6. Supervised Techniques III

DS-GA 1015, Text as Data
Arthur Spirling

March 16, 2021

Where Are We?

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The Lost Realm of
ARNOR
Carn Dûm
Here was of old the Witch-realm of Angmar
Ettenmoors
Fornost
Weather Hills
RHUDAUR
Last Bridge
East-West Road
Bree
South Downs
CARDOLAN
ERIADOR
THE SHIRE
Hobbiton
Sarn Ford
Harlond
Forlond
Forlindon
Gulf of Lhûn
Harlindon
MINHIRIATH
Tharbad
Nîn-in-Eilph (Swanfleet)
ENEDWAITH
Lond Daer
Dunland
Isengard
FANGORN
Helm's Deep
Edoras
Angren (Isen)
Adorn
Anfalas (Langstrand)
Pinnath Gelin
Andrast (Ras Morthil)
The Realm Of
MIDDLE EARTH
Miles
50 100 150 200

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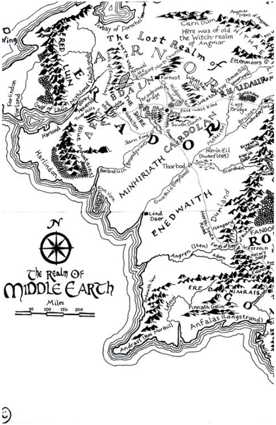

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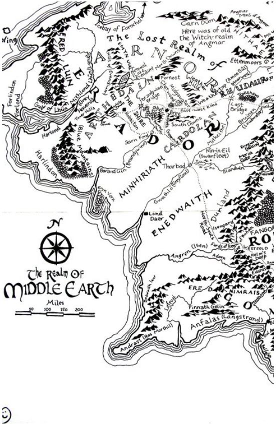

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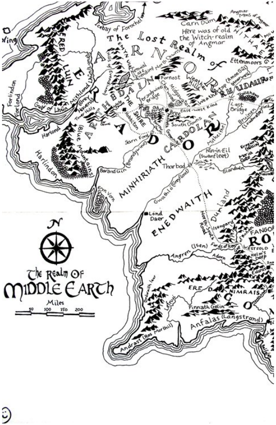

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These techniques involve some important decisions about the bias-variance tradeoff, and the use of (cross) validation in checking model performance and selecting the best model.

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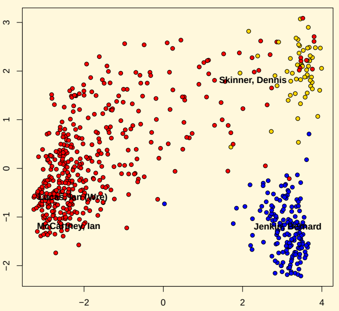

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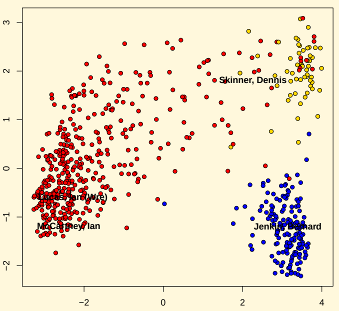

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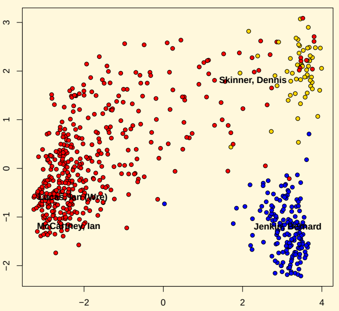

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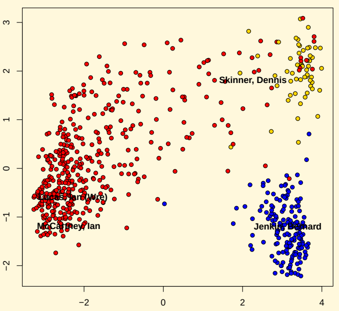

