

Robustifying the Viterbi Algorithm

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A Toy Example

Weather estimation



?



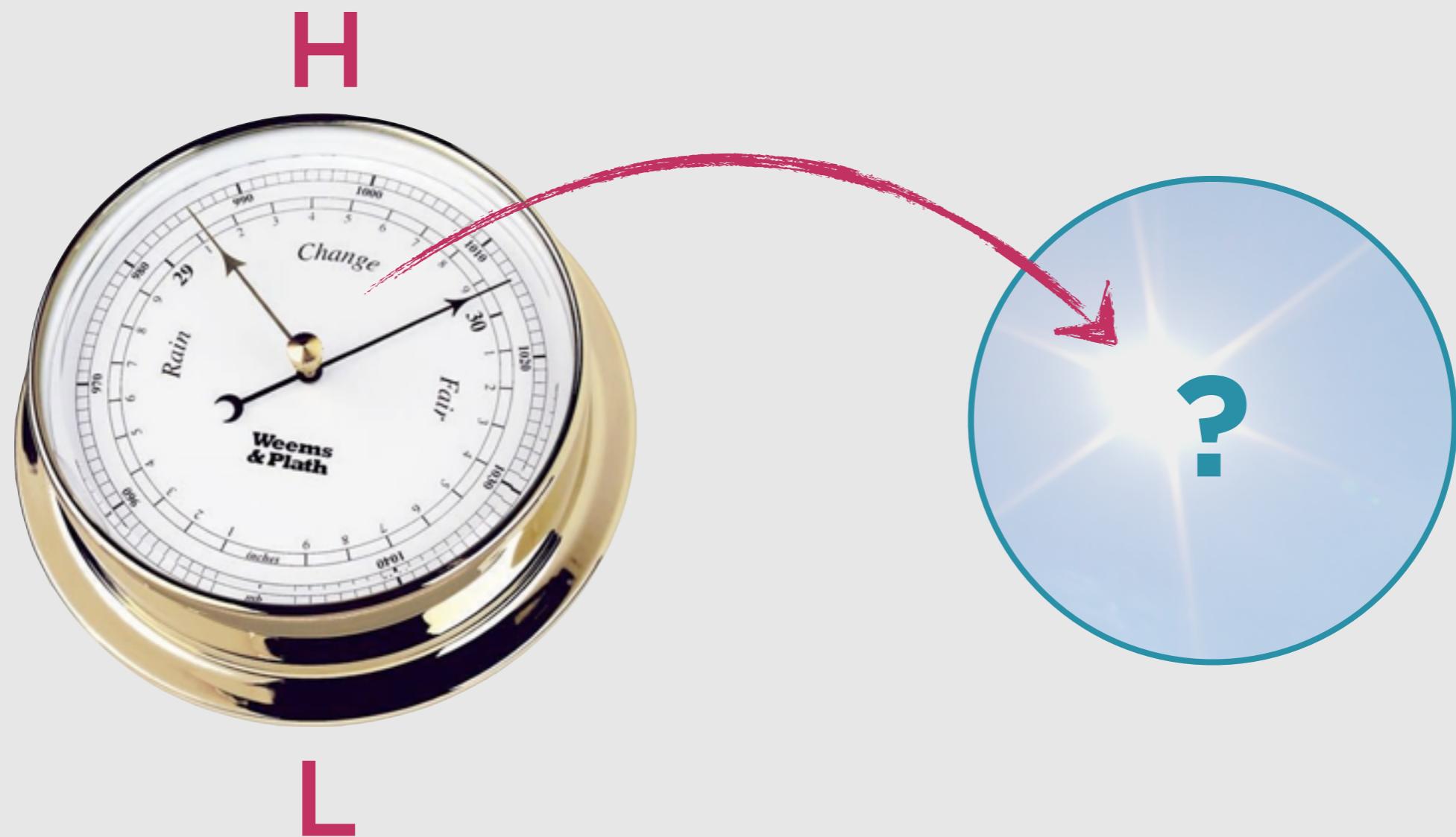
A Toy Example

Weather estimation



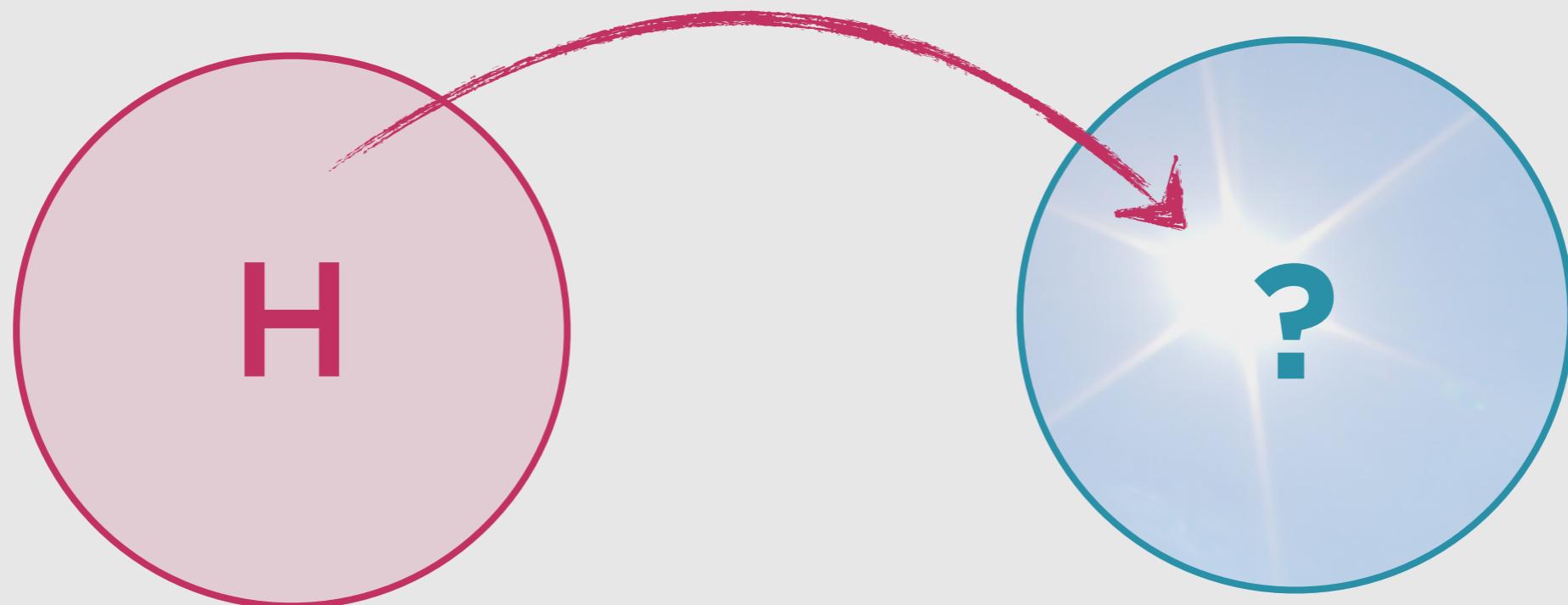
A Toy Example

Weather estimation



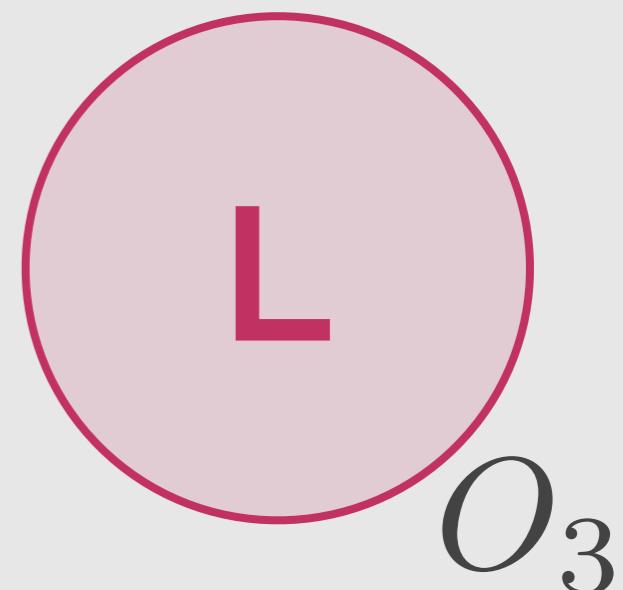
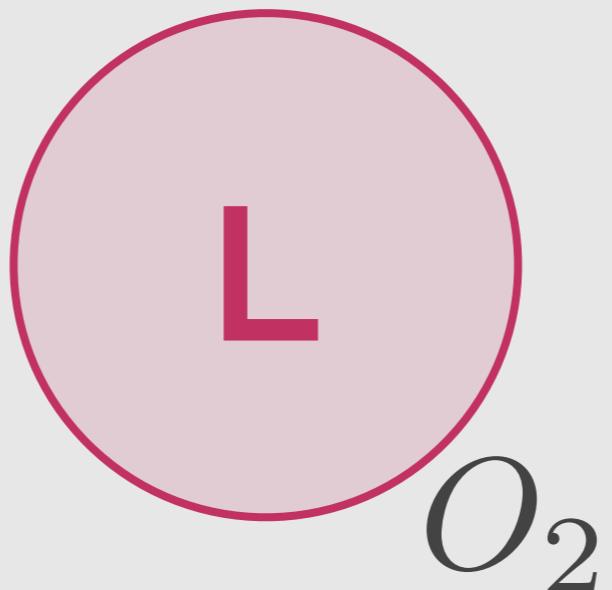
A Toy Example

