

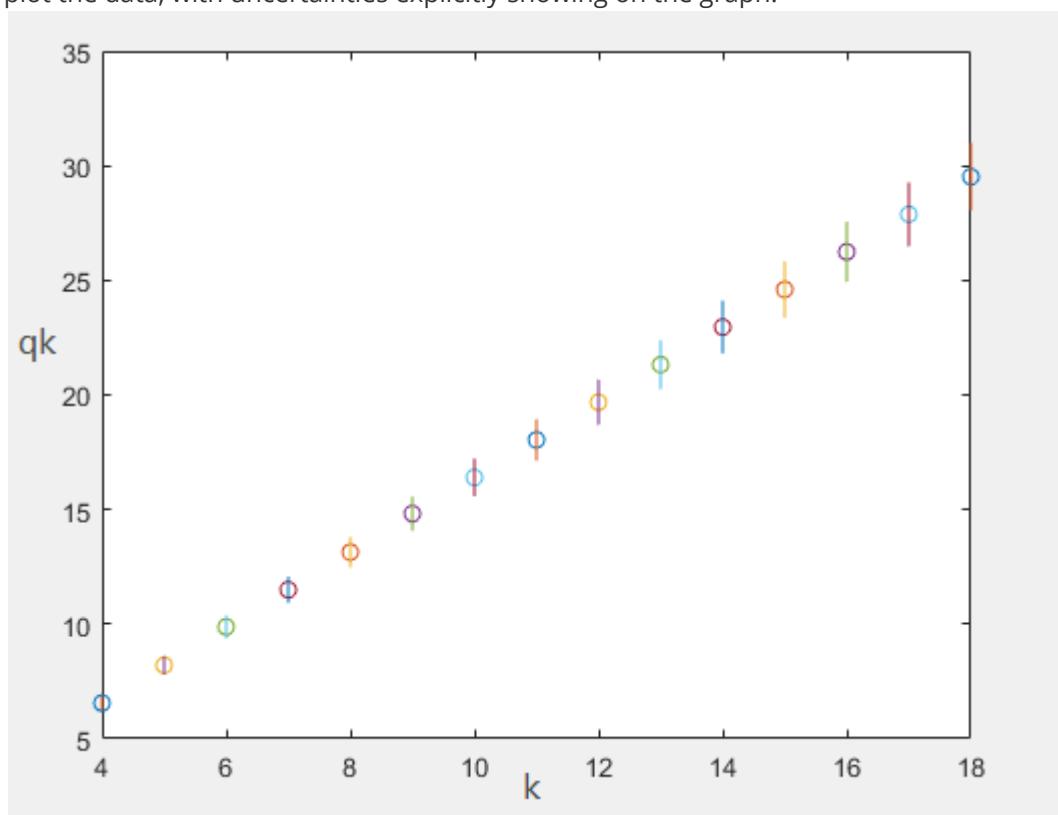
第三次作业

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1

(1)

Firstly we plot the data, with uncertainties explicitly showing on the graph.



And we apply linear regression method to this problem, using Round-off error susceptible version.

Code:

```
X=4:1:18;
Y=
[6.558,8.206,9.880,11.50,13.14,14.82,16.40,18.04,19.68,21.32,22.96,24.60,26.24,27.88,29.52];
U=0.05*Y;
for i=1:15
    plot(X(i),Y(i),'-o',[X(i),X(i)],[Y(i)-U(i),Y(i)+U(i)]);
    hold on;
end
S=sum(1./U./U);
Sx=sum(X./U./U);
Sy=sum(Y./U./U);
```

```

t=(X-Sx/S)./U;
Stt=sum(t.*t);
a1=sum(t.*Y./U)/Stt;
a0=(Sy-Sx*a1)/S;
u2a0=(1+Sx*Sx/S/Stt)/S;
u2a1=1/Stt;
Cov=-Sx/S/Stt;
r=Cov/sqrt(u2a0)/sqrt(u2a1);
Y1=a0+a1*X;
Chi2=sum((Y-Y1).*(Y-Y1)./U./U);
[a0,u2a0,a1,u2a1,Chi2,Cov,r]

```

Result:

$$a_0 = 0.01175, \sigma_{a_0} = 0.3838, a_1 = 1.640, \sigma_{a_1} = 0.04749$$

$$X^2 = 0.01088, Cov(a_0, a_1) = -0.01631, r = -0.8949$$

We get $q_k = 0.01175(\pm 0.3838) + 1.640(\pm 0.047)k$. The intercept has rather big uncertainty in the experiment $0.01175(\pm 0.3838)$, while the slope is relatively accurate $1.640(\pm 0.047)$. Here X^2 is about 0.01088, which is small. And coefficient of correlation is -0.8949, showing a fairly strong anticorrelation of error between a_0 and a_1 .

