 States and estimates when noise power of thetadot = 0.009

I notice that even a slight noise power of thetadot can lead to very noisy results. When noise power =0.0001, the system is almost unstable. When noise power = 0.009, estimates blow up to infinity. It is caused by eigenvalues I placed when designing. To allow bigger noise power, I have to choose eigenvalues more carefully, this part will be discussed in Noise rejection .

REDESIGN CONTROLLER

Stability

To improve the performance of stability, I would like to see the system goes to stable in minimum time. Therefore, I choose theta initial condition to be 0.36 radian (the largest value without becoming unstable), and try to place eigenvalues for better stability.

First, I plot states and estimates when eigenvalues = 0.95, 0.94, 0.93. The system goes to stable about 2.5t.

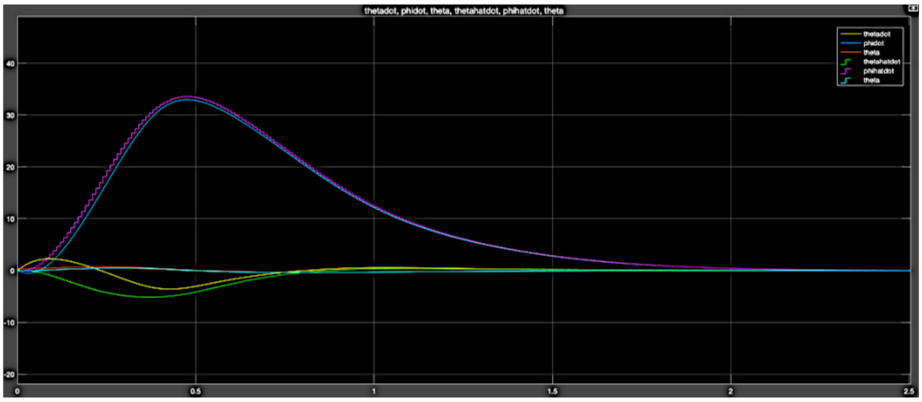

Figure 9 States and estimates when λ=0.95, 0.94, 0.93

Then I try other eigenvalues

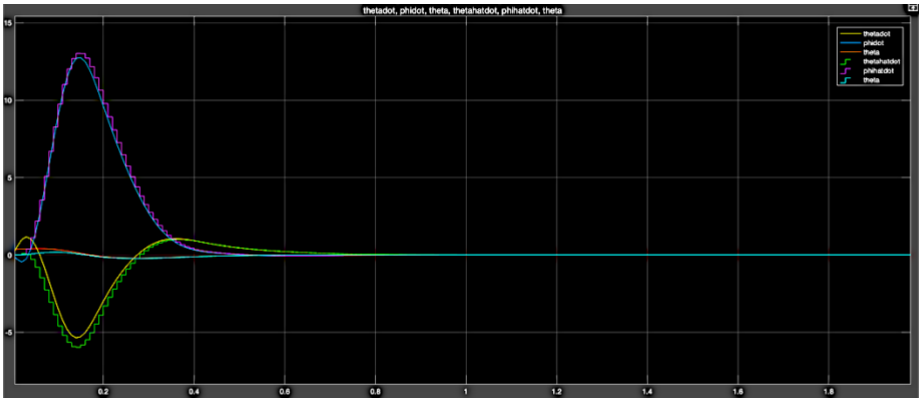

Figure 10 States and estimates when λ=0.82, 0.83, 0.84. Goes to stable 0.8t

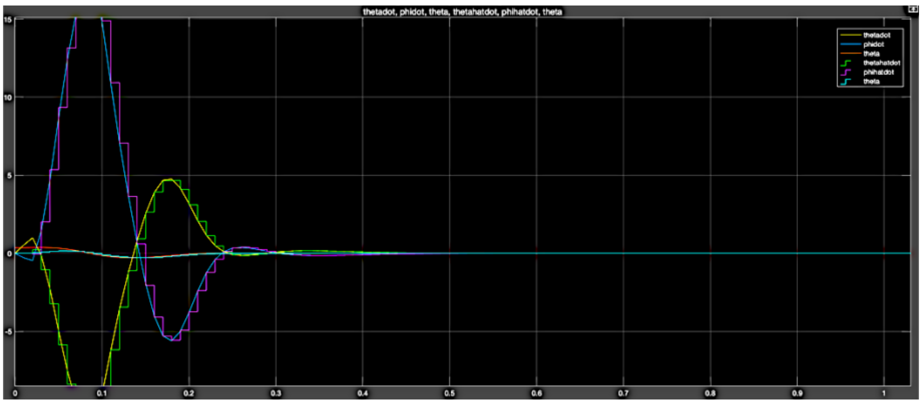

Figure 11 States and estimates when λ=0.69, 0.70, 0.71. Goes to stable 0.4t

Response time

When applied load hit ± 6V, it is the response time. Let's see the response time when eigenvalues are 0.95, 0.94, 0.93

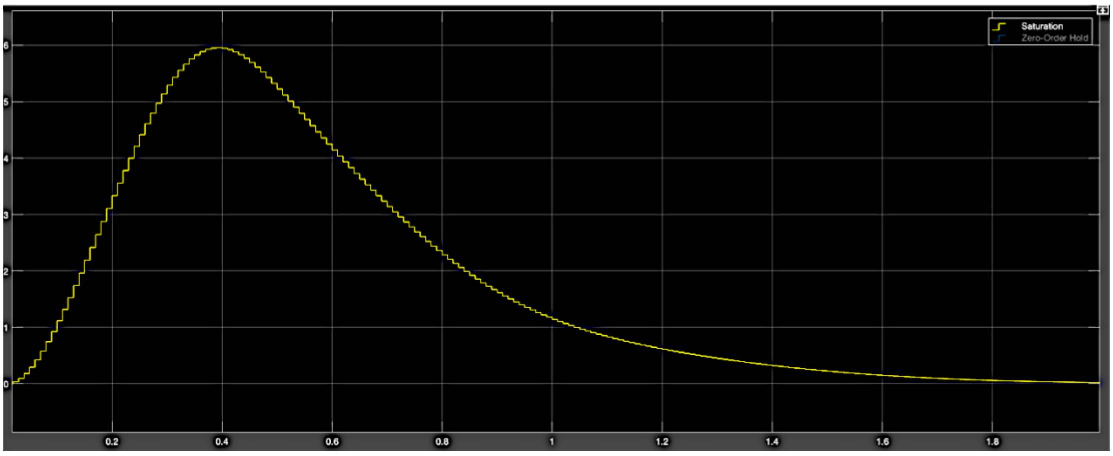

Figure 12 Applied load when eva=0.95,0.94,0.93. Response time=0.4t

Since when eigenvalues = [0.69,0.70,0.71] has the best performance of stability, let's plot its response time and compare with Figure 12.