Weather estimation



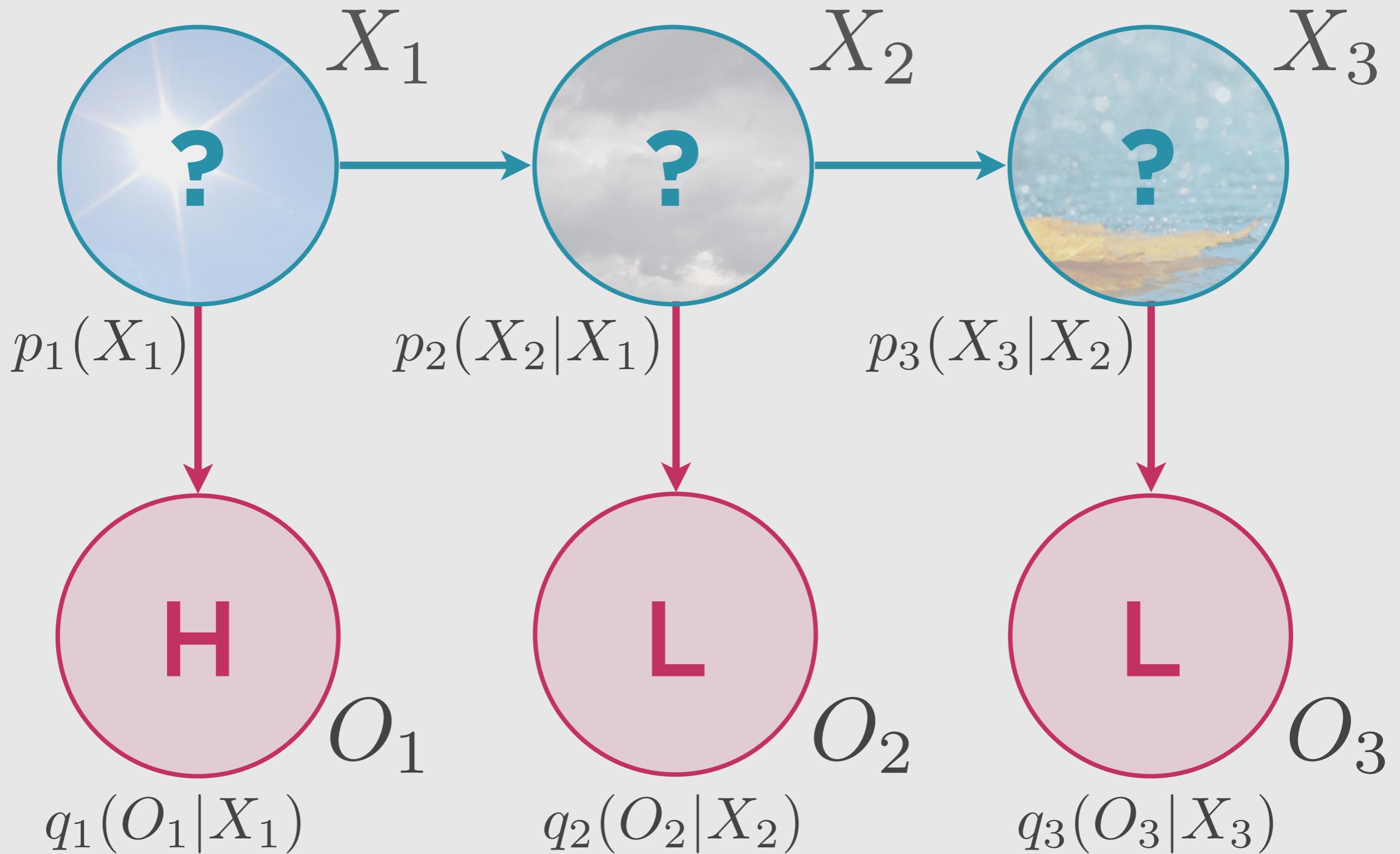
A Toy Example

Weather estimation



Hidden Markov Model

Local models



Hidden Markov Model

Local models



Local models

$$p_1(X_1) \quad p_2(X_2|X_1) \quad p_3(X_3|X_2)$$

$$q_1(O_1|X_1) \quad q_2(O_2|X_2) \quad q_3(O_3|X_3)$$

Hidden Markov Model

Global model



Local models

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$$q_1(O_1|X_1) \quad q_2(O_2|X_2) \quad q_3(O_3|X_3)$$



Global model

$$p(X_{1:3}, O_{1:3}) = p_1(X_1)q_1(O_1|X_1)$$

$$\prod_{i=2}^3 p_i(X_i|X_{i-1})q_i(O_i|X_i)$$

Hidden Markov Model

Estimating the hidden sequence



Global model

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$$\arg \max_{x_{1:3}} p(x_{1:3}|o_{1:3})$$

Hidden Markov Model

Estimating the hidden sequence



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$$\arg \max_{x_{1:3}} p(x_{1:3}|o_{1:3}) = \arg \max_{x_{1:3}} \frac{p(x_{1:3}, o_{1:3})}{p(o_{1:3})}$$

Hidden Markov Model

Estimating the hidden sequence



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$$= \arg \max_{x_{1:3}} p(x_{1:3}, o_{1:3})$$

Hidden Markov Model

Estimating the hidden sequence



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Viterbi algorithm (1967)

Hidden Markov Model

Viterbi algorithm

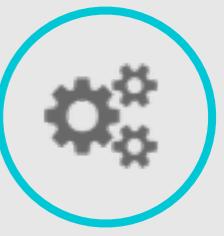


Viterbi algorithm (1967)

➤ Recursive

Hidden Markov Model

Viterbi algorithm

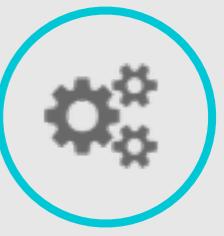


Viterbi algorithm (1967)

- Recursive
- Complexity $O(nm^2)$
 - n : length of the sequence
 - m : size of state space

Hidden Markov Model

Viterbi algorithm

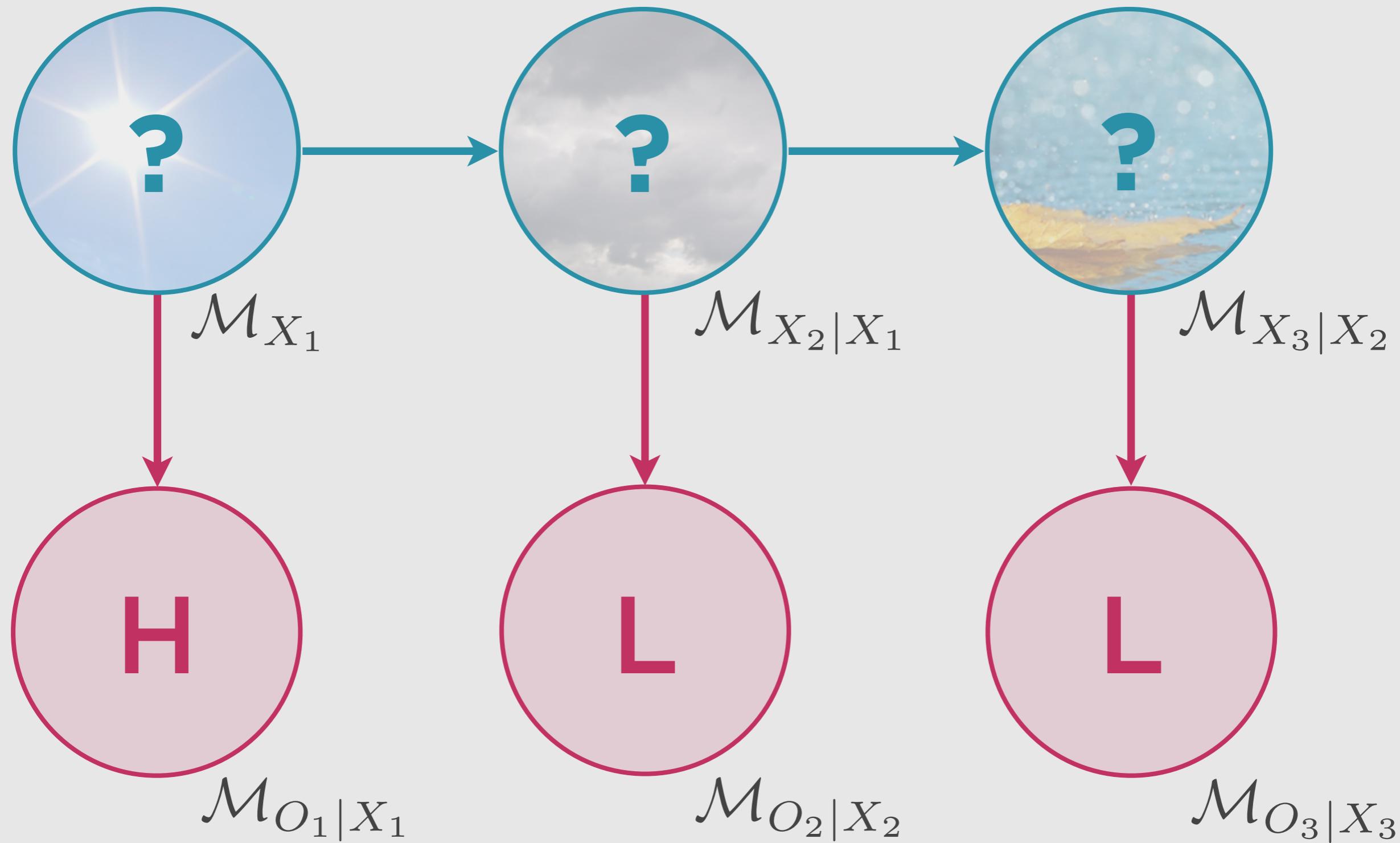


Viterbi algorithm (1967)

- Recursive
- Complexity $O(nm^2)$
 - n : length of the sequence
 - m : size of state space
- Extendible to k -best Viterbi

Imprecise Hidden Markov Model

Local models



Imprecise Hidden Markov Model

Local models



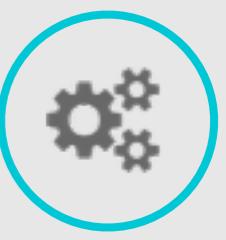
Local models

$$\mathcal{M}_{X_1} \quad \mathcal{M}_{X_2|X_1} \quad \mathcal{M}_{X_3|X_2}$$

$$\mathcal{M}_{O_1|X_1} \quad \mathcal{M}_{O_2|X_2} \quad \mathcal{M}_{O_3|X_3}$$

Imprecise Hidden Markov Model

Global model



Local models

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$$\mathcal{M}_{O_1|X_1} \quad \mathcal{M}_{O_2|X_2} \quad \mathcal{M}_{O_3|X_3}$$



Global model

$$\begin{aligned} \mathcal{M} = \left\{ \prod_{i=1}^3 p_i(X_i|X_{i-1}) q_i(O_i|X_i) : \right. \\ (\forall k \in \{1, 2, 3\}) p_k(\cdot|X_{k-1}) \in \mathcal{M}_{X_k|X_{k-1}}, \\ \left. q_k(\cdot|X_k) \in \mathcal{M}_{O_k|X_k} \right\} \end{aligned}$$

Imprecise Hidden Markov Model

Global model



Local models

$$\mathcal{M}_{X_1} \quad \mathcal{M}_{X_2|X_1} \quad \mathcal{M}_{X_3|X_2}$$

$$\mathcal{M}_{O_1|X_1} \quad \mathcal{M}_{O_2|X_2} \quad \mathcal{M}_{O_3|X_3}$$



Global model

May contain infinitely
many precise models!

$$\mathcal{M} = \left\{ \prod_{i=1}^3 p_i(X_i|X_{i-1}) q_i(O_i|X_i) : \right.$$

↑

$$(\forall k \in \{1, 2, 3\}) p_k(\cdot|X_{k-1}) \in \mathcal{M}_{X_k|X_{k-1}},$$
$$q_k(\cdot|X_k) \in \mathcal{M}_{O_k|X_k} \left. \right\}$$

Imprecise Hidden Markov Model

Estimating the hidden sequence



Partial order

$$x_{1:3} \succ \hat{x}_{1:3} \Leftrightarrow (\forall p \in \mathcal{M}) \ p(x_{1:3}|o_{1:3}) > p(\hat{x}_{1:3}|o_{1:3})$$

Imprecise Hidden Markov Model

Estimating the hidden sequence



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Set of maximal solutions

$$\text{opt}_{\max}(\mathcal{X}_{1:3}) \triangleq \{\hat{x}_{1:3} \in \mathcal{X}_{1:3} : (\forall x_{1:3} \in \mathcal{X}_{1:3}) x_{1:3} \not\succ \hat{x}_{1:3}\}$$

Imprecise Hidden Markov Model

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Indecision

There may be multiple maximal solutions.

Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

$$x_{1:n} \succ \hat{x}_{1:n} \Leftrightarrow (\forall p \in \mathcal{M}) \ p(x_{1:n}|o_{1:n}) > p(\hat{x}_{1:n}|o_{1:n})$$

Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

$$\begin{aligned}x_{1:n} \succ \hat{x}_{1:n} &\Leftrightarrow (\forall p \in \mathcal{M}) \ p(x_{1:n}|o_{1:n}) > p(\hat{x}_{1:n}|o_{1:n}) \\&\Leftrightarrow (\forall p \in \mathcal{M}) \ p(x_{1:n}, o_{1:n}) > p(\hat{x}_{1:n}, o_{1:n})\end{aligned}$$

Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

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Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

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Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

$$x_{1:n} \succ \hat{x}_{1:n} \Leftrightarrow (\forall p \in \mathcal{M}) p(x_{1:n}|o_{1:n}) > p(\hat{x}_{1:n}|o_{1:n})$$

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$$\Leftrightarrow (\forall p \in \mathcal{M}) \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} > 1$$

$$\Leftrightarrow \min_{p \in \mathcal{M}} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} > 1$$



What if $p(\hat{x}_{1:n}, o_{1:n})$ becomes zero?

Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

$$x_{1:n} \succ \hat{x}_{1:n} \Leftrightarrow \min_{p \in \mathcal{M}} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} > 1$$

Imprecise Hidden Markov Model

Rewriting the solution set



Partial order

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Imprecise Hidden Markov Model

Rewriting the solution set



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Imprecise Hidden Markov Model

Rewriting the solution set



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Imprecise Hidden Markov Model

Rewriting the solution set



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Can be calculated in advance

Imprecise Hidden Markov Model

Rewriting the solution set



Set of maximal solutions

$$\text{opt}_{\max}(\mathcal{X}_{1:n}) \triangleq \{\hat{x}_{1:n} \in \mathcal{X}_{1:n} : (\forall x_{1:n} \in \mathcal{X}_{1:n}) x_{1:n} \not> \hat{x}_{1:n}\}$$



$$\max_{x_{1:n} \in \mathcal{X}_{1:n}} \prod_{k=1}^n \chi_k(x_k, x_{k-1}, \hat{x}_k, \hat{x}_{k-1}) \omega_k(x_k, \hat{x}_k, o_k) \leq 1$$

Imprecise Hidden Markov Model

Rewriting the solution set



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How do we calculate this set?

Imprecise Hidden Markov Model

Rewriting the solution set



Set of maximal solutions

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$$\max_{x_{1:n} \in \mathcal{X}_{1:n}} \prod_{k=1}^n \chi_k(x_k, x_{k-1}, \hat{x}_k, \hat{x}_{k-1}) \omega_k(x_k, \hat{x}_k, o_k) \leq 1$$

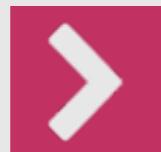
How do we calculate this set?



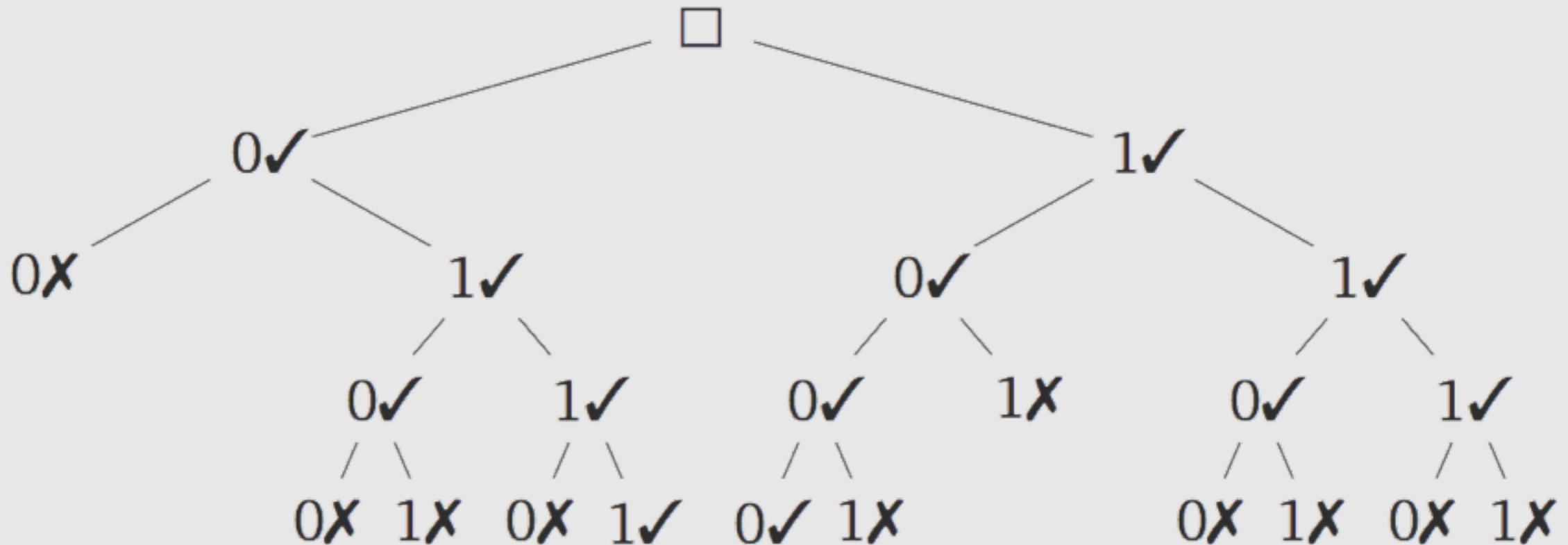
MaxiHMM algorithm

MaxiHMM algorithm

General overview

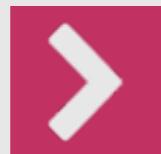


MaxiHMM algorithm

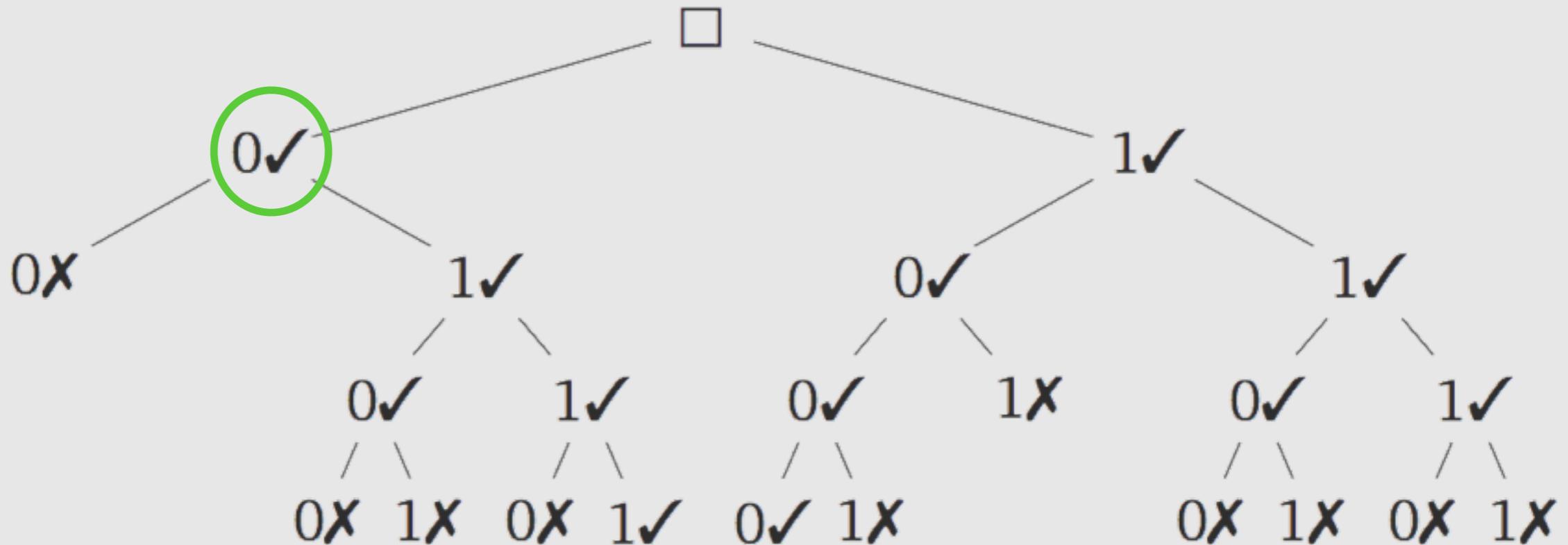


MaxiHMM algorithm

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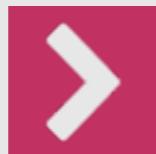


MaxiHMM algorithm

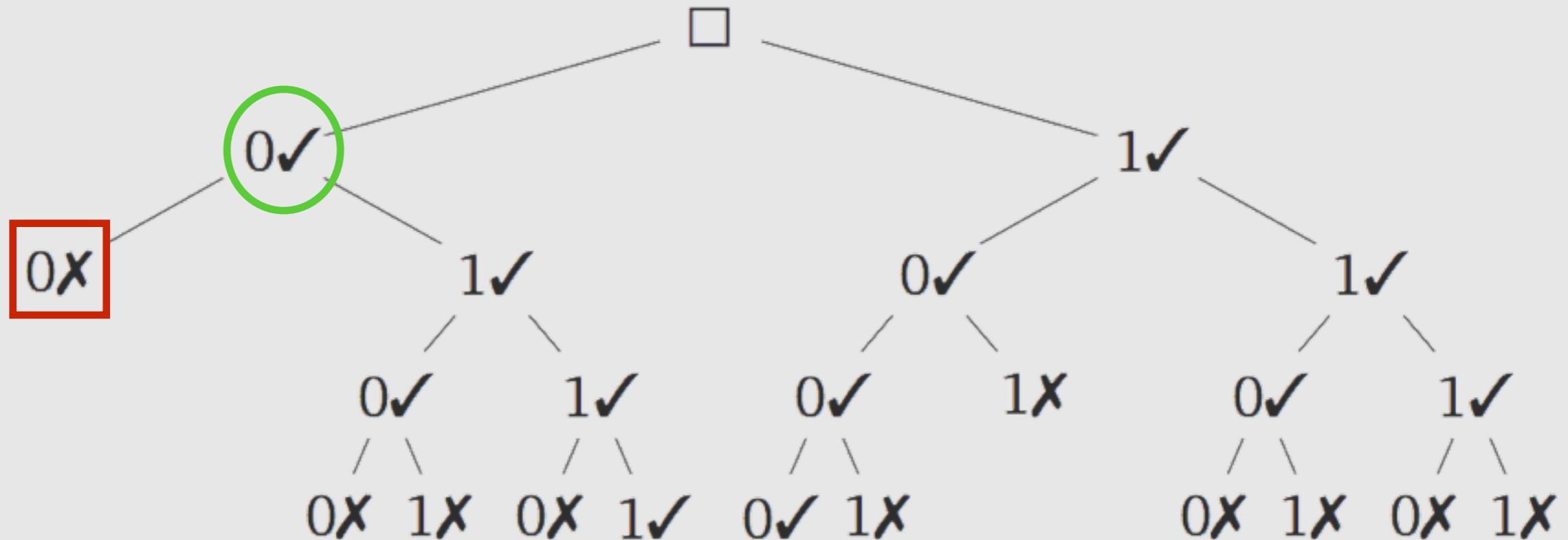


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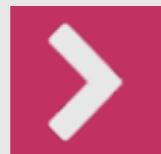