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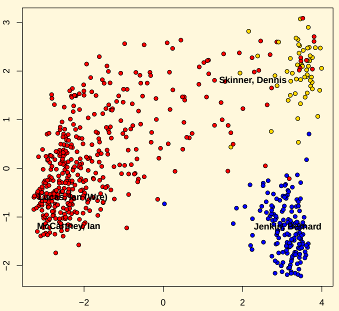

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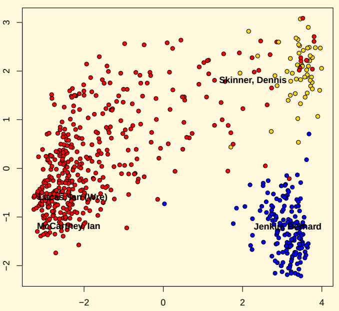

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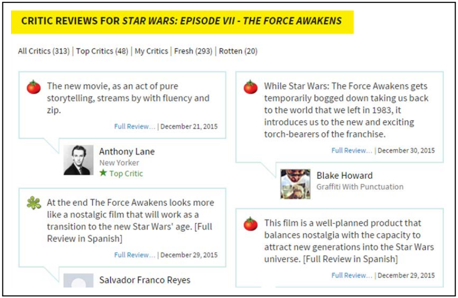

Workflow of Supervised Learning: Bias/Variance Tradeoff

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So techniques may require careful tuning of *regularization parameters* to obtain good performance.

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So managing the *bias-variance tradeoff* is a key element of supervised learning, and we may need to tune our algorithms with that in mind.

Bias-Variance Tradeoff (Hastie et al, p38)

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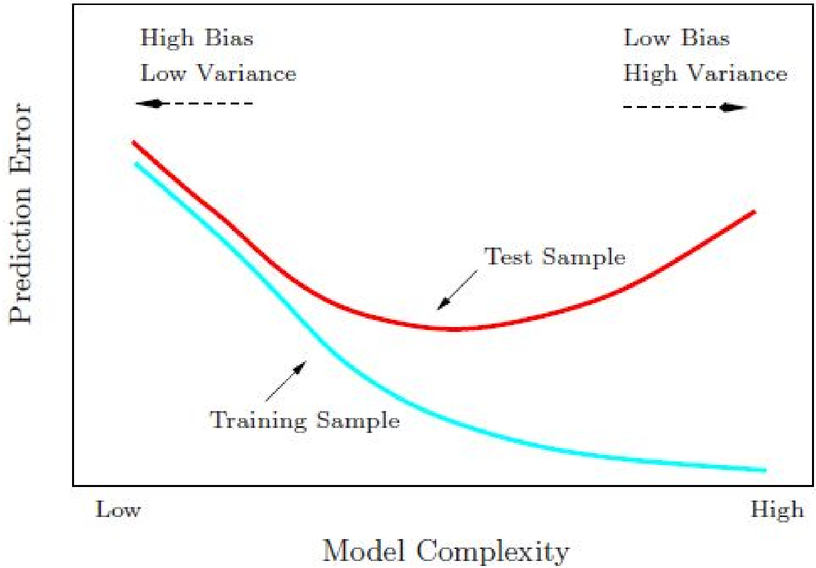

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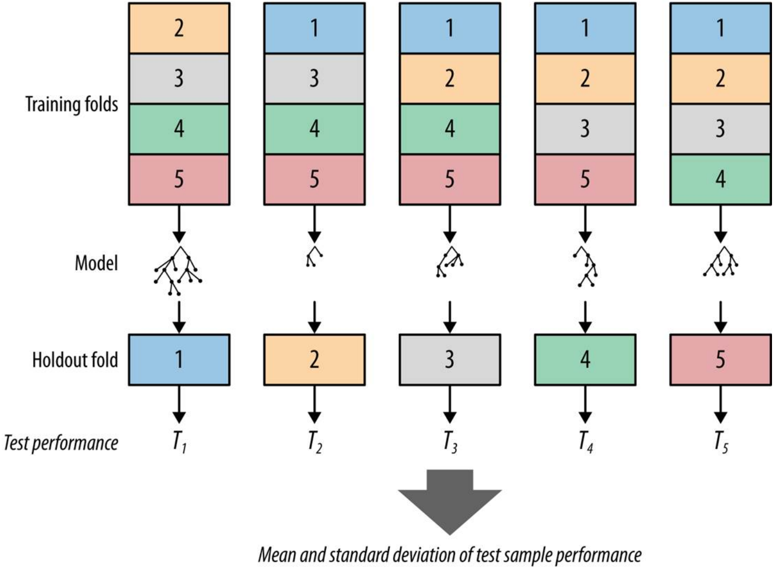