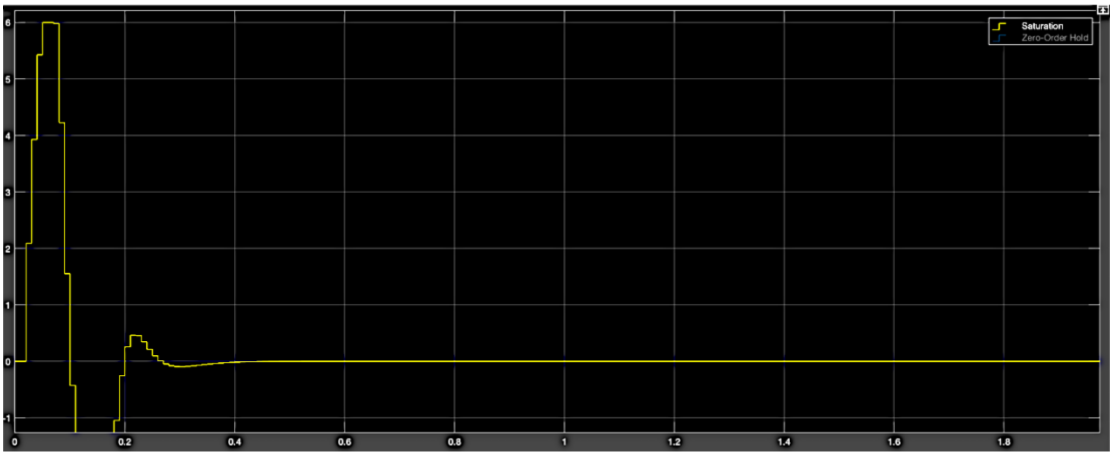

Figure 13 Applied load when eva=0.69,0.70,0.71. Response time=0.4t

Until now, these eigenvalues 0.69, 0.70, 0.71 actually improve the stability and response time. However, there are more problems need to be concerned when designing a state-estimate feedback controller.

Noise rejection

As I mentioned in Noise power, the eigenvalues can impact noise power severely if we do not choose them carefully. Let's try my new eigenvalues and see what will happen if applying noise power.

To compare with the original design, I simply set theta initial condition = 0, noise power of thetadot = 0.0001.

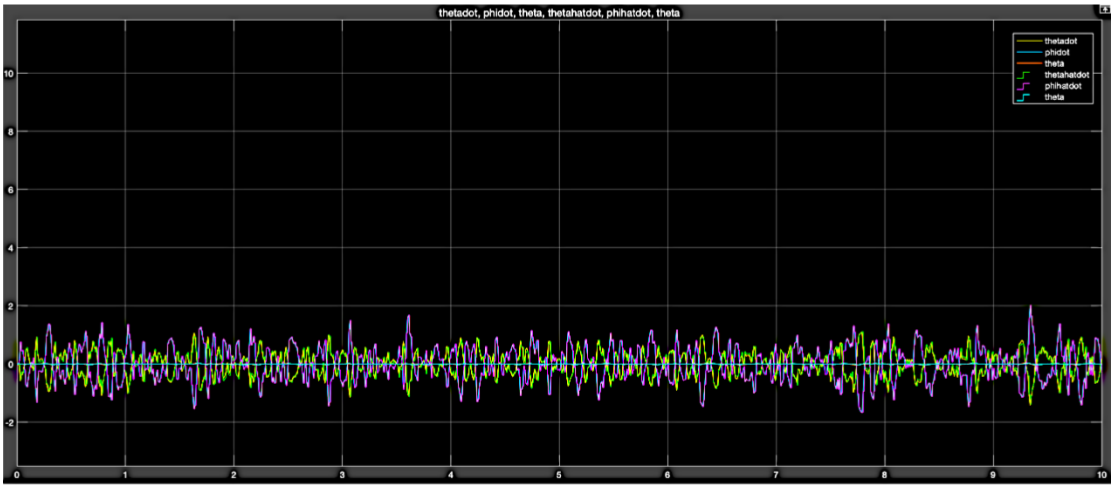

Figure 14 States and estimates when eva=0.69,0.70,0.71, noise power of thetadot=0.0001

Let's now compare Figure 14 with Figure 7. In Figure 7, noise of each curve is smaller than in Figure 14. However, in Figure 7, noise power of thetadot leads phidot curve goes to unstable.

Now, I apply noise power of phidot.