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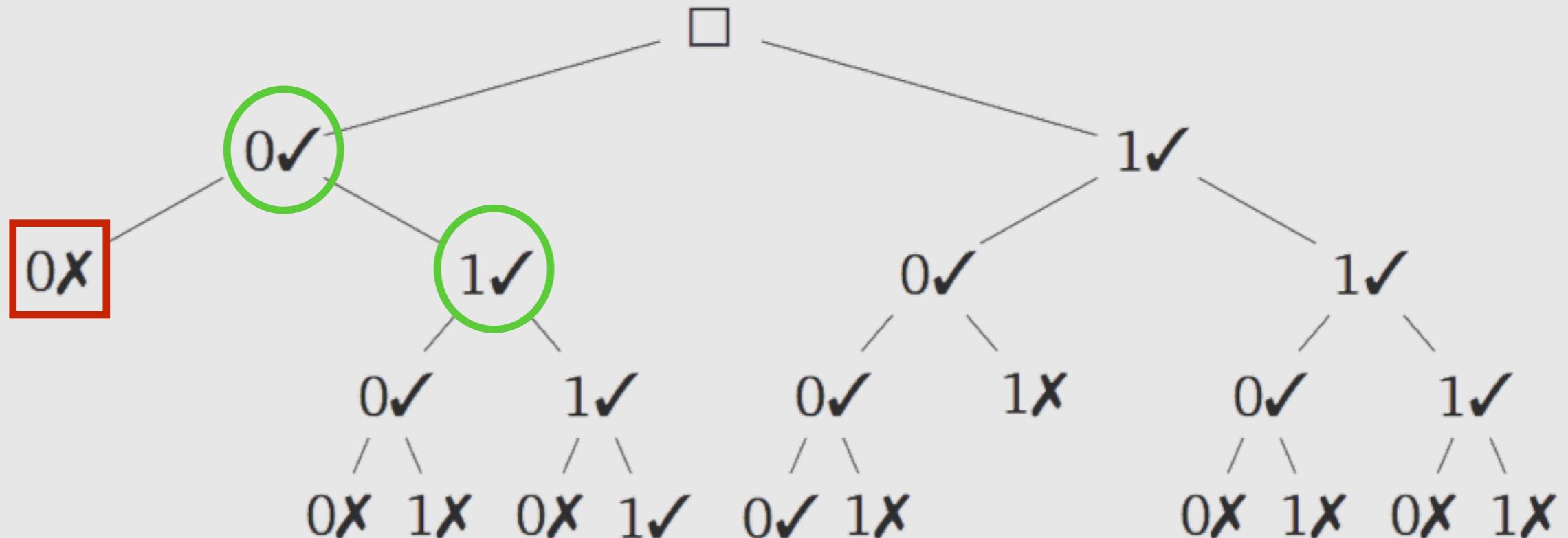


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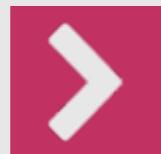


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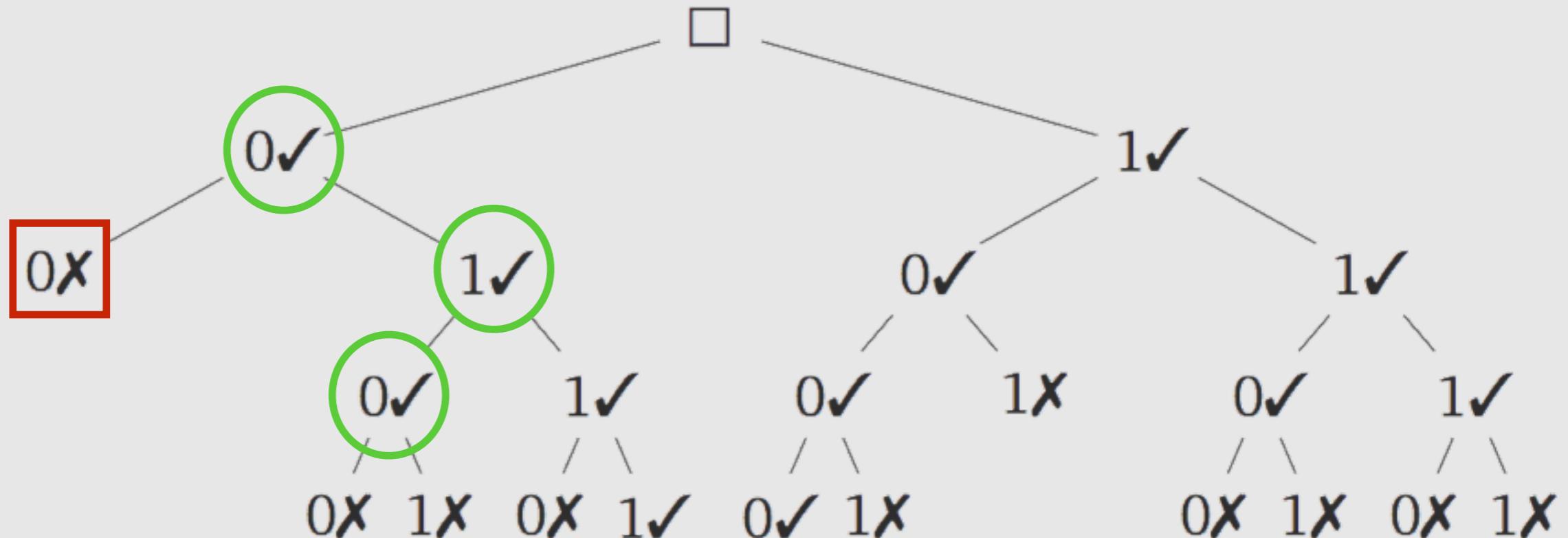


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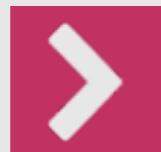


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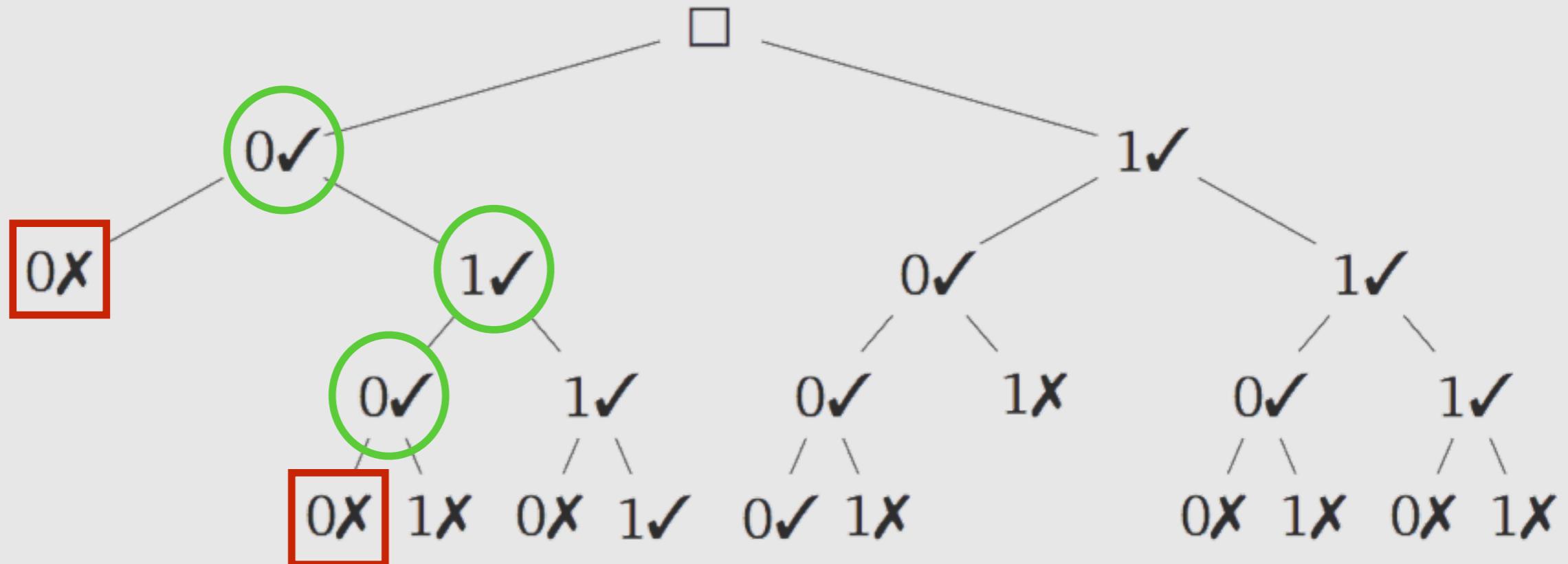


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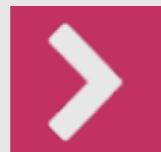