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Assume that the observations are linearly separable: if we plot the Senators in two dimensions (x_1, x_2), we can divide them (perfectly) into the two parties using a straight line.

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Here We have 10 Senators that belong to one of two classes: five Republicans, five Democrats. $y_i \in \{-1, +1\}$

Each senator has a number of p features, which are the term weights from their speeches. To make things simple, suppose that $p = 2$: there are (only) two features, x_1 and x_2 per observation.

Assume that the observations are linearly separable: if we plot the Senators in two dimensions (x_1, x_2), we can divide them (perfectly) into the two parties using a straight line.

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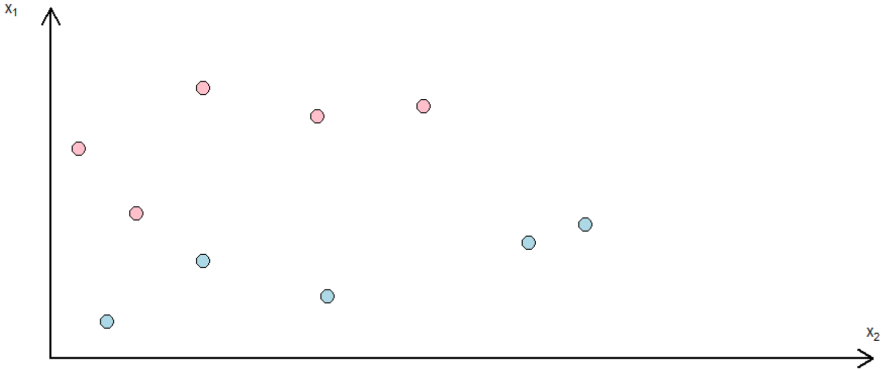

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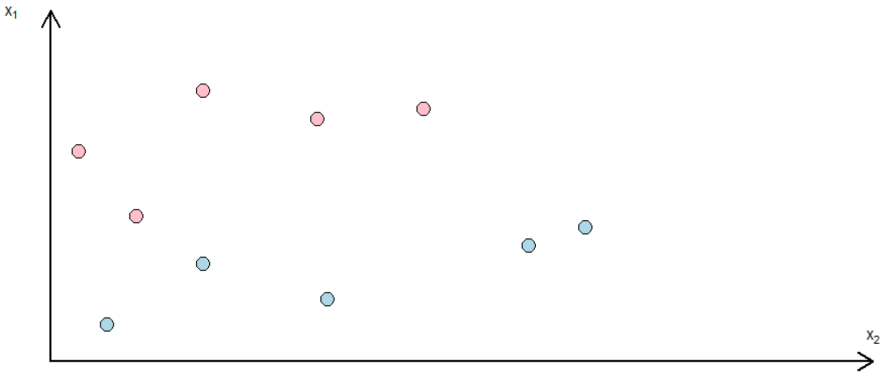

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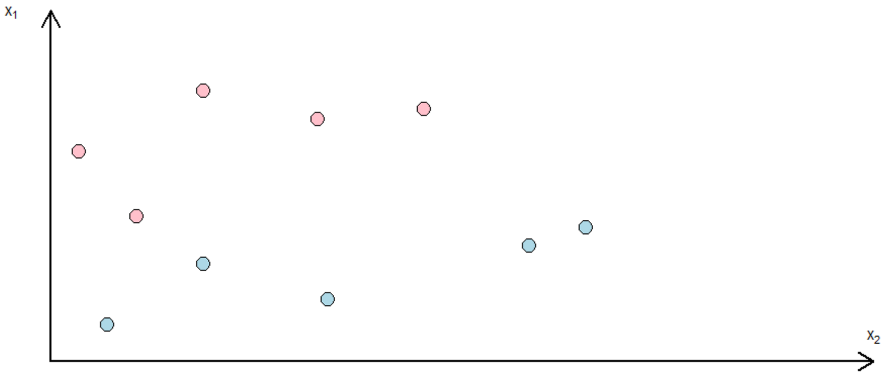

As the parties linearly separably? Where could you draw the line?

