Lecture 7B. Unsupervised Techniques I

DS-GA 1015, Text as Data
Arthur Spirling

March 23, 2021

Where Are We?

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The Realm of Middle Earth
Miles
50 100 150 200
The Lost Realm of Arnor
Here was of old the Witch-realm of Angmar
Carn Dûm
Angmar
Mtns of Angmar
Icebay of Forochel
Ettenmoors
Eriador
Ered Luin
Forlindon
Forlond
Gulf of Lhûn
Mithlond
(Grey Havens)
Tower Hills
Harlond
Harlindon
(Blue Mountains)
Arthedain
Emyn Uial
Nenuial
(Lake Evendim)
North Downs
Fornost
Weather Hills
Weathertop
(Amon Sûl)
Rhudaur
Trollshaws
Last Bridge
Bruinen
(Loudwater)
The Shire
Hobbiton
White Downs
Brandywine Bridge
Bree
Old Forest
East-West Road
South Downs
Midgewater
Chetwood
Sarn Ford
Baranduin
(Brandywine)
Cardolan
Mitheithel
Eregion
(Hollin)
Moria Gate
Nîn-in-Eilph
(Swanfleet)
Tharbad
Glanduin
Minhiriath
Eryn Vorn
Gwathlo (Greyflood)
Enedwaith
Dunland
Lond Daer
North-South Road
Isengard
Fangorn
Fords of Isen
Angren (Isen)
Adorn
Westfold
Helm's Deep
Edoras
Ered Nimrais
Gondor
Pinnath Gelin
Lefnui
Anfalas (Langstrand)
Andrast (Ras Morthil)

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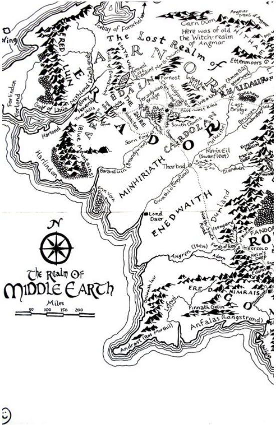

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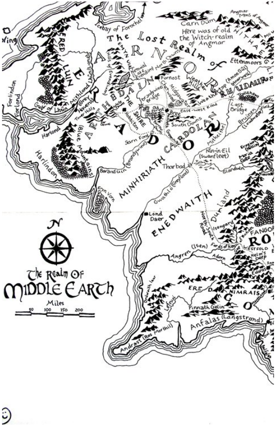

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and demonstrate challenges that emerge in interpreting the results.

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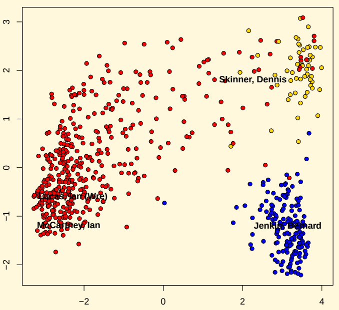

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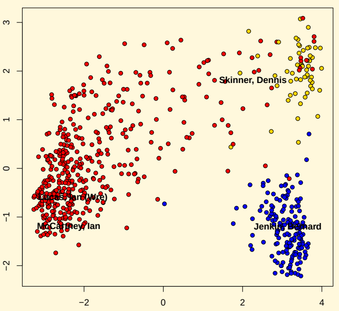

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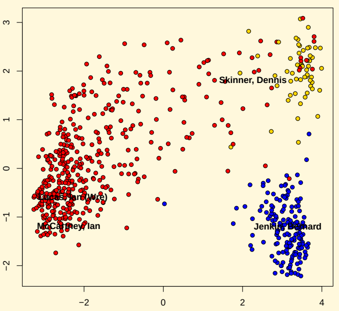