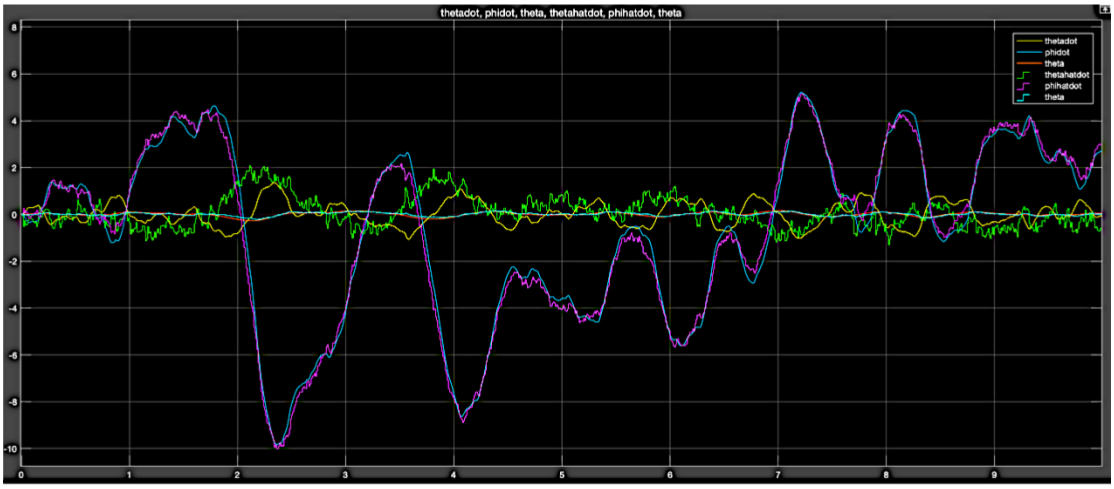

Figure 15 States and estimates when eva=0.95, 0.94, 0.93, noise power of phidot=0.003

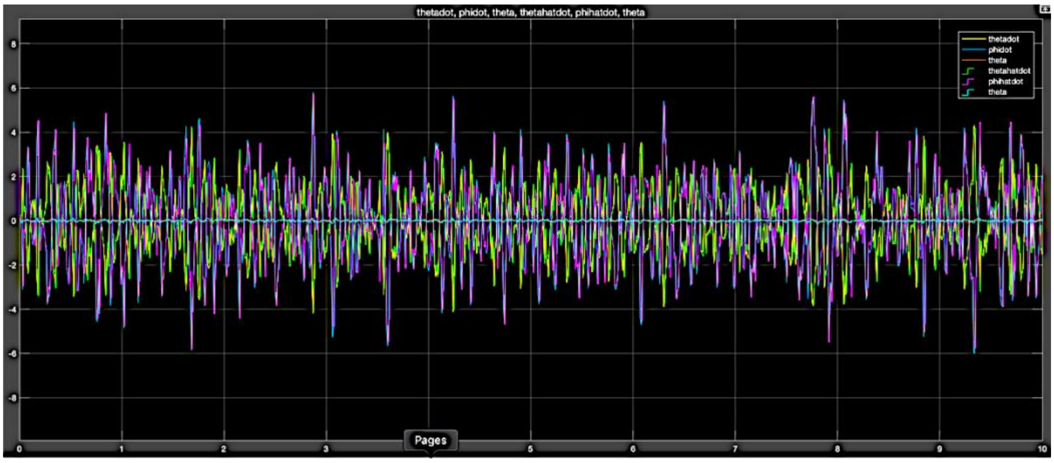

Figure 16 States and estimates when eva=0.69, 0.70, 0.71, noise power of phidot=0.003

Applying noise power of phidot has the same problem as applying thetadot. Although new eigenvalues make states and estimates stay in zero more likely, the noise of each curve is pretty large.

Now let's plot states and estimates by applying noise power with theta initial condition= 0.2 radian.

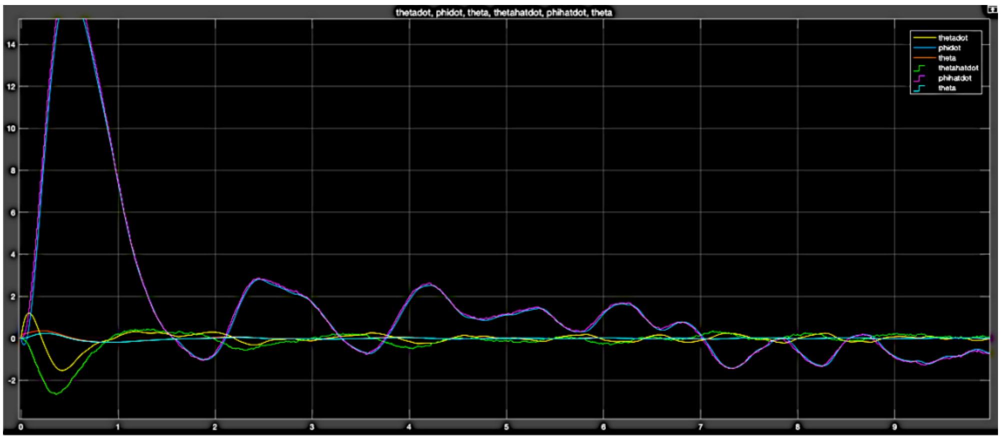

Figure 17 States and estimates when eva=0.93,0.94,0.95, noise power of thetadot=0.0001

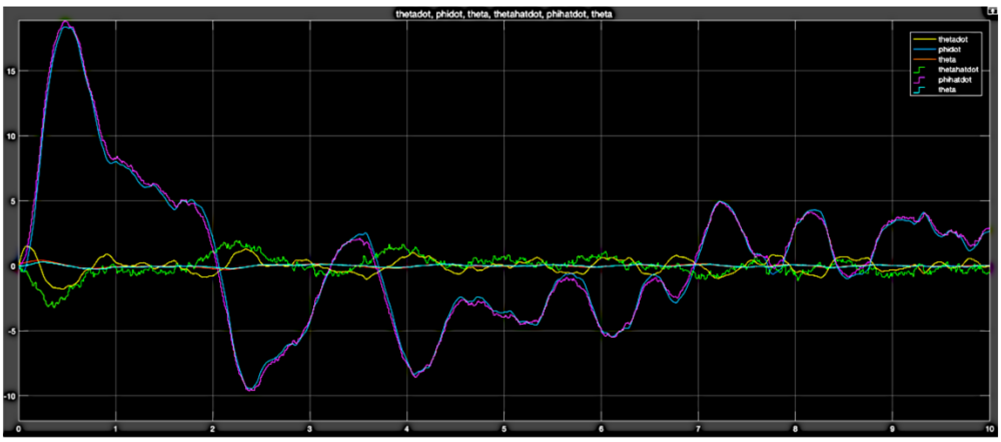

Figure 18 States and estimates when eva=0.93,0.94,0.95, noise power of phidot=0.003