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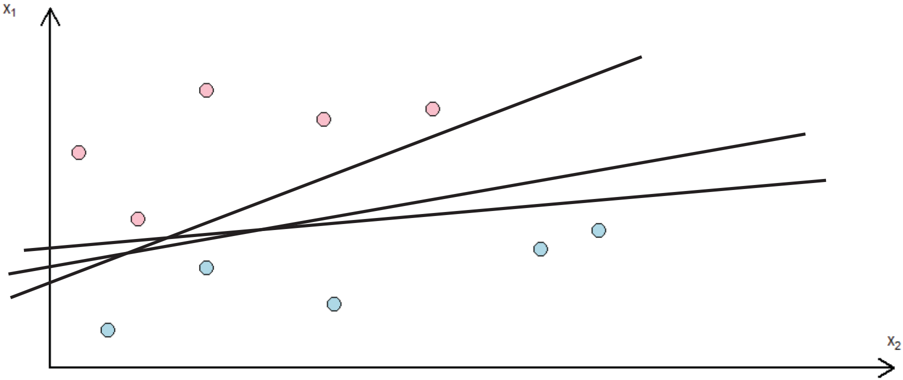

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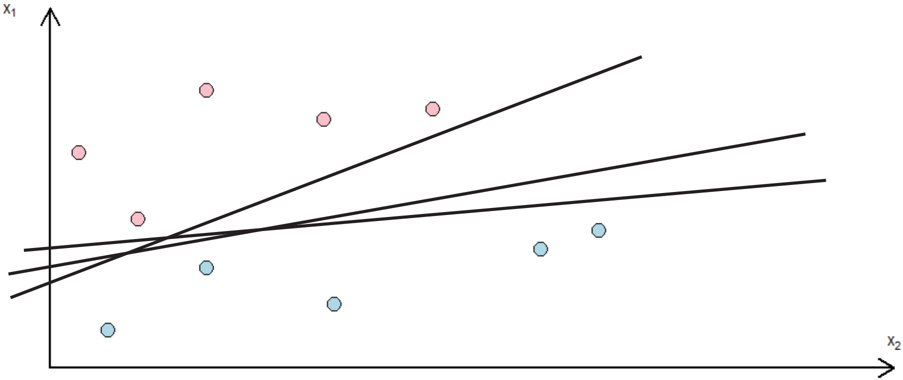

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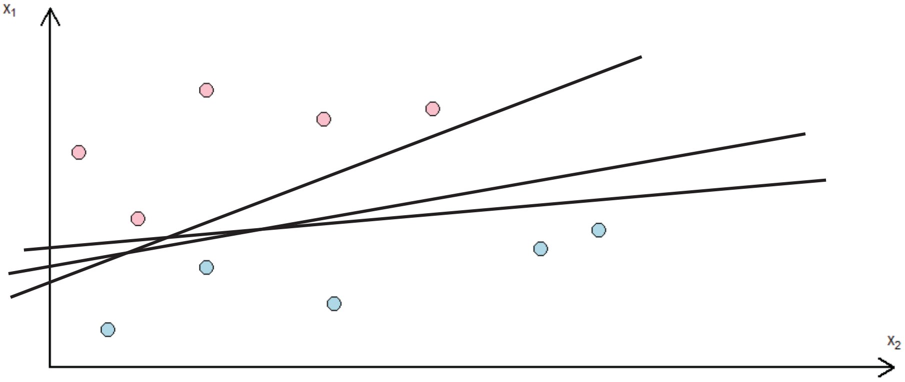

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$\rightarrow$ That optimal line—the separating hyperplane—is the maximum margin hyperplane. It will maximize the margin of the training data.