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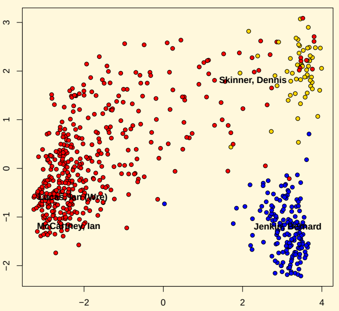

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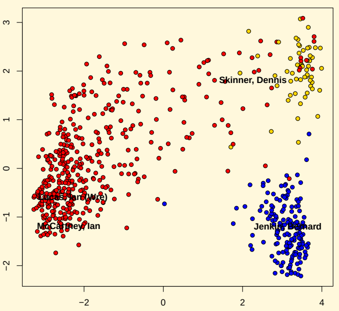

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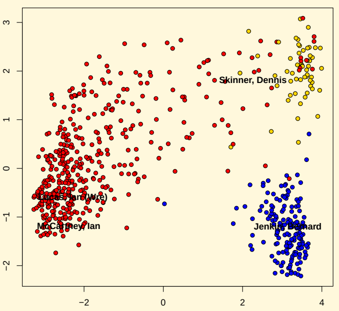

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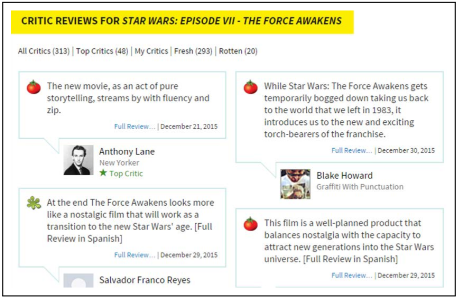

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(not "what is the recall/precision/accuracy?")

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Ainsworth, Peter (E S)	Con	NA	1	NA	...
Alexander, Douglas	Lab	NA	0	0	...
Allan, Richard	LD	1	0	1	...
Allen, Graham	Lab	0	0	0	...
Amess, David	Con	1	1	NA	...
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Principal Components Analysis

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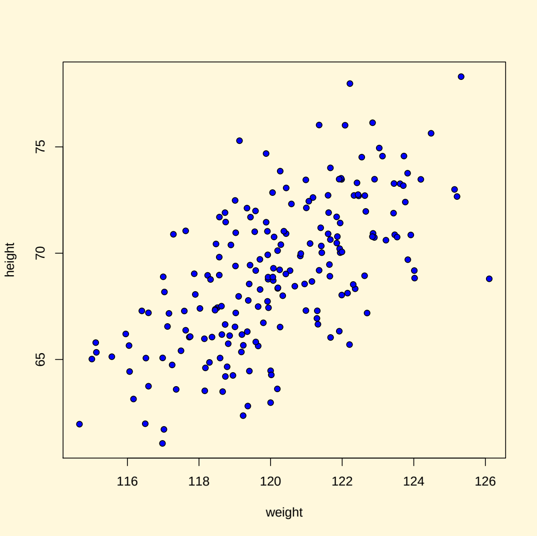

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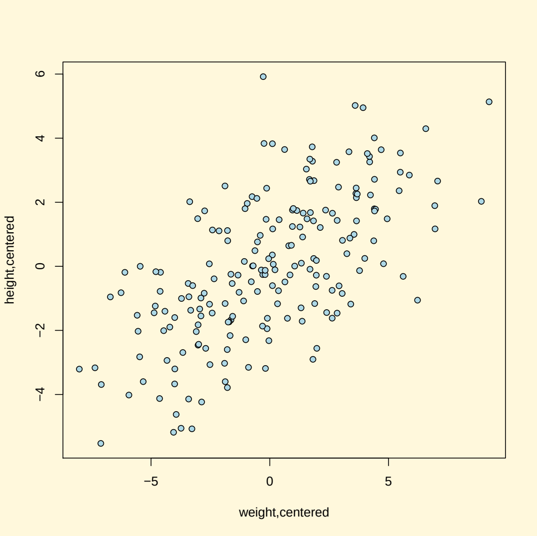