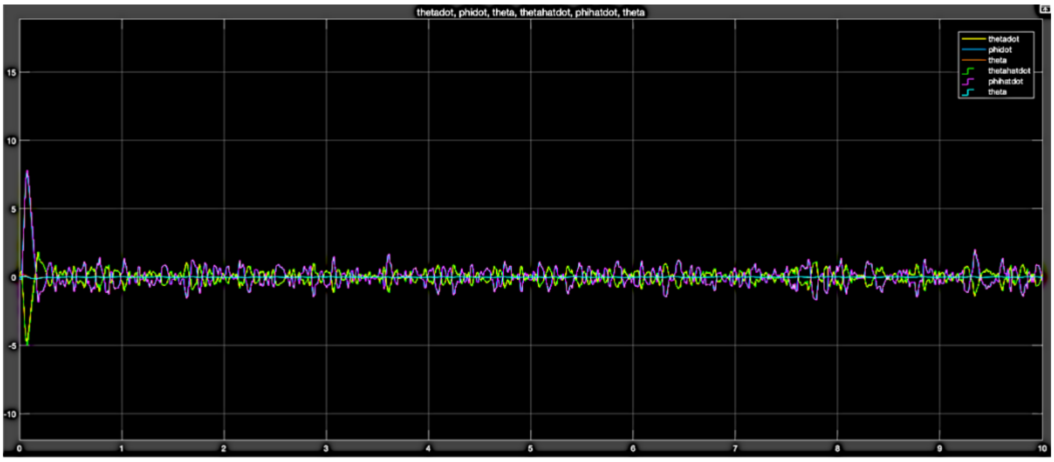

Figure 19 States and estimates when eva=0.69,0.70,0.71, noise power of thetadot=0.0001

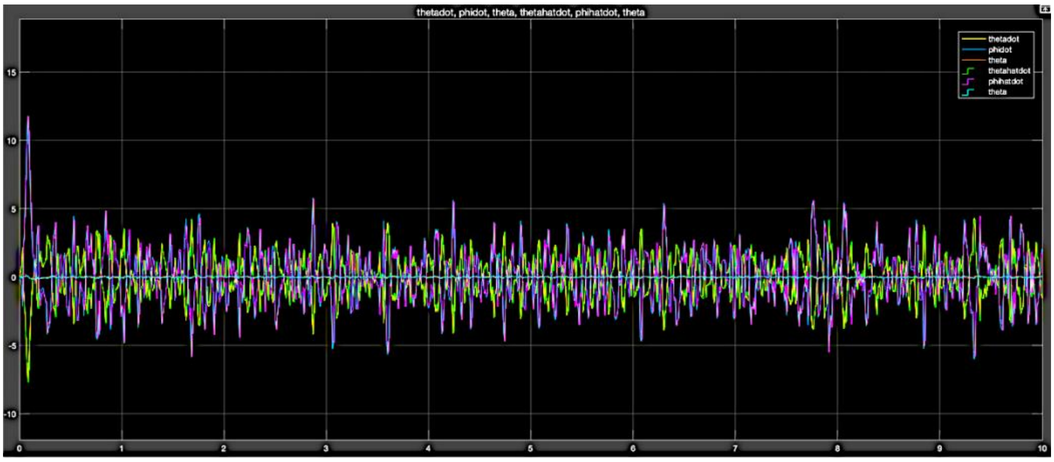

Figure 20 States and estimates when eva=0.69,0.70,0.71, noise power of phidot=0.003

From Figure 17, Figure 18, Figure 19 and Figure 20, I notice that although I improve the performance of stability and response time, the noise rejection goes worse. This is an understandable result, since the faster the system reach stable, the noisier the system is. However, the curve of phidot deformed severely in Figure 17 and Figure 18. To figure out what leads to these results, I decide to look into Amine and Lmine for eigenvalues=0.95,0.94,0.93

<u>States-estimate feedback controller when eva=0.93,0.94,0.95:</u>

$$\text{Amine}=\begin{bmatrix} 1.0564 & 0.3336 & -1.0478 \\ -0.0032 & 0.8419 & 2.5160 \\ 0.0079 & 0.0186 & 0.9944 \end{bmatrix} \quad \text{Lmine}=\begin{bmatrix} -0.2548 & -0.3520 \\ 0.2686 & 0.1821 \\ 0.001 & -0.0186 \end{bmatrix}$$

I notice $Amine_{11}$ is much larger than $Amine_{21}$, I guess it is the reason of curve deformation. Therefore, I change $Amine_{11}$=0.005 and plot equilibrium situation when applying noise power of thetadot=0.0001.

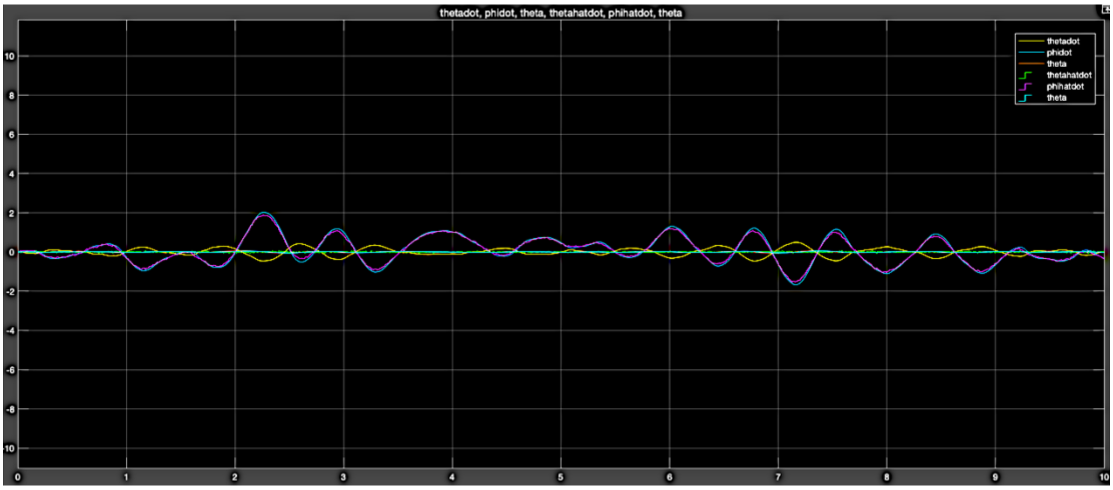

Figure 21 States and estimates by replacing Amine11 into 0.005

Comparing Figure 21 with Figure 7, after I change Amine11, the curve stays in zero more likely. Therefore, when designing a controller, elements in Amine matrix play important roles in performance. I will use this concept in Final redesign controller.