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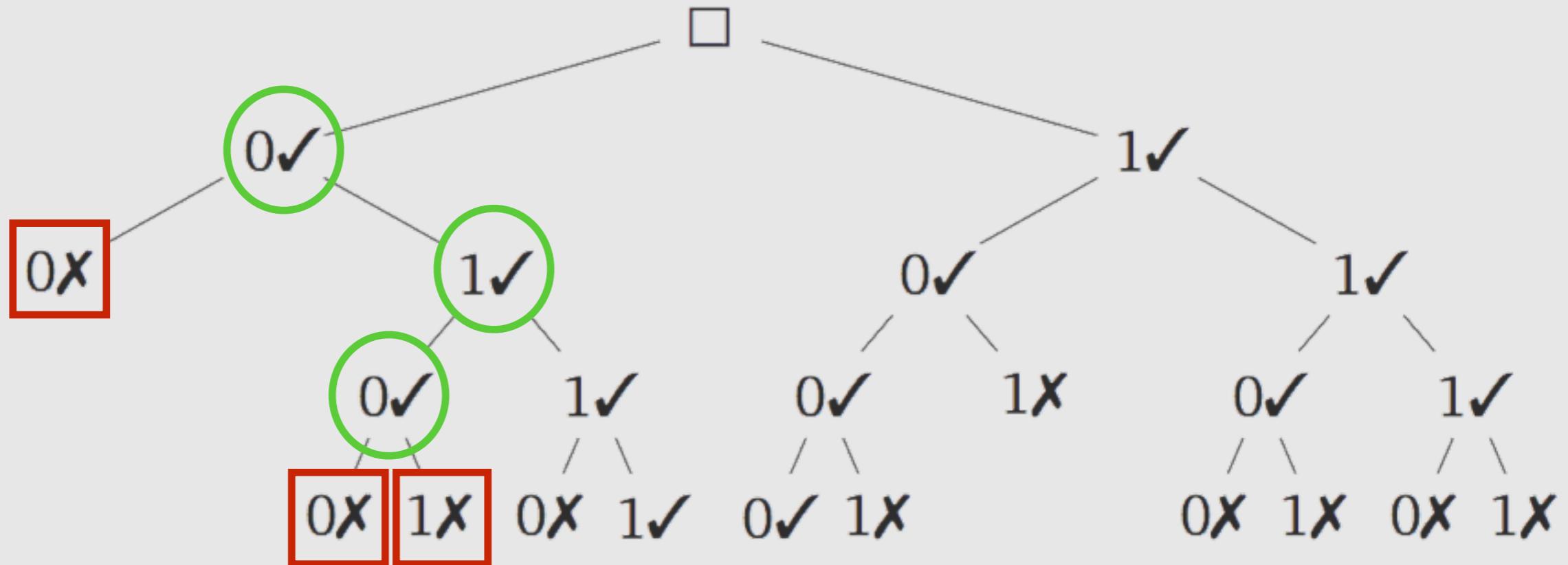


MaxiHMM algorithm

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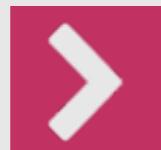


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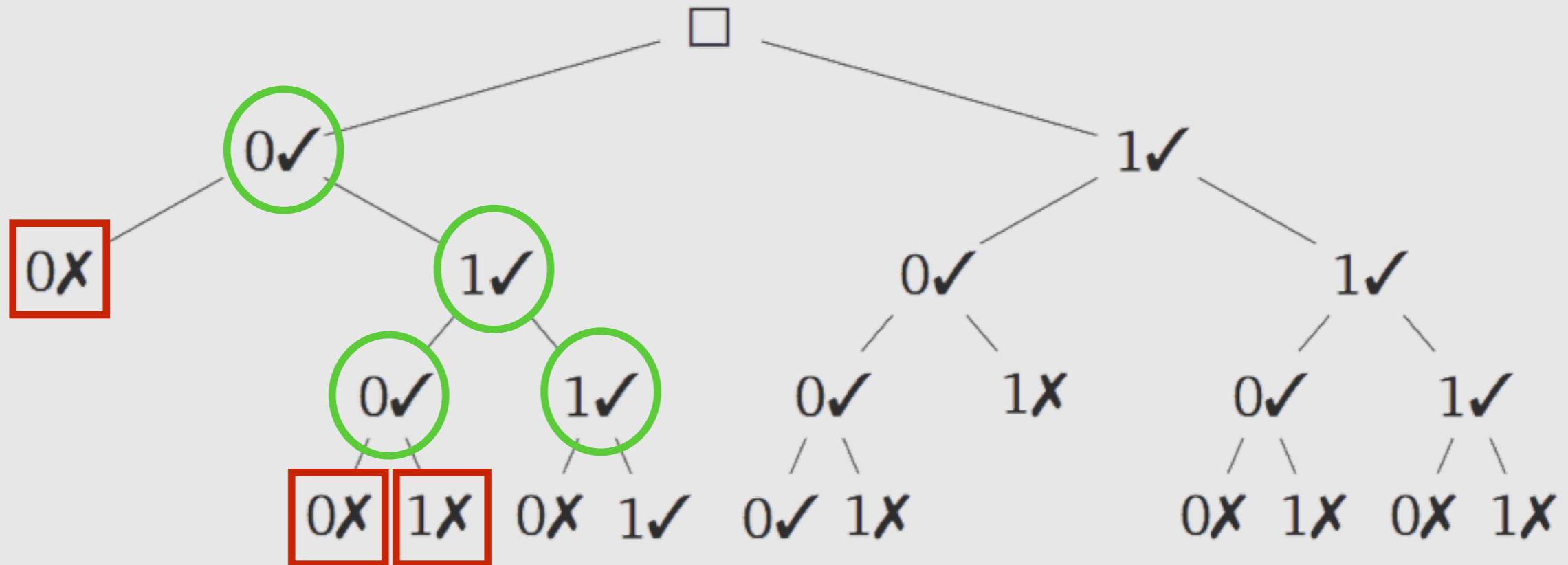


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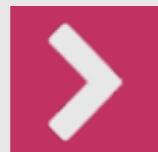


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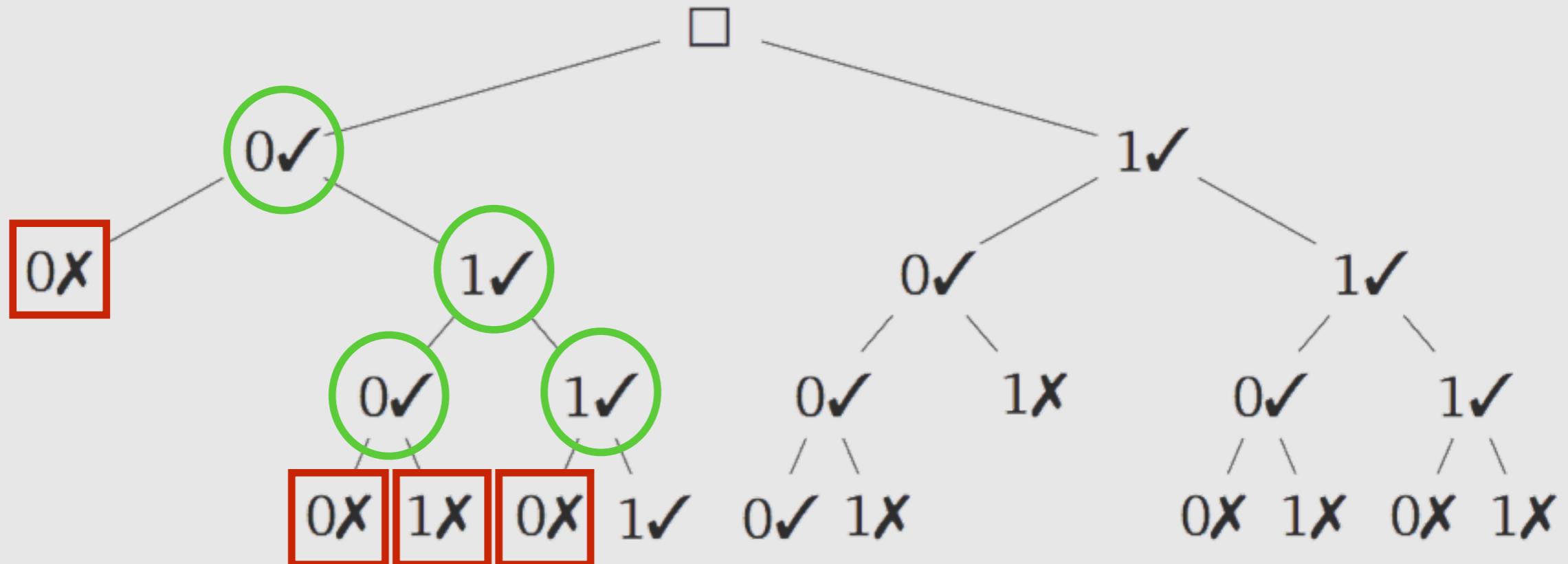


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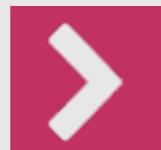


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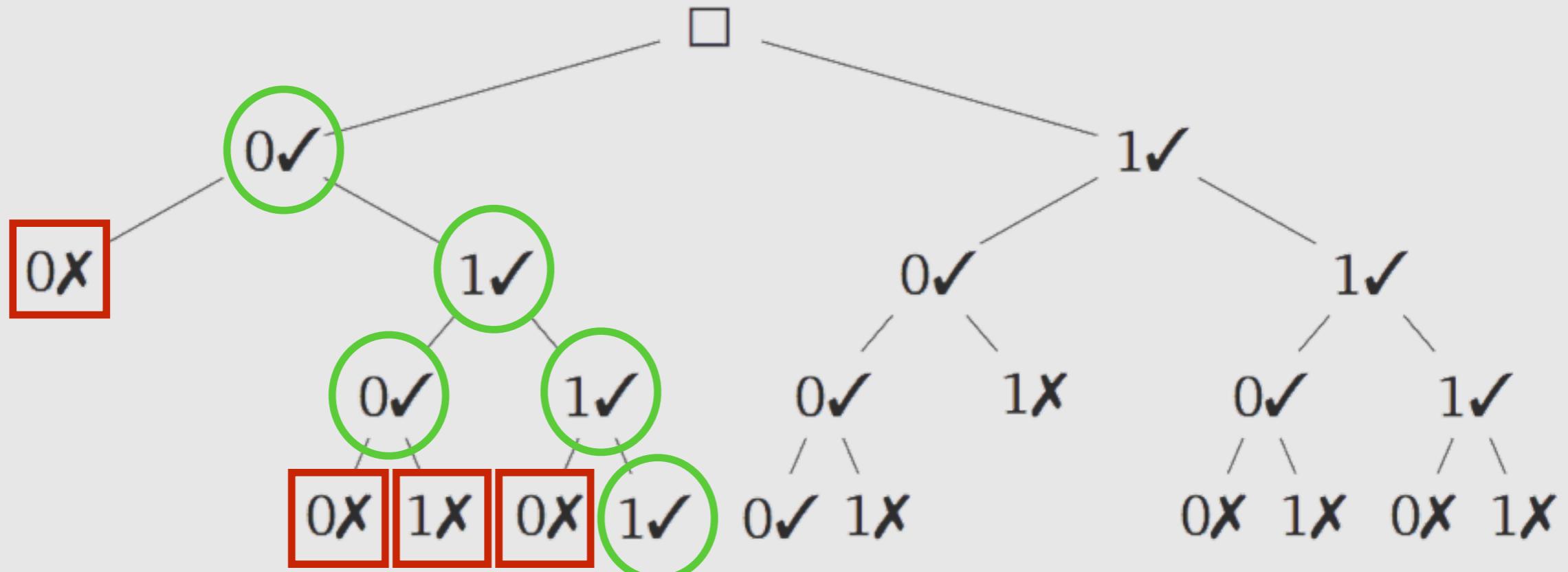