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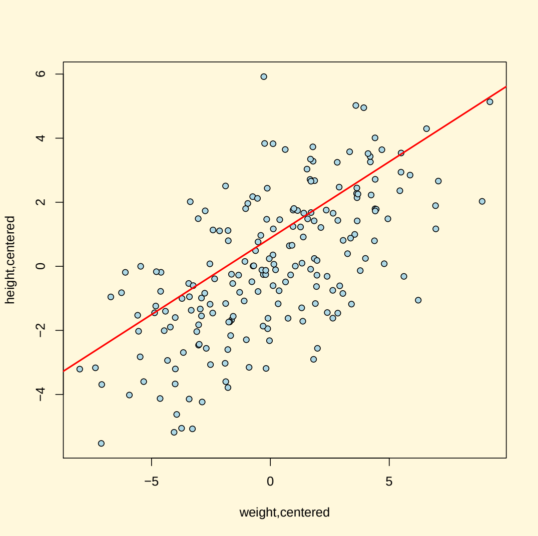

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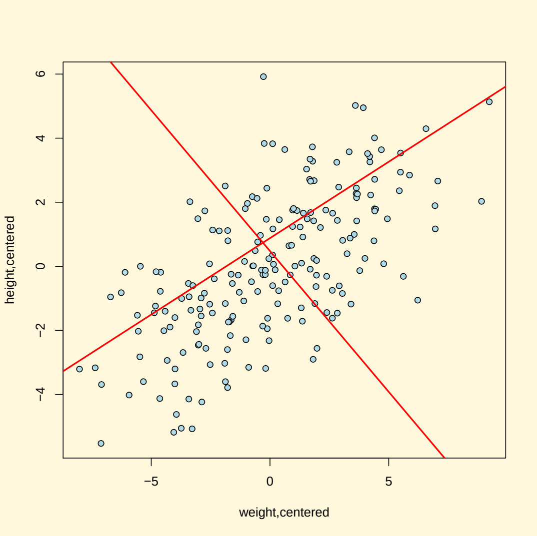

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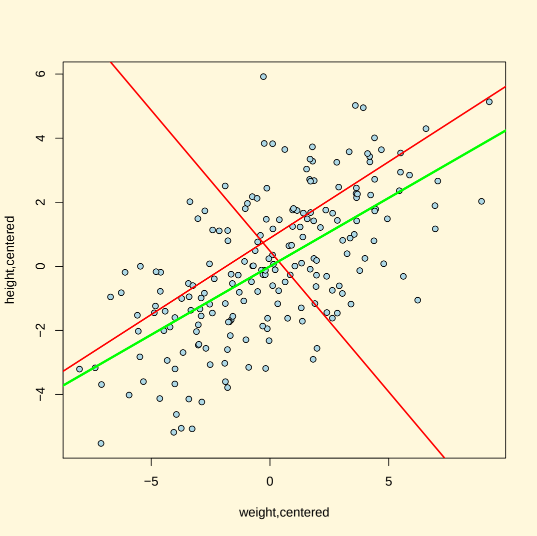

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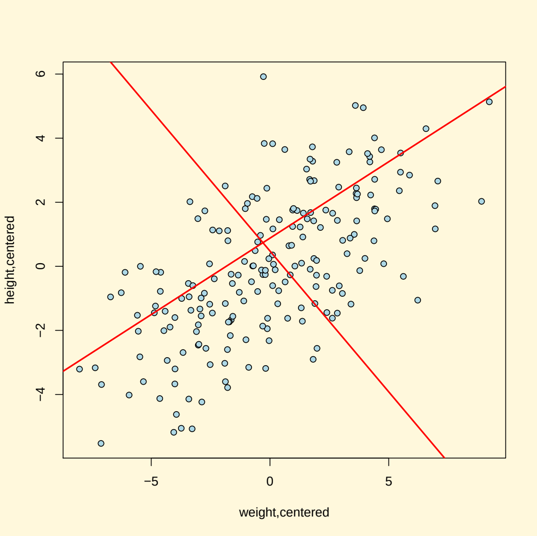

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Subsequent PCs are found subject to the constraint that $\mathrm{cov}(z_k, z_l) = 0$ and that $a_k' a_k = 1$.