Region of attraction

Since the battery behind the Mip can only apply 12V, thus we devide into $\pm 6V$ and make it our saturation. To find region of attraction, I tried only some initial conditions, not much, and applied load blows up when thetaic = 0.37 just like the original one. Therefore, by replacing eigenvalues to 0.69,0.70,0.71 does not improve the performance of region attraction.

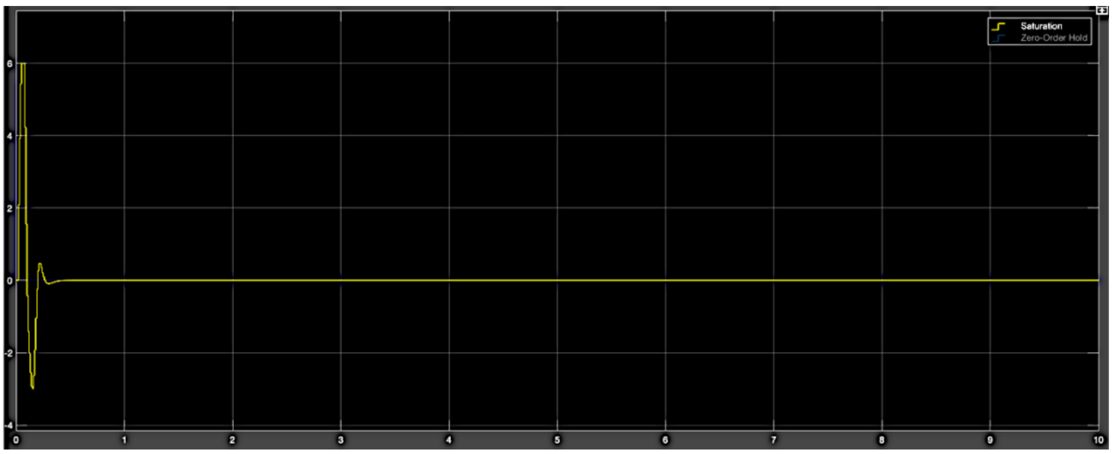

Figure 22 Applied load when eva=0.69,0.70,0.71, thetaic=0.37

Final redesign controller

After trying several eigenvalues, I choose eigenvalues to be 0.80, 0.81, 0.82 for states estimates feedback controller. Here are some figures comparing this final design with other two.