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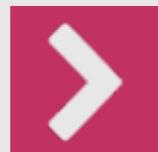


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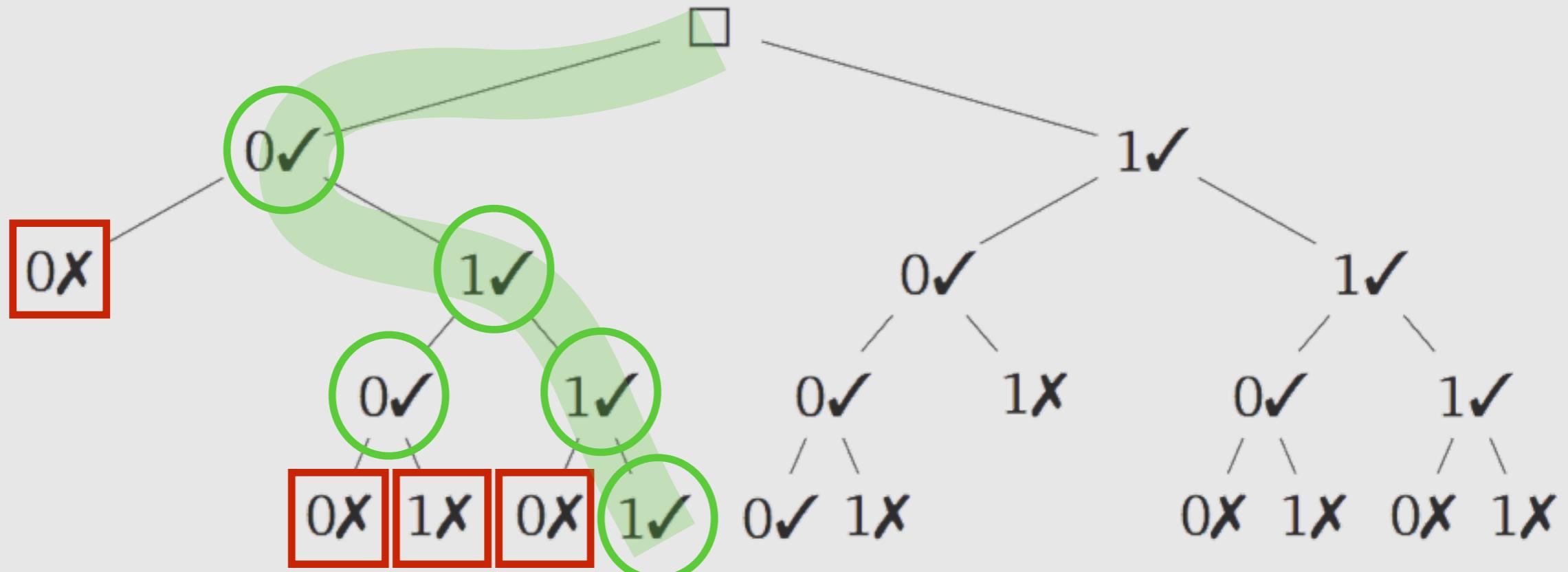


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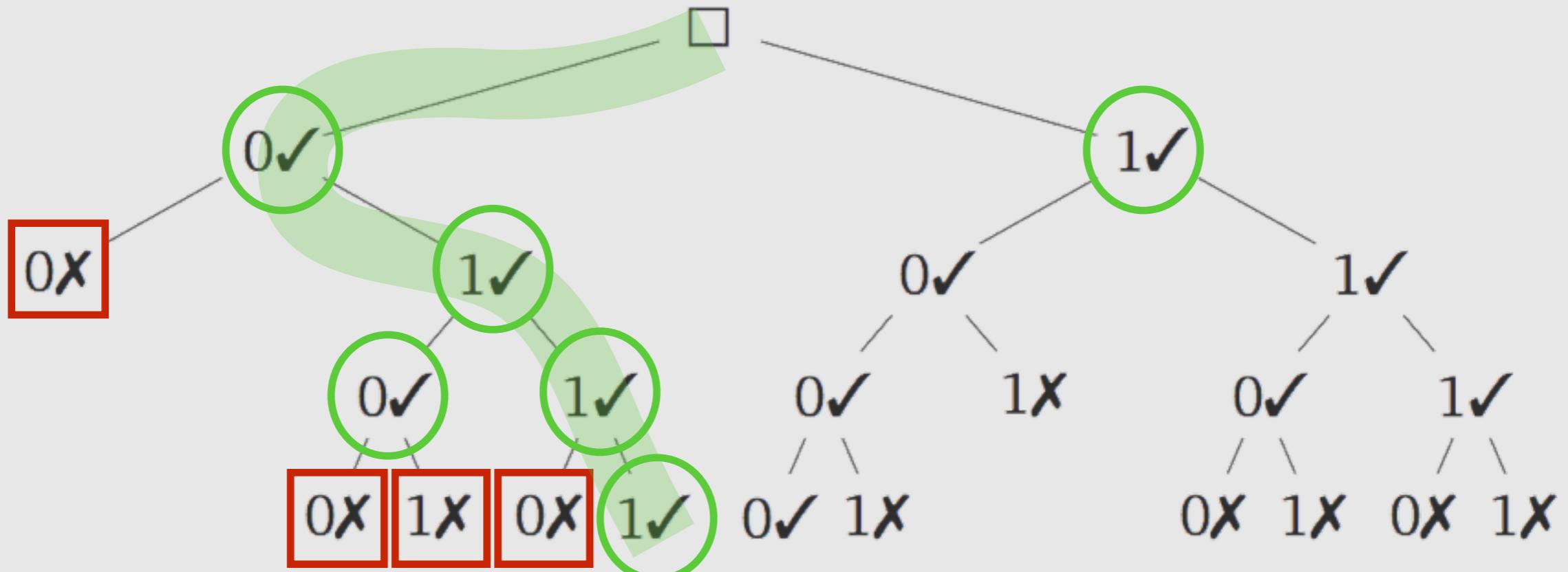
MaxiHMM algorithm



General overview



MaxiHMM algorithm



MaxiHMM algorithm

General overview



We recursively define two parameters:

MaxiHMM algorithm

General overview



We recursively define two parameters:

$$\gamma_k(x_k, \hat{x}_k) \triangleq \min_{\hat{x}_{k+1} \in \mathcal{X}_{k+1}} \max_{x_{k+1} \in \mathcal{X}_{k+1}} \chi_{k+1}(x_{k+1}, x_k, \hat{x}_{k+1}, \hat{x}_k)$$
$$\omega_{k+1}(x_{k+1}, \hat{x}_{k+1}, o_{k+1}) \gamma_{k+1}(x_{k+1}, \hat{x}_{k+1})$$
$$\gamma_n(x_n, \hat{x}_n) \triangleq 1$$

MaxiHMM algorithm

General overview



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... and ...

$$\delta_k(x_k, \hat{x}_{1:k}) \triangleq \max_{x_{k-1} \in \mathcal{X}_{k-1}} \chi_k(x_k, x_{k-1}, \hat{x}_k, \hat{x}_{k-1})$$
$$\omega_k(x_k, \hat{x}_k, o_k) \delta_{k-1}(x_{k-1}, \hat{x}_{1:k-1})$$

$$\delta_1(x_1, \hat{x}_1) \triangleq \chi_1(x_1, \hat{x}_1) \omega(x_1, \hat{x}_1, o_1)$$

MaxiHMM algorithm

General overview



We want to be able to check whether:

$$(\forall \hat{x}_{1:n} \in \text{opt}_{\max}(\mathcal{X}_{1:n})) \hat{x}_{1:k} \neq \hat{x}_{1:k}^*$$



some initial segment

MaxiHMM algorithm

General overview



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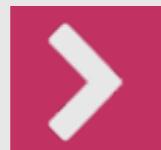


some initial segment

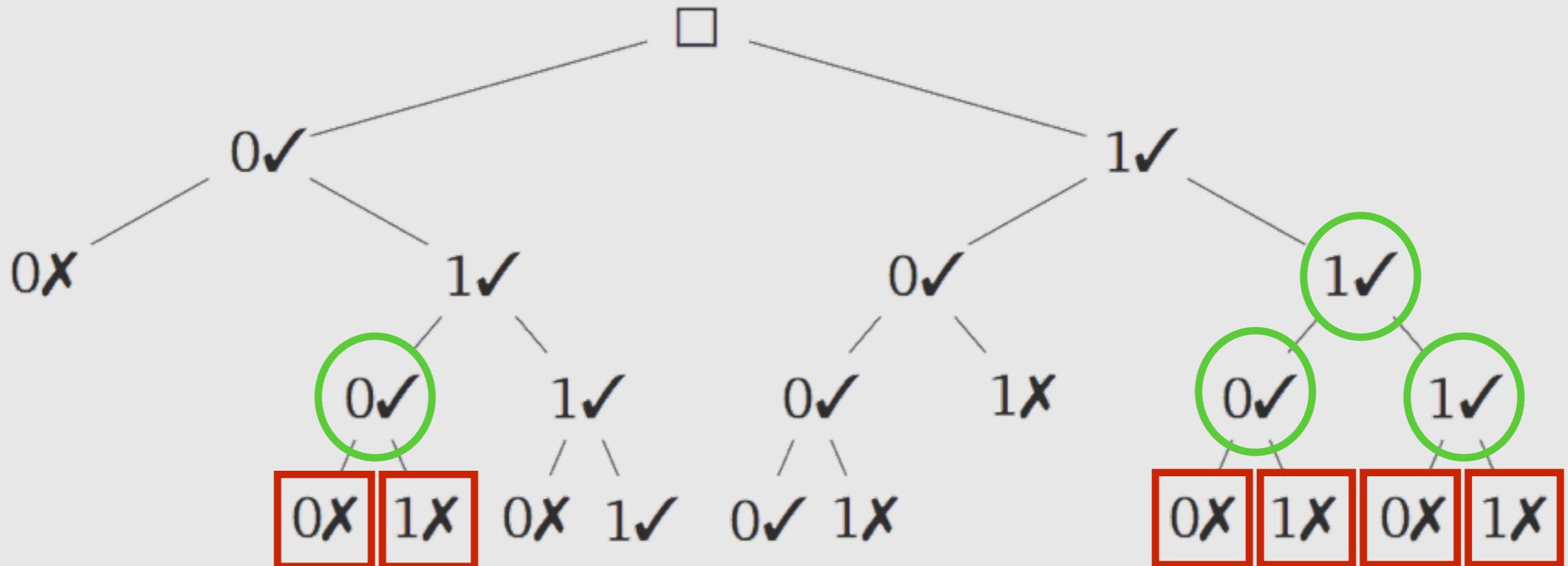
$$\max_{x_k \in \mathcal{X}_k} \delta_k(x_k, \hat{x}_{1:k}^*) \gamma_k(x_k, \hat{x}_k^*) > 1$$