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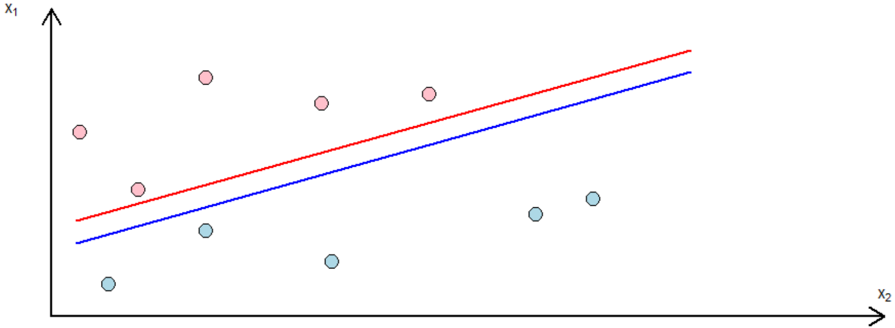

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NB The hyperplane cannot be anywhere other than equidistant because then it will break the rule about ensuring the largest minimum distance.

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Turns out that the minimization of $||\mathbf{w}||$ is amenable to quadratic programming methods.

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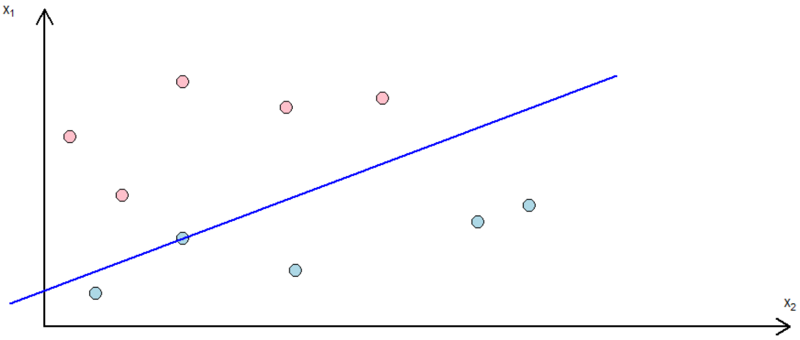

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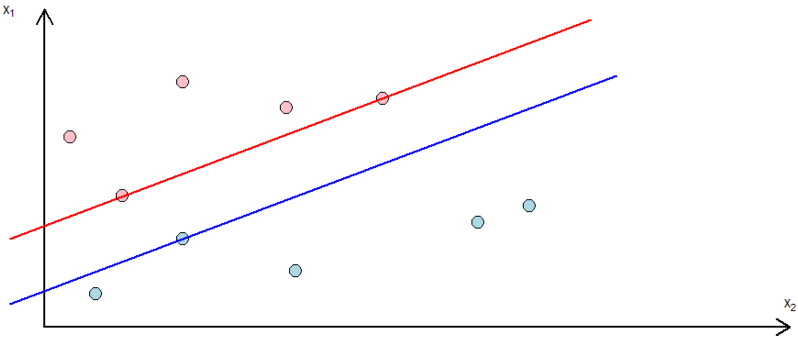

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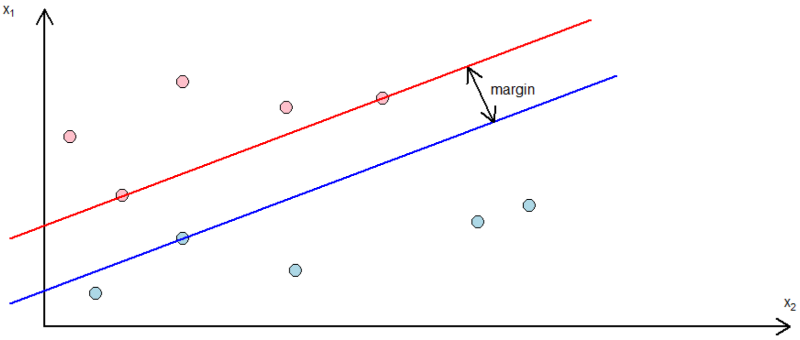

