第一次作业

计算物理

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1. Consider the 32-bit single-precision floating-point number A

	s	e	f
Bit position	31	30...23	22...0
value	0	0000 1110	1010 0000 0000 0000 0000 000

Determine the full value of A.

$s = 0\, e = 00001110 = 2^3 + 2^2 + 2 = (14)_{10}\, p = e - 127 = -113$
$f = 1.10100000000000000000000 = (1 + 0.5 + 0.125)_{10} = 1.625$
$(-1)^s \times 1.625 \times 2^p = 1.565 \times 10^{-34}$

2. Sometimes the loss of significance error can be avoided by rearranging terms in the function using a known identity from trigonometry or algebra. Find an equivalent formula for the following functions that avoids a loss of significance.

   (a) $\ln(x+1) - \ln(x)$ for large x
   (b) $\sqrt{x^2+1} - x$ for large x
   (c) $\cos^2(x) - \sin^2(x)$ for $x \approx \pi/4$
   (d) $\sqrt{\frac{1+\cos(x)}{2}}$ for $x \approx \pi$

(a). $\ln(x+1) - \ln(x) = \ln(1 + \frac{1}{x})$ (b). $\sqrt{x^2+1} - x = \frac{1}{\sqrt{x^2+1}+x}$ (c). $\cos^2(x) - \sin^2(x) = \cos(2x)$ (d). $\sqrt{\frac{1+\cos(x)}{2}} = \cos(\frac{x}{2})$

3.Writer a program to determine the under- and overflow limits.

over-flow: 8.9885e+307 under-flow: 4.9407e-324

code:

```
%Write a programme to determine the under and over-flow limits.

under = 1;
over = 1;
a = 0;
```

```
    a = under;
    under = under/2
end

while b~=over
    b = over;
    over = over*2
end
```

4.Write a program to determine your machine precision and double-precision floats.

single-precision is: 5.9605e-08 double-precision is: 1.1102e-16

code:

```
clear all

one = single(0);
sing_eps = single(1);
while one ~= 1
    sing_eps = single(sing_eps/2);
    one = single(1 + sing_eps);
end

fprintf (1,'single-precision is: \n')
single_eps = sing_eps

dou_one = 0;
dou_eps = 1;
while dou_one ~= 1
    dou_eps = dou_eps/2;
    dou_one = 1 + dou_eps;
end
fprintf (1,'double-precision is: \n')
double_eps = dou_eps
```

5. The value of π can be calculated with the series:

$$\pi = 4\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{2n-1} = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots\right)$$

- Describe your algorithm that calculates the value of π by using n terms of the series.
- Write a program to implement your algorithm and calculate the corresponding true relative error.
- Use the program to calculate π and the true relative error for: (a) $n = 10$, (b) $n = 20$, (c) $n = 40$ (d) $n > 50$ of your choice.
- Comment on your results of relative errors.

(1)

1. $a = 0$
2. 当N为偶数时，$n = N - 1$; 当N为奇数时，$n = N - 2$
3. $a = a + \frac{1}{2n-1} - \frac{1}{2n+1} = \frac{2}{4n^2-1}$
4. $n = n - 2$
5. 重复步骤3、4直至$n = 1$
6. N为偶数时输出$\pi = 4a$；N为奇数时，输出$\pi = 4(a + \frac{1}{2N-1})$

(2)code:

```
clear all
clc

fprintf(1,' 输入级数N:  \n ');
N = input('N = ');
a = 0;

if mod(N,2) == 0
    n = N-1;
    while n>=1
        a = a +2/(4*n^2-1);
        n = n - 2;
    end
else
    n = N-2;
    while n>=1
        a = a +2/(4*n^2-1);
        n = n - 2;
    end
    a = a + 1/(2*N-1);
end
a = 4*a;
```

(3)

n	π估计值	相对误差
10	3.041840	-3.175238%
20	3.091624	-1.590558%
40	3.116597	-0.795650%
100	3.131593	-0.318302%

(4)

绘制N=10、20、40、50、100、200 级数——误差双对数图，可以看到，随着n的增大，相对误差会逐渐减小，双对数图斜率拟合为-0.994，误差主要是approximation errors