2

(1)

$$Y = y, X = \frac{1}{x} \text{ then, } Y = AX + B$$

(2)

$$Y = \frac{1}{y}, X = x, A = \frac{1}{D}, B = CA \text{ then, } Y = AX + B$$

(3)

$$Y = \frac{1}{y}, X = x \text{ then, } Y = AX + B$$

(4)

$$\frac{1}{y} = \frac{A}{x} + B \quad Y = \frac{1}{y}, X = \frac{1}{x} \text{ then, } Y = AX + B$$

(5)

$$Y = y, X = \ln x \text{ then, } Y = AX + B$$

(6)

$$\ln y = \ln C + A \ln x \quad Y = \ln y, X = \ln x, B = \ln C \text{ then, } Y = AX + B$$

(7)

$$y^{-0.5} = Ax + BY = y^{-0.5}, X = x \text{ then } Y = AX + B$$

(8)

$$\ln y = \ln C + \ln x - Dx \ln \frac{y}{x} = \ln C - Dx Y = \ln \frac{y}{x}, X = x, A = -D, B = \ln C \text{ then } Y = AX + B$$

3

$$y = 1 + Ae^{-\alpha t}, A < 0, \alpha > 0 \ln(1 - y) = \ln(-A) - \alpha t \quad Y = \ln(1 - y), B = \ln(-A) \text{ then } Y = B - \alpha t$$