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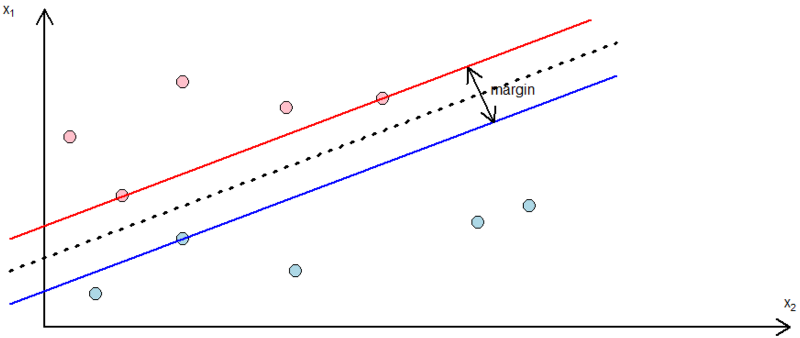

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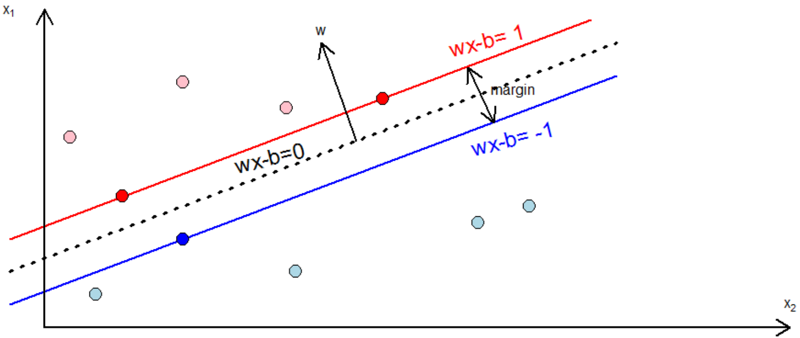

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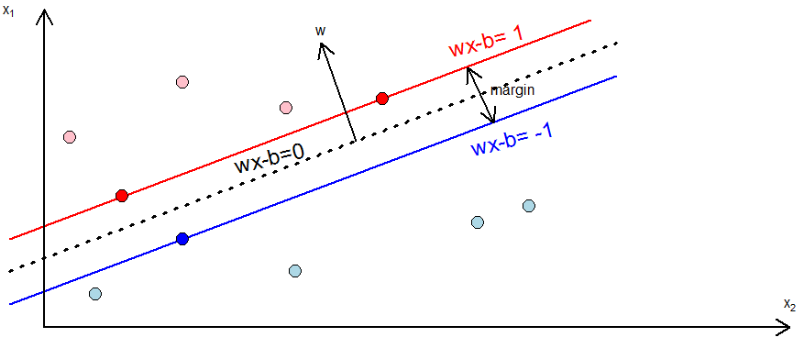

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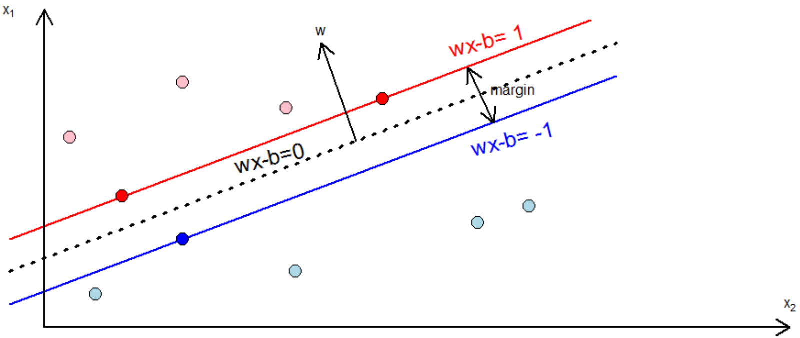