Stability

compare with Figure 9 and Figure 11

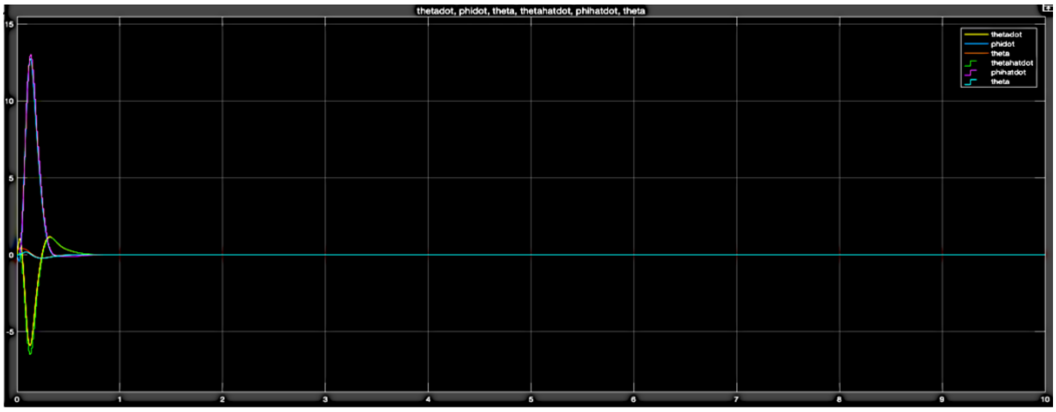

Figure 23 Final design – stability

Response time

compare with Figure 12 and Figure 13

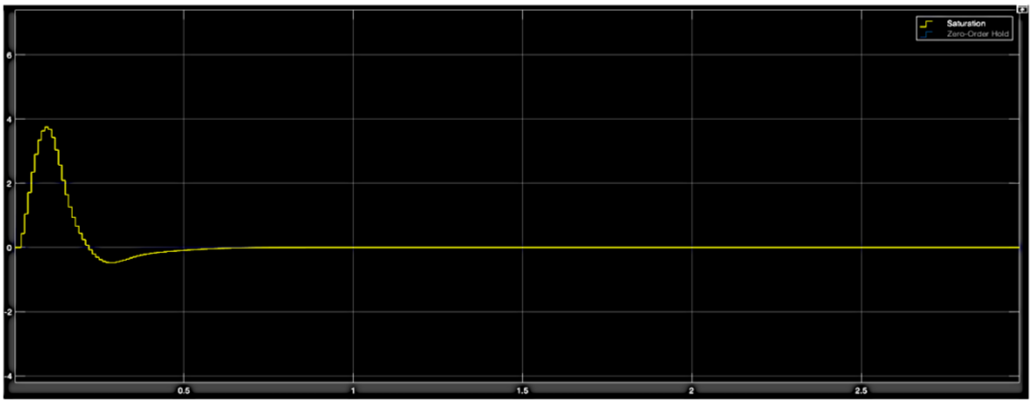

Figure 24 Final design – response time

Noise rejection

Equilibrium-noise power of thetadot=0.0001. compare with Figure 6 & Figure 14

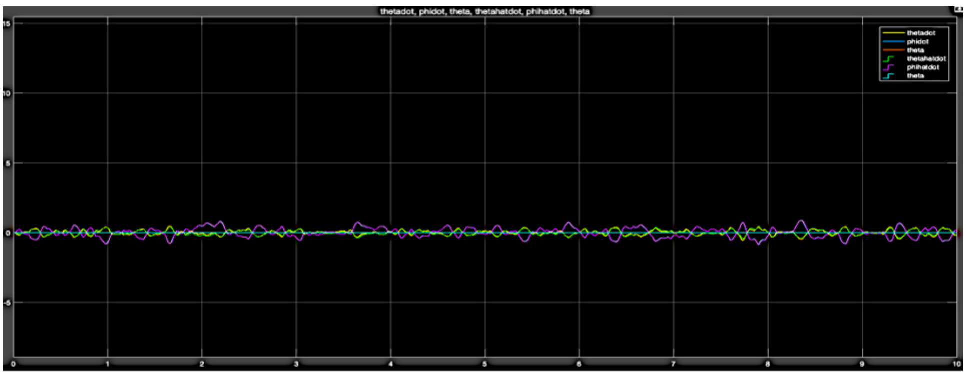

Figure 25 Final design – noise rejection to thetadot

Equilibrium-noise power of phidot=0.003. compare with Figure 15 & Figure 16

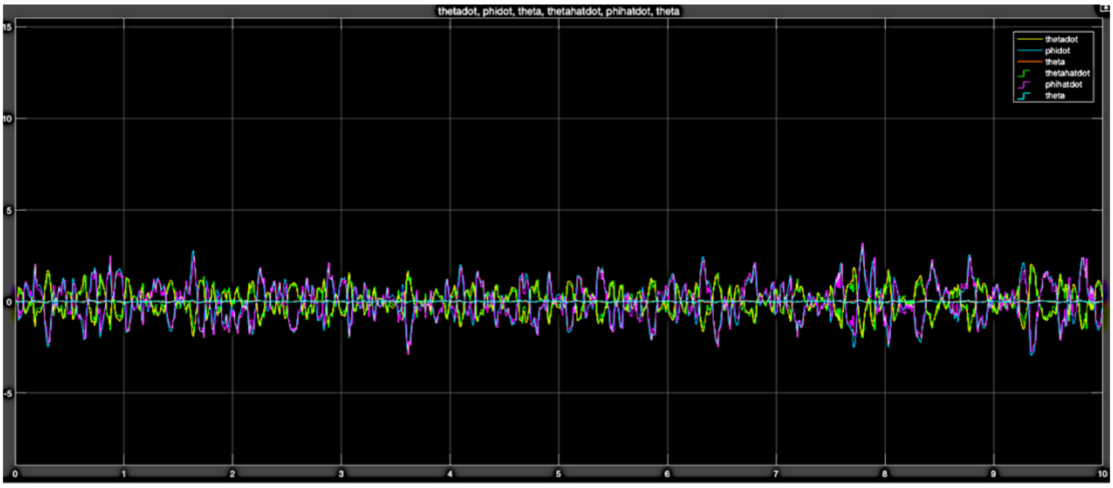

Figure 26 Final design – noise rejection to phidot

Theta initial condition = 0.36 radian with noise power of thetadot = 0.0001 compare with Figure 17 & Figure 19

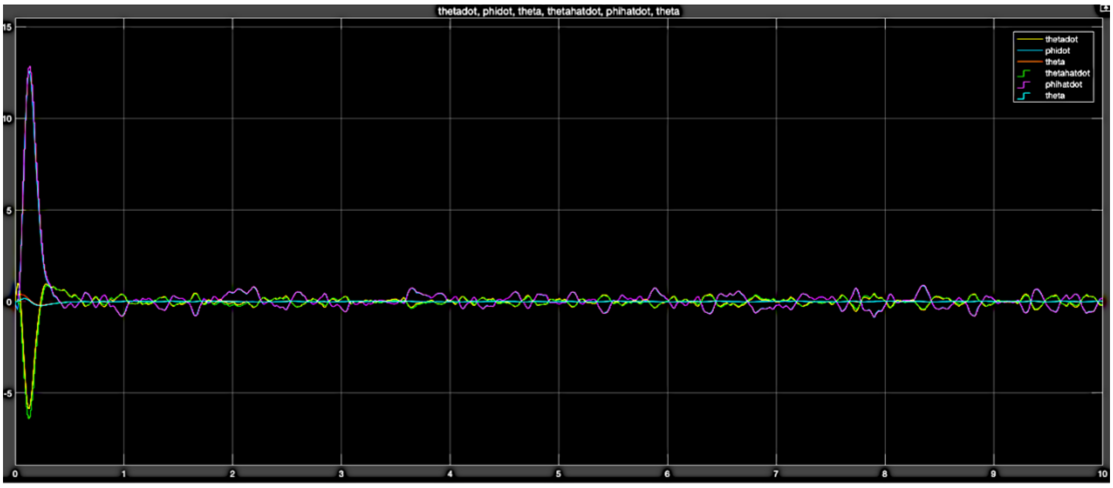

Figure 27 Final design – noise rejection to thetadot with thetaic

Theta initial condition = 0.36 radian with noise power of thetadot = 0.0001 compare with Figure 18 & Figure 20