MaxiHMM algorithm

General overview



MaxiHMM algorithm



MaxiHMM algorithm

General overview



Algorithm 1: MaxiHMM

Data: the local parameters χ_k and ω_k , an output sequence $o_{1:n}$, and the corresponding global parameters γ_k

Result: the set $\text{opt}(\mathcal{X}_{1:n}|o_{1:n})$ of all maximal state sequences

```
1 opt( $\mathcal{X}_{1:n}|o_{1:n}$ )  $\leftarrow \emptyset$ 
2 for  $\hat{x}_1 \in \mathcal{X}_1$  do
3   for  $x_1 \in \mathcal{X}_1$  do
4      $\delta_1(x_1, \hat{x}_1) \leftarrow \chi_1(x_1, \hat{x}_1)\omega_1(x_1, \hat{x}_1, o_1)$ 
5     if  $\max_{x_1 \in \mathcal{X}_1} \delta_1(x_1, \hat{x}_1)\gamma_1(x_1, \hat{x}_1) \leq 1$  then Recur(1,  $\hat{x}_1$ ,  $\delta_1(\cdot, \hat{x}_1)$ )
6 return opt( $\mathcal{X}_{1:n}|o_{1:n}$ )
```

Procedure Recur($k, \hat{x}_{1:k}, \delta_k(\cdot, \hat{x}_{1:k})$)

```
1 if  $k = n$  then
2   add  $\hat{x}_{1:n}$  to  $\text{opt}(\mathcal{X}_{1:n}|o_{1:n})$                                  $\triangleright$  We found a solution!
3 else
4   for  $\hat{x}_{k+1} \in \mathcal{X}_{k+1}$  do
5      $\hat{x}_{1:k+1} \leftarrow (\hat{x}_{1:k}, \hat{x}_{k+1})$        $\triangleright$  Append  $\hat{x}_{k+1}$  to the end of  $\hat{x}_{1:k}$ 
6     for  $x_{k+1} \in \mathcal{X}_{k+1}$  do
7        $\delta_{k+1}(x_{k+1}, \hat{x}_{1:k+1}) \leftarrow \max_{x_k \in \mathcal{X}_k} \chi_{k+1}(x_{k+1}, x_k, \hat{x}_{k+1}, \hat{x}_k)$ 
8        $\omega_{k+1}(x_{k+1}, \hat{x}_{k+1}, o_{k+1})$ 
9        $\delta_k(x_k, \hat{x}_{1:k})$ 
10      if  $\max_{x_{k+1} \in \mathcal{X}_{k+1}} \delta_{k+1}(x_{k+1}, \hat{x}_{1:k+1})\gamma_{k+1}(x_{k+1}, \hat{x}_{k+1}) \leq 1$  then
11        Recur( $k + 1, \hat{x}_{1:k+1}, \delta_1(\cdot, \hat{x}_{1:k+1})$ )
```

MaxiHMM algorithm

Properties



MaxiHMM algorithm

➤ Recursive

MaxiHMM algorithm

Properties



MaxiHMM algorithm

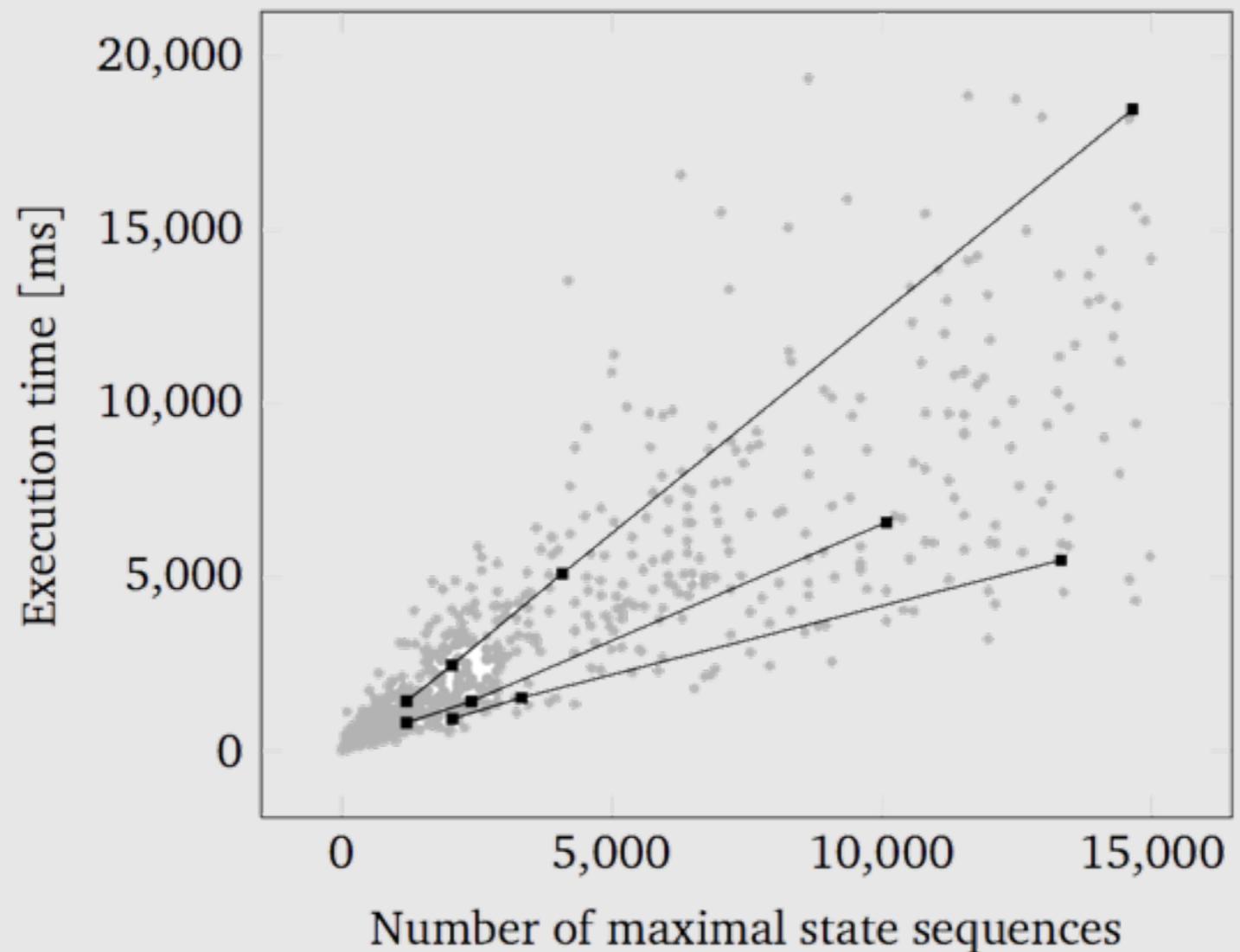
- Recursive
- Heuristic complexity $O(Snm^2)$
 - S: number of solutions
 - n: length of the sequence
 - m: size of state space

MaxiHMM algorithm

Properties

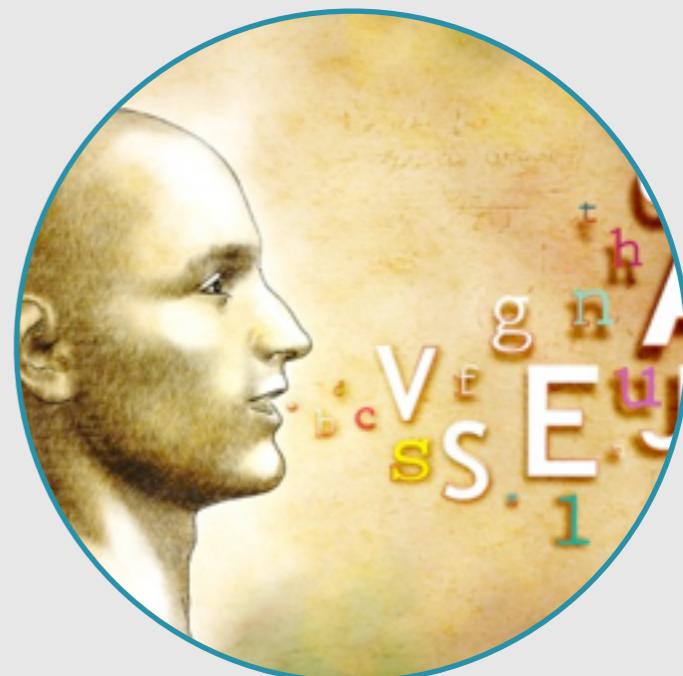


MaxiHMM algorithm



MaxiHMM algorithm

Applications





...the Third
political changes
to those apparent in
developing countries of various
United Nations. The first is
Third World, a common model
apparent if nations experience simil-
ar political change, and this is
on urban change and its causes is
in each Third World city, we suspect
greater diversity in the rates of urban
The second is the fact that there are
which, without a major modification to
have no hope of developing prosperous
As such, they can hardly be expected to
model which depends on very large capital
economic changes and a very considerable level
their citizens are never likely to have the
choose where to live, based on anything but
economic base for their lives.
to go beyond local and often inaccessible
urban change in the Third World. Now
rural and urban are needed in analysis
and spatial distribution of non-agricultural
The concept that there is a urban bias
must be tested on a nation by nation
basis, it is not evident that power groups
thus centres are benefiting more than the
studies of urban change must build from
city and sub-city studies.
few mega-cities of 7 million or more
In 1990, they are likely to house
World's population with less than
0 million or more inhabitants,
to avoid the results of the
early 1990s. If our concern
ments and aid agencies
and in controlling
is when special