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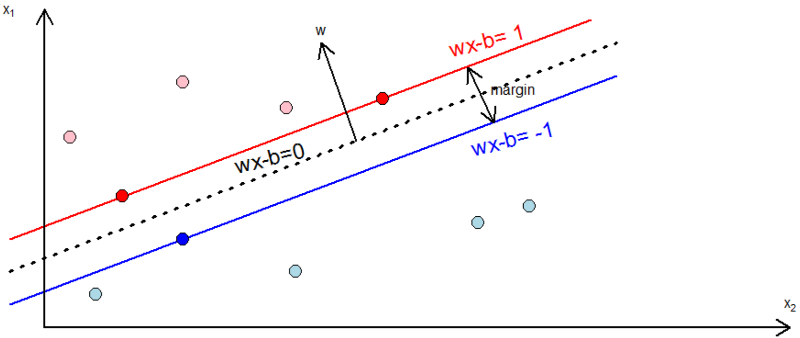

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Words			
Liberal		Conservative	
FAS: −199.49	SBA: −113.10	habeas: 193.55	homosexual: 103.07
Ethanol: −198.92	Nursing: −109.38	CFTC: 187.16	everglades: 102.87
Wealthiest: −159.74	Providence: −108.73	surtax: 151.81	tower: 101.67
Collider: −142.28	Arctic: −108.30	marriage: 145.79	tripartisan: 101.23
WIC: −140.14	Orange: −107.98	cloning: 141.71	PRC: 102.90
ILO: −139.89	Glaxo: −107.81	tritium: 133.49	scouts: 97.55
Handgun: −129.01	Libraries: −107.70	ranchers: 132.95	nashua: 99.32
Lobbyists: −128.95	Disabilities: −106.44	BTU: 121.92	ballistic: 97.22
Enron: −127.71	Prescription: −106.31	grazing: 121.59	salting: 94.28
Fishery: −127.30	NIH: −105.52	unfunded: 120.82	abortion: 91.94
Hydrogen: −122.59	Lobbying: −105.35	catfish: 120.82	NTSB: 93.81
Souter: −121.40	NRA: −105.20	IRS: 114.91	Haiti: 97.28
PTSD: −119.87	Trident: −104.15	unborn: 111.88	PAC: 92.85
Gun: −119.52	RNC: −103.46	Taiwan: 111.13	taxing: 90.39

Beyond basic (hard margin) SVM

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BTW RLR can cope well with noise, and (hard margin) SVM will struggle if there is no linear seperability...

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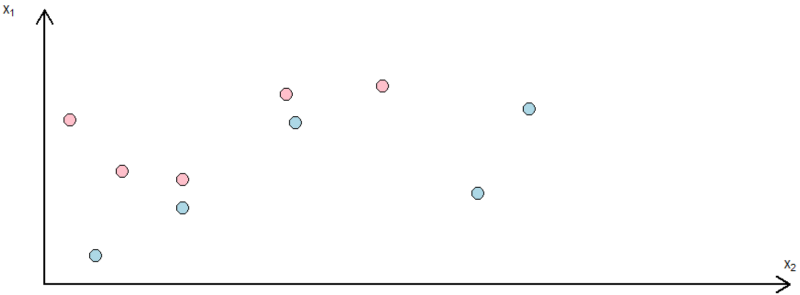

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Hyperplane(s) will be drawn in way that is more sensitive to 'bigger' mistakes in classification.