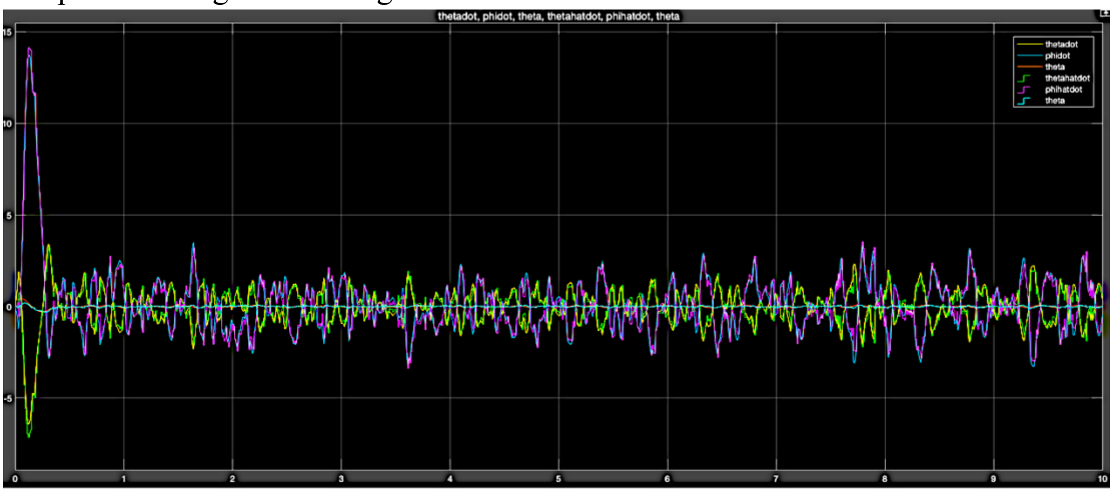

Figure 28 Final design – noise rejection to phidot with thetaic

Recalling I mentioned in Noise rejection, now I would like to check the elements in Amine when eigenvalues are 0.80,0.81,0.82

$$\text{Amine}=\begin{bmatrix} -0.3807 & -0.3749 & -10.1270 \\ 1.5592 & 1.5467 & 14.3998 \\ -0.0064 & 0.0383 & 0.9467 \end{bmatrix}$$

Although Amine21 is roughly four times of Amine11, it is much better than the original Amine (roughly 330 times). Therefore, the curve of states and estimates in final design are more likely to be zero, and that is what I glad to see.

<u>Region of attraction:</u>

Final design's region of attraction: Thetaic=$\pm$ 0.54 radian

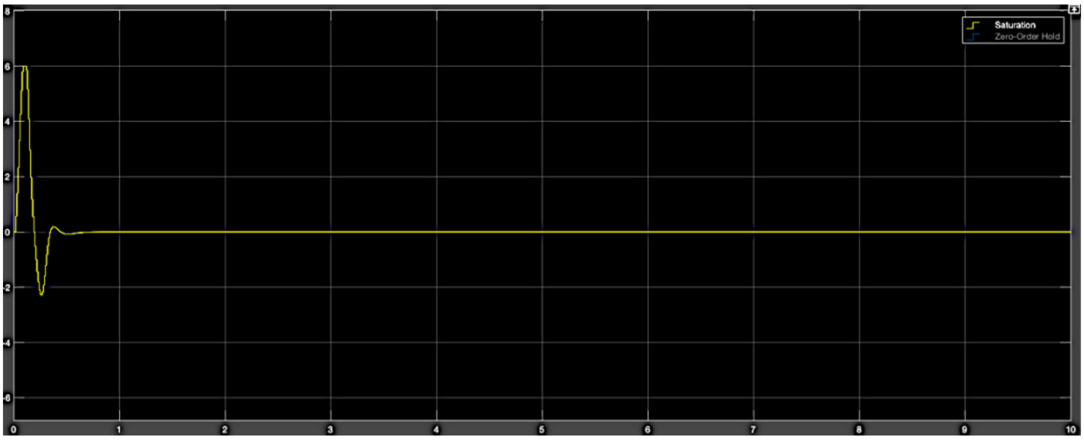

Figure 29 Applied load when thetaic =0.54

SETTING INITIAL CONDITION IN CONTROLLER BLOCK

By setting initial value in controller block, it means the system starting input $\dot{\theta}$, $\dot{\emptyset}$ with initial values. If I set initial value in controller block as 0.1, the curve of $\dot{\theta}$, $\dot{\emptyset}$ start at 0.1, we can see that from states and estimates graph.