Formally

Obtain the first PC as the solution of $z_1 \equiv \mathbf{a_1'X} = \sum_{i=1}^{p} a_{i1} x_i$

where $\mathbf{X}$ is the $p \times n$ data matrix (here, 2×200), $\mathbf{a_1} = (a_1, a_2, \ldots, a_p)$ is $1 \times p$ vector (here 1×2) picked such that $\mathrm{var}(z_1)$ is maximized

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btw Presumably, we wouldn't fit two components to two variables (why?)

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- $\boldsymbol{\Sigma}$ is diagonal matrix of singular values (related to eigenvalues of covariance matrix).
- $\rightarrow$ $\mathbf{U}\boldsymbol{\Sigma} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}'\mathbf{V} = \mathbf{X}\mathbf{V}$ are the principal component scores.
- $\mathbf{V}$ has columns which are the right singular vectors of $\mathbf{X}$

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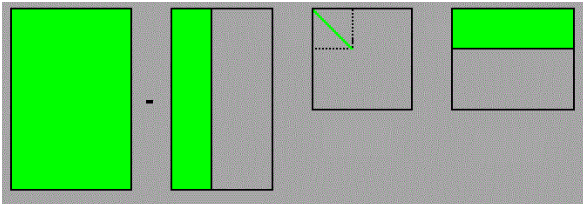

(from `http://web.eecs.utk.edu/~mberry/`)

Narcissistic Personality Disorder

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Assessed by Narcissistic Personality Inventory: 40 'forced choice' questions.

	A	B
1.	I have a natural talent for influencing people.	I am not good at influencing people.
2.	Modesty doesn't become me.	I am essentially a modest person.
3.	I would do almost anything on a dare.	I tend to be a fairly cautious person.
4.	When people compliment me I sometimes get embarrassed.	I know that I am good because everybody keeps telling me so.
5.	The thought of ruling the world frightens the hell out of me.	If I ruled the world it would be a better place.
6.	I can usually talk my way out of anything.	I try to accept the consequences of my behavior.
7.	I prefer to blend in with the crowd.	I like to be the center of attention.
8.	I will be a success.	I am not too concerned about success.
9.	I am no better or worse than most people.	I think I am a special person.
10.	I am not sure if I would make a good leader.	I see myself as a good leader.
11.	I am assertive.	I wish I were more assertive.
12.	I like to have authority over other people.	I don't mind following orders.
13.	I find it easy to manipulate people.	I don't like it when I find myself manipulating people.
14.	I insist upon getting the respect that is due me.	I usually get the respect that I deserve.

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and Squared factor loading is percent of variance in that variable explained by the factor