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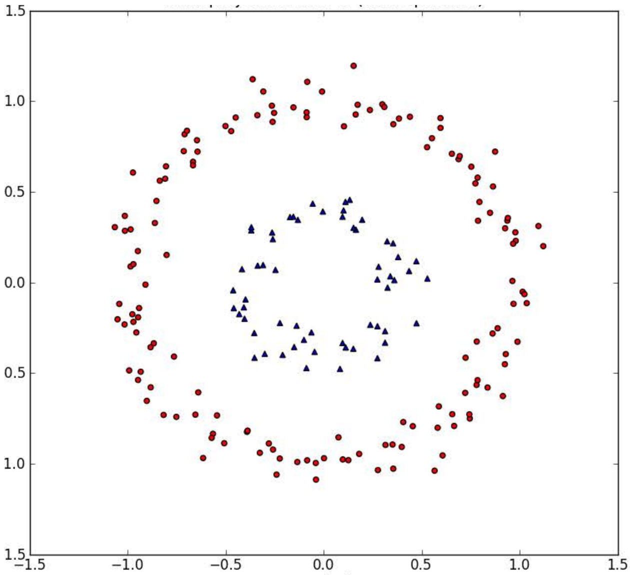

from `www.eric-kim.net`

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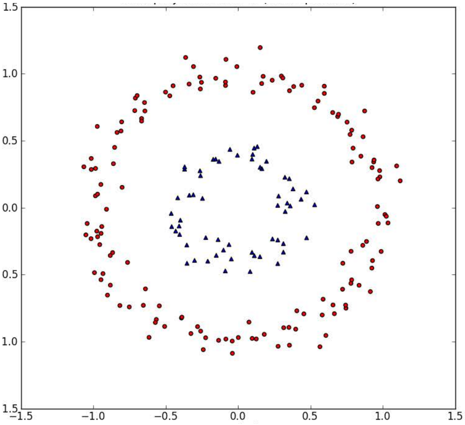

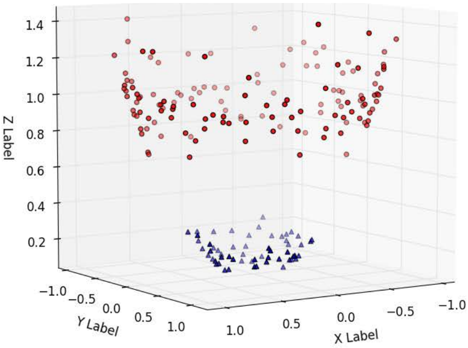

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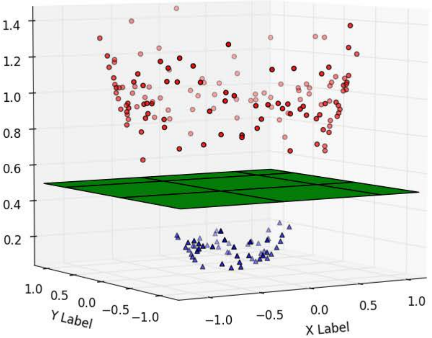
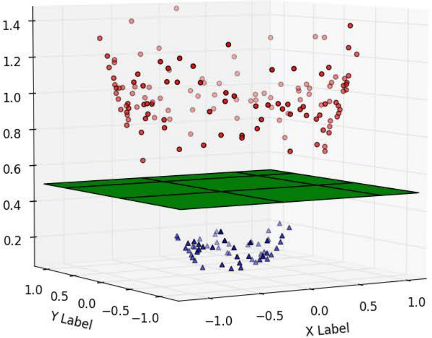
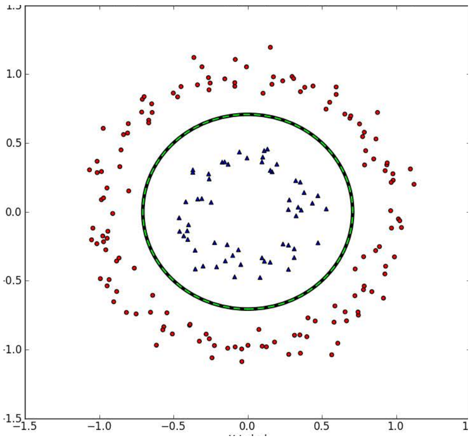


1.4
1.2
1.0
0.8
0.6
0.4
0.2
1.0 0.5 0.0 −0.5 −1.0
Y Label
−1.0 −0.5 0.0 0.5 1.0
X Label





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For text analysis, string kernels use a function $K(a, b)$ to implicity calculate the distance between strings of characters via the number of subsequences they have in common.