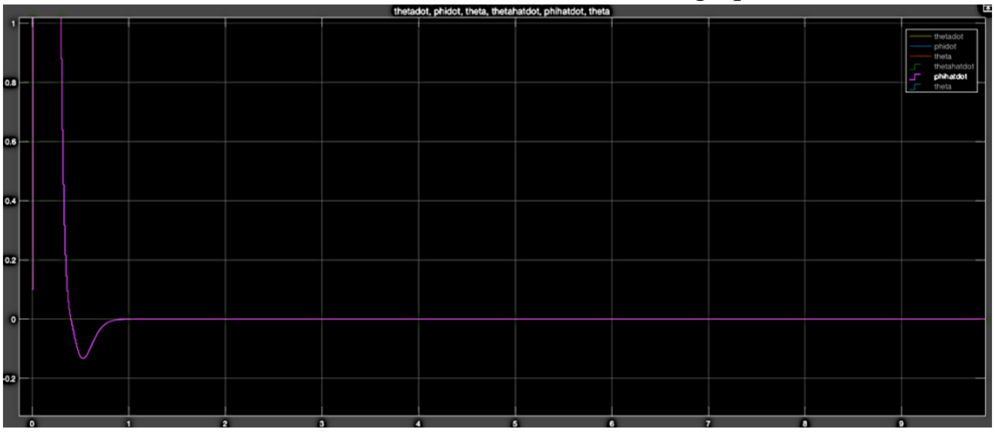

Figure 30 Graph of phidot in linear system with initial value 0.1

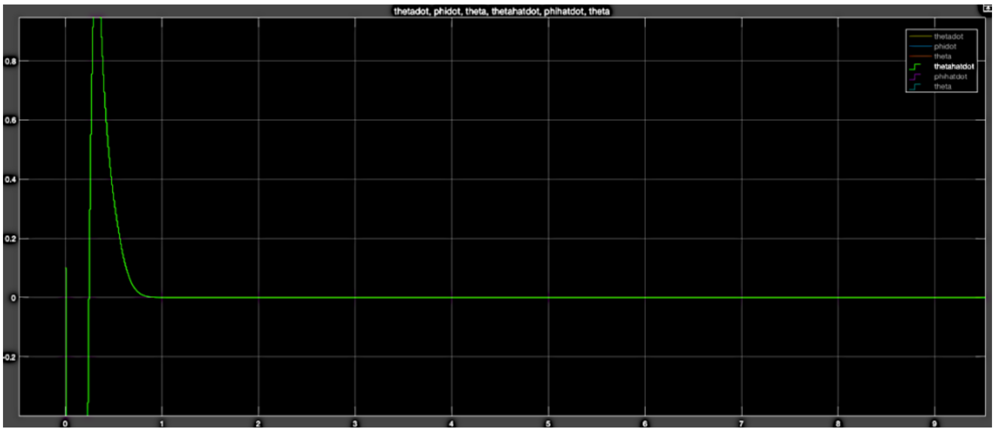

Figure 31 Graph of thetadot in linear system with initial value 0.1

I also find when applying initial condition in controller block, applied load will be affected as well. The figure is applied load when initial condition in controller block = 0.1, thetaic = 0.53 (region of attraction).

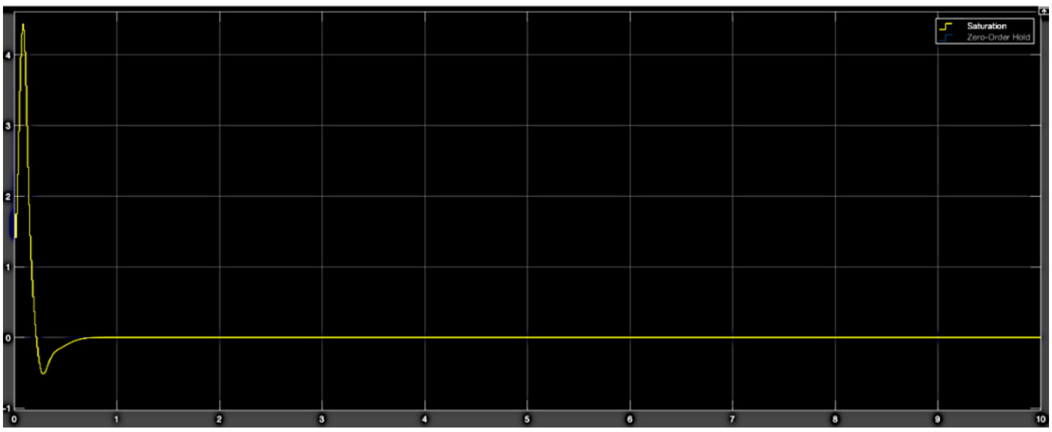

Figure 32 Applied load when initial condition = 0.1, thetaic = 0.53

By setting initial condition, the system allow larger initial angle of Mip, i.e., bigger region of attraction.

When initial condition in controller block = 0.1, system allows region of attraction up to ± 0.65 radian.

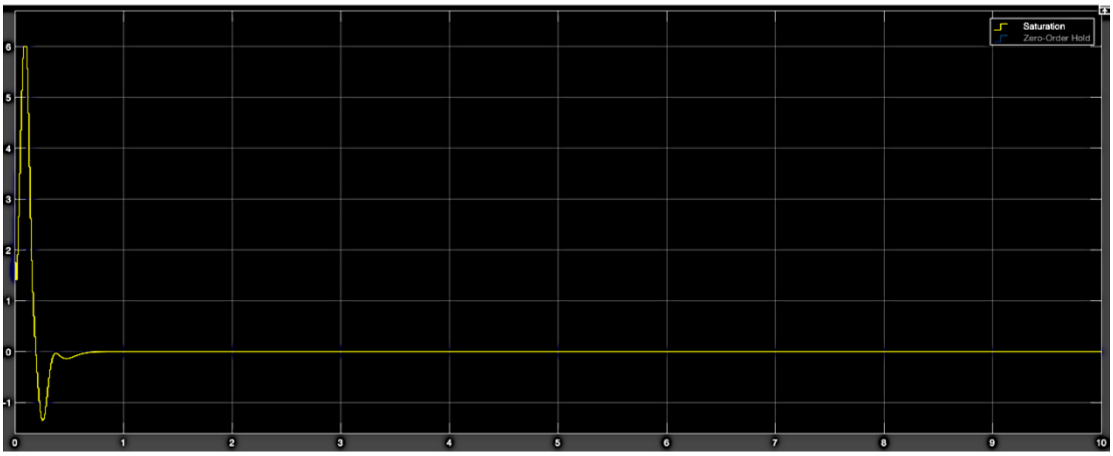

Figure 33 Applied load when initial condition in controller block = 0.1, thetaic = 0.65

ALTERING DESIGN FOR $\emptyset$

To make $\emptyset$ what we want, we have to add $\emptyset$ to the input of the system. When designing the controller above, I set u= –K$\hat{x}$, now u have to be –K$\hat{x}$+w, where w is the input of $\emptyset$.

$$\begin{cases} \dot{x} = Ax + Bu \\ y=Cx \end{cases}$$

$$\begin{cases} \dot{\hat{x}}=(A-LC)\hat{x}+Bu+Ly \\ u = -K\hat{x}+w \\ w = \emptyset \end{cases}$$

$$\begin{cases} \dot{\hat{x}}=(A-LC-BK)\hat{x}+[L \quad B] \cdot \begin{bmatrix} y \\ w \end{bmatrix} \\ u= -K\hat{x}+w \\ w=\emptyset \end{cases}$$

CONCLUSION

After completing this final report, I totally understand the tough side of designing a controller. By considering different objective, there are so many ways can design a controller. There is no best controller, you can only design the controller you want depends on what you concern the most. For example, the tradeoff between response time and noise is unavoidable. Therefore, I choose the controller that have all performance good, but not great, to prevent other performances become extremely bad.