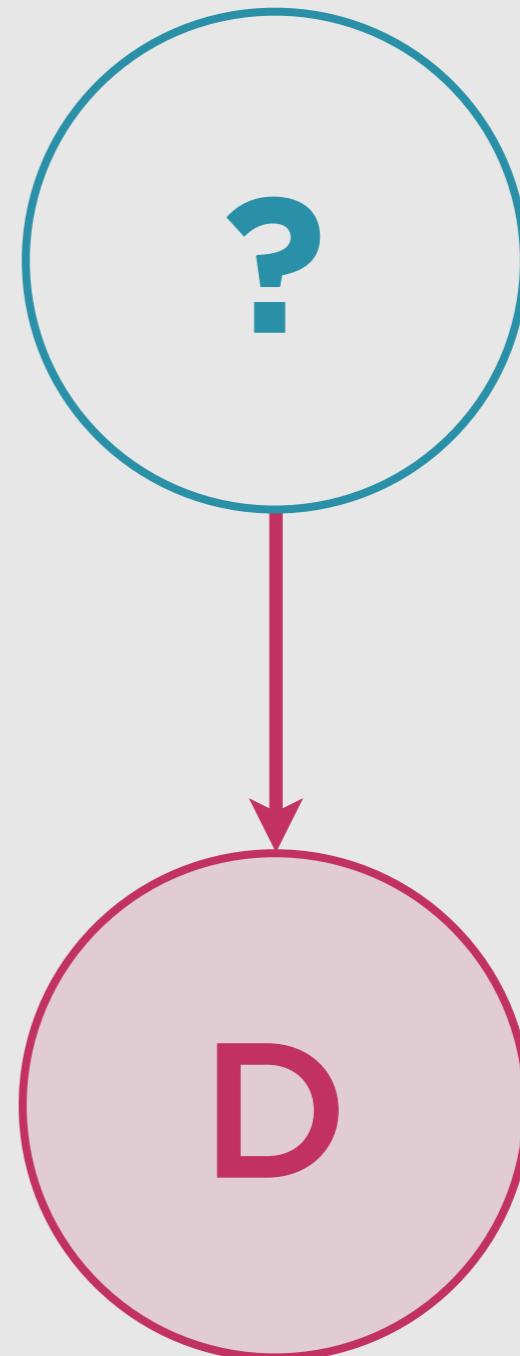
VAN DEN VOS REYNAERDE

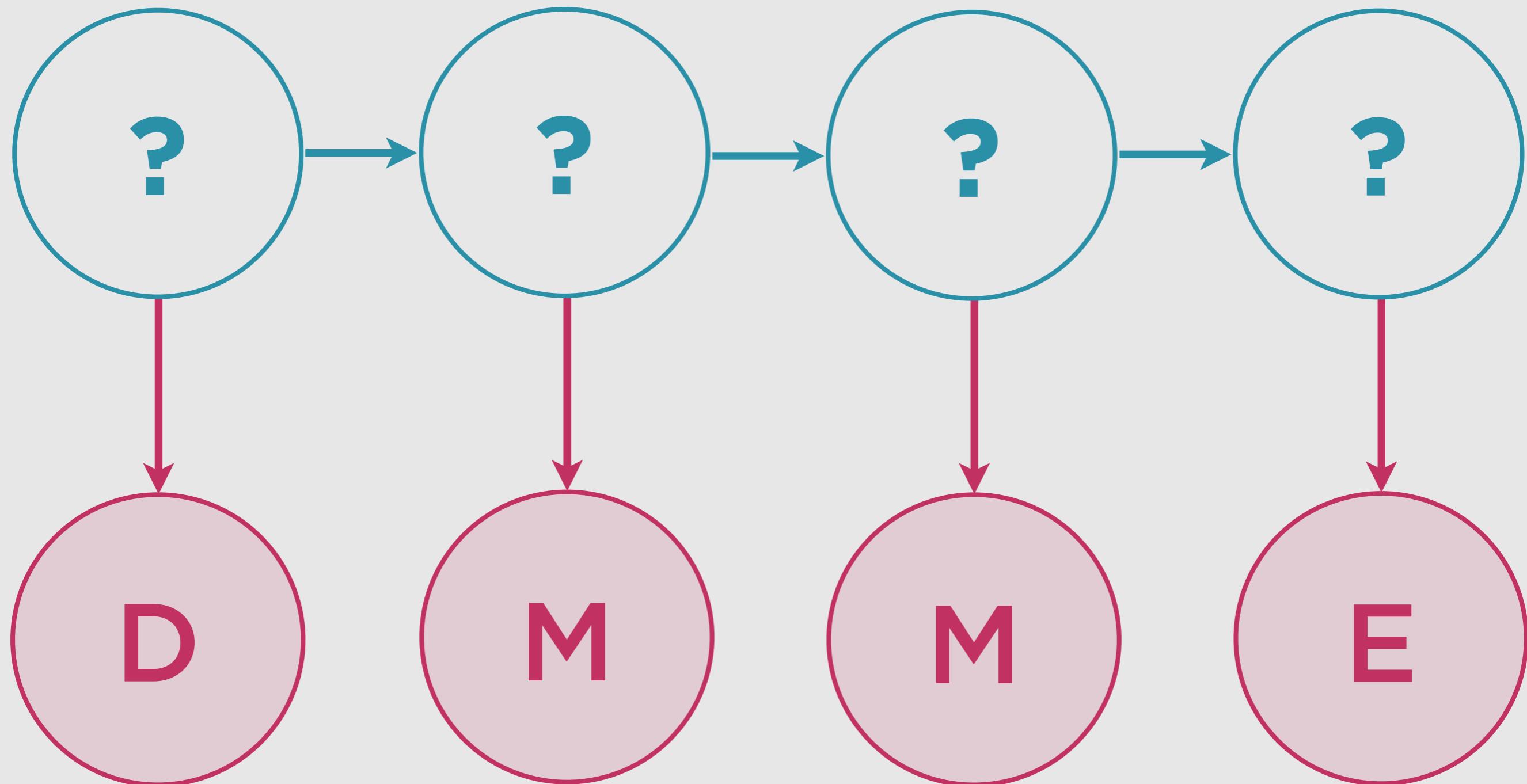
Willem die vele bouke maecte,
Daer hi dicken omme waecte,
Hem vernoyde so haerde
Dat die avonture van Reynaerde
In Dietsche onghemaket bleven

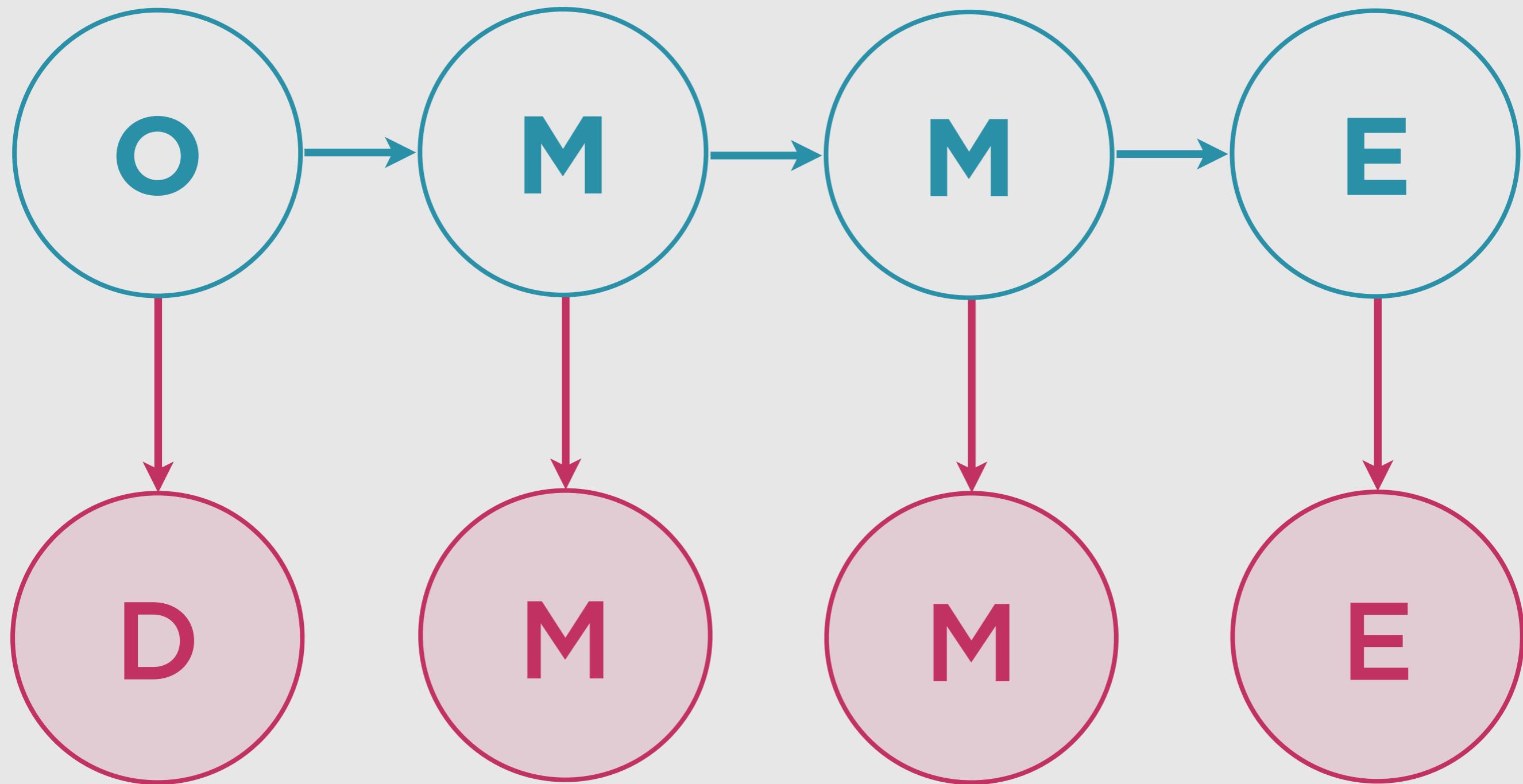
- Die Willem niet hevet vulscreven -
Dat hi die vijte van Reynaerde soucken
Ende hise na den Walschen boucken
In Dietsche dus hevet begonnen.



Willem, die vele bouke maecte,
Daer hi dicken omme waecte,
Hem vernoyde so haerde
Dat die avonture van Reynaerde
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Dat hi die vijte van Reynaerde soucken
Ende hise na den Walschen boucken
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Van den Vos Reynaerde

▶ Medieval Dutch (13th century)





Van den Vos Reynaerde

- Medieval Dutch (13th century)
- Often little data on hand





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- Medieval Dutch (13th century)
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- Building very accurate **precise** models is impossible





Van den Vos Reynaerde

- Medieval Dutch (13th century)
- Often little data on hand
- Building very accurate **precise** models is impossible
- Use **imprecise** models





 BEWAENT

read as BEWAEHT

 3-best Viterbi solutions

BEWAENT BEWAERT BEWAEHT

 MaxiHMM solutions

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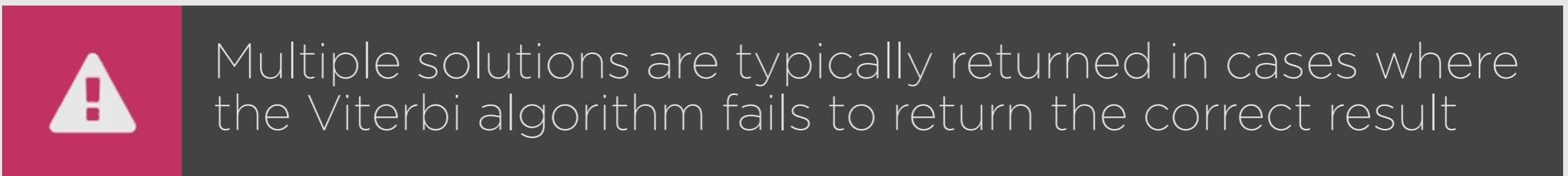
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 3-best Viterbi solutions

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 MaxiHMM solutions

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Imprecision takes care of problems with small data sets

Questions?