Narcissistic Personality Inventory Items and Principal-Component Loadings

Items	Loadings 1	2	3	4	5	6	7
47. I would prefer to be a leader.	.83	.00	−.07	.04	−.12	.07	.22
15. I see myself as a good leader.	.83	.16	.09	−.12	.06	.03	−.14
13. I will be a success.	.67	.00	−.09	−.14	−.14	.17	.26
46. People always seem to recognize my authority.	.66	.02	.06	−.06	.06	.00	.20
2. I have a natural talent for influencing people.	.66	−.15	.02	−.02	.29	.03	−.24
16. I am assertive.	.56	.18	−.02	.22	−.02	−.03	−.27
17. I like to have authority over other people.	.56	.08	−.08	.18	.08	.05	.24
50. I am a born leader.	.35	.20	.22	.00	.09	−.14	−.01
30. I rarely depend on anyone else to get things done.	.02	.61	−.17	.04	.04	.10	−.11
23. I like to take responsibility for making decisions.	.28	.59	−.23	.23	−.12	.00	.02
53. I am more capable than other people.	−.19	.57	.16	.07	.11	.01	.20
45. I can live my life in any way I want to.	−.13	.46	.29	−.02	.05	.05	−.03
29. I always know what I am doing.	.15	.46	−.14	−.03	.30	.01	−.09
48. I am going to be a great person.	.05	.43	.39	.04	−.03	−.05	.00
54. I am an extraordinary person.	.06	.22	.69	−.07	−.06	.01	.06
7. I know that I am good because everybody keeps telling me so.	−.18	.01	.69	.00	.21	.01	.15
36. I like to be complimented.	.00	−.28	.67	.06	.00	.11	−.17
14. I think I am a special person.	.08	.16	.64	−.02	−.09	.17	−.01
51. I wish somebody would someday write my biography.	−.06	−.01	.57	.06	−.22	.09	.00
28. I am apt to show off if I get the chance.	−.04	−.02	.04	.71	−.03	.06	.06
3. Modesty doesn't become me.	−.01	.19	−.01	.69	−.16	−.06	.14
52. I get upset when people don't notice how I look when I go out in public.	−.16	.04	.10	.51	.09	.25	.17

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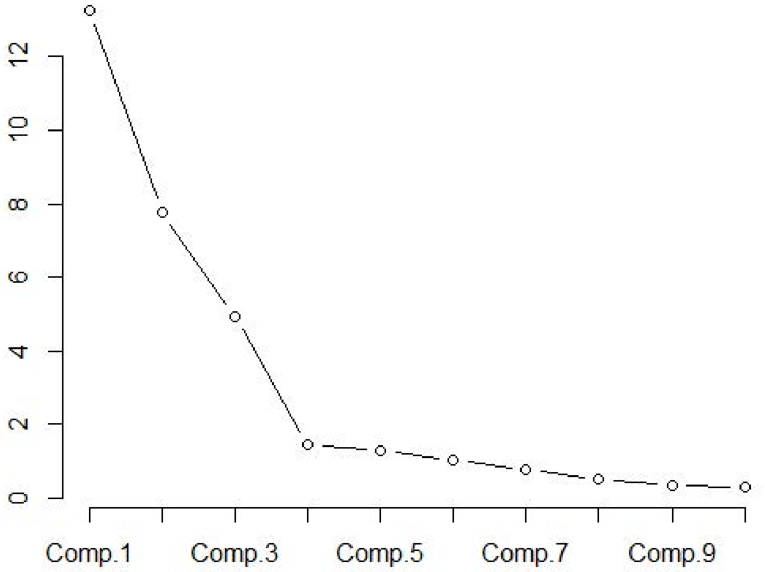
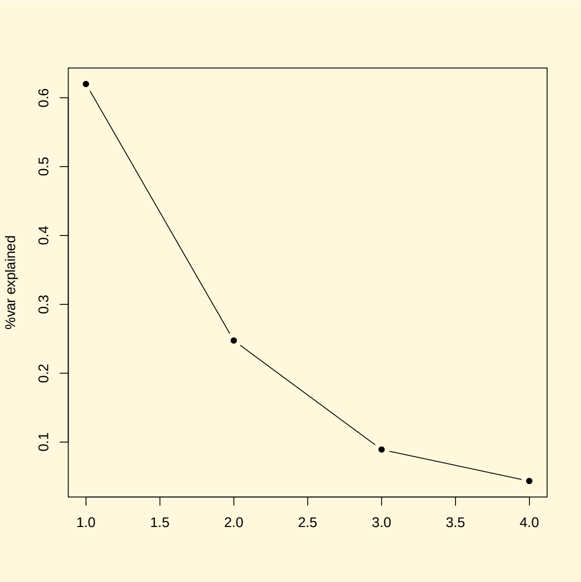
btw generally like to see an 'elbow'

Good



Bad



Ugly