$$t_i \mid y_i \mid Y_i$$

- $\mid \therefore \mid \therefore$

$$0.5 \mid 0.4 \mid -0.5108$$

$$1 \mid 0.5 \mid -0.6931$$

$$2 \mid 0.7 \mid -1.2040$$

$$3 \mid 0.8 \mid -1.6094$$

$$4 \mid 0.9 \mid -2.3026$$

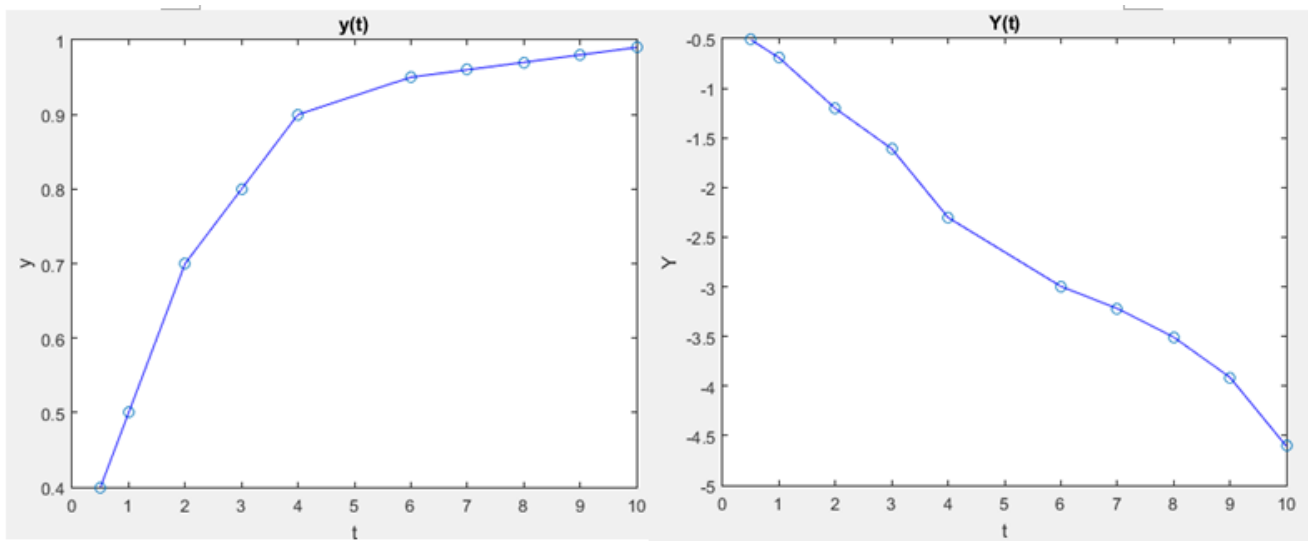
$$6 \mid 0.95 \mid -2.9957$$

$$7 \mid 0.96 \mid -3.2189$$

$$8 \mid 0.97 \mid -3.5066$$

$$9 \mid 0.98 \mid -3.9120$$

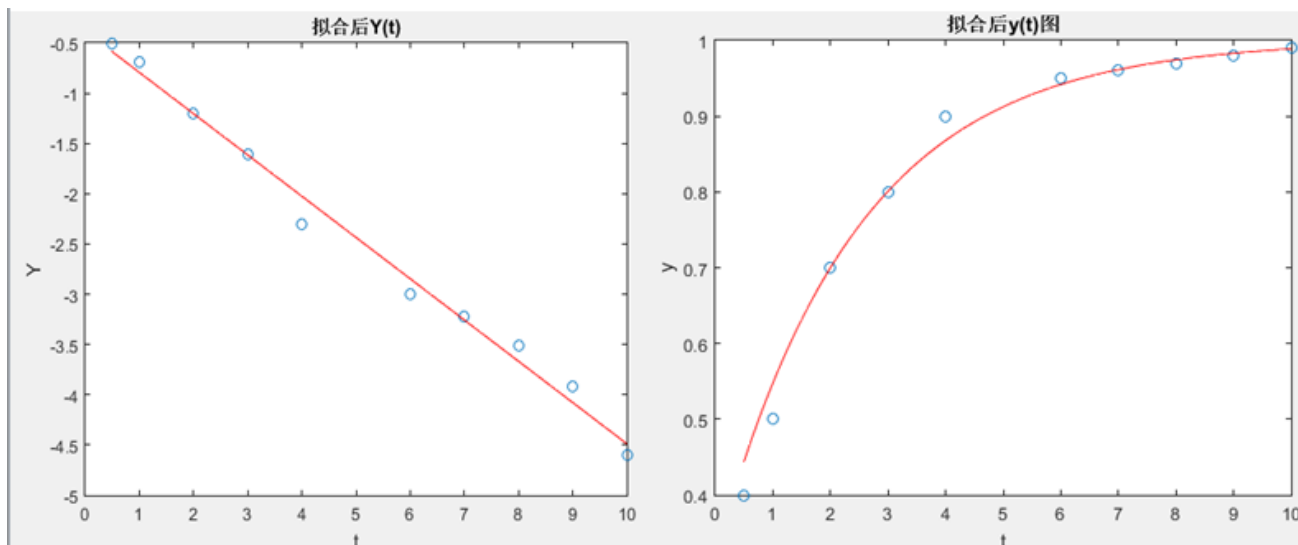
$$10 \mid 0.99 \mid -4.6052$$



$$S_t = \sum_{i=1}^{10} t_i = 50.5 \quad S_Y = \sum_{i=1}^{10} Y_i = -24.5583 \quad S_t Y = \sum_{i=1}^{10} t_i Y_i = -167.2141 \quad S_t t = \sum_{i=1}^{10} t_i t_i = 360.25$$

$$\text{So, } \alpha = -\frac{10S_{tY} - S_t S_Y}{10S_{tt} - S_t^2} = 0.410497 \quad B = \frac{S_{tt} S_Y - S_t S_{tY}}{10S_{tt} - S_t^2} = -0.382825 \quad A = -e^B = -0.681932$$

$$y = 1 - 0.681932e^{-0.410497t}$$



code:

```
%计算物理 第三次作业 第3题
%王潇卫 515072910032
clear all
clc

t = [0.5,1,2,3,4,6,7,8,9,10];
y = [0.4,0.5,0.7,0.8,0.9,0.95,0.96,0.97,0.98,0.99];
Y = log(1-y);
%关系式变形为 Y = B - at

%% 作图
figure(1) % y、t图
plot(t,y,'o',t,y,'b')
xlabel('t')
ylabel('y')
title('y(t)')
hold on

figure(2) % Y、t图
plot(t,Y,'o',t,Y,'b')
xlabel('t')
ylabel('Y')
title('Y(t)')
hold on

%% 拟合
S_t = sum(t);
S_Y = sum(Y);
tY = t.*Y;
S_tY = sum(tY);
S_tt = sum(t.^2);

alpha = (S_t*S_Y-10*S_tY)/(10*S_tt-(S_t)^2);
```

```

B = (S_tt*S_Y-S_t*S_tY)/(10*S_tt-(S_t)^2);
A = -exp(B);

fprintf(1,'Fitting the curve gets Y = %f - %f t \n',B,alpha);
fprintf(1,'Equals to y = 1 %f e^(-%f t) \n',A,alpha);

%% 作图
t1 = 0.5:0.1:10;
y1 = 1 - 0.681932*exp(-0.410497*t1);
Y1 = -0.382825 - 0.410497*t1;

figure(3) % 拟合后y、t图
plot(t,y,'o',t1,y1,'r')
xlabel('t')
ylabel('y')
title('拟合后y(t)图')
hold on

figure(4) % 拟合后Y、t图
plot(t,Y,'o',t1,Y1,'r')
xlabel('t')
ylabel('Y')
title('拟合后Y(t)')
hold on

```

4

We use the method of linearization of nonlinear data to fit the dataset.

(i)

For the power fitting function, $k=aT^b$, Define $Y=\ln(k)$, $X=\ln(T)$, then we get $Y=a_1X + a_0$, and we have the relation: $b=a_1, a=\exp(a_0)$;

(ii)

For the exponential fitting function, $k=ae^{bT}$, Define $Y=\ln(k)$, $X=T$, then we get $Y=a_1X + a_0$, and we have the relation: $b=a_1, a=\exp(a_0)$;

(iii)

For the saturation fitting function, $k=\frac{T}{aT+b}$, Define $Y=\frac{1}{k}$, $X=\frac{1}{T}$ then we get $Y=a_1X + a_0$, and we have the relation: $b=a_1, a=a_0$;

【Code:Q4.m】

```

%this script is used to solve Question 4.
clear all; clc;

%% the dataset:

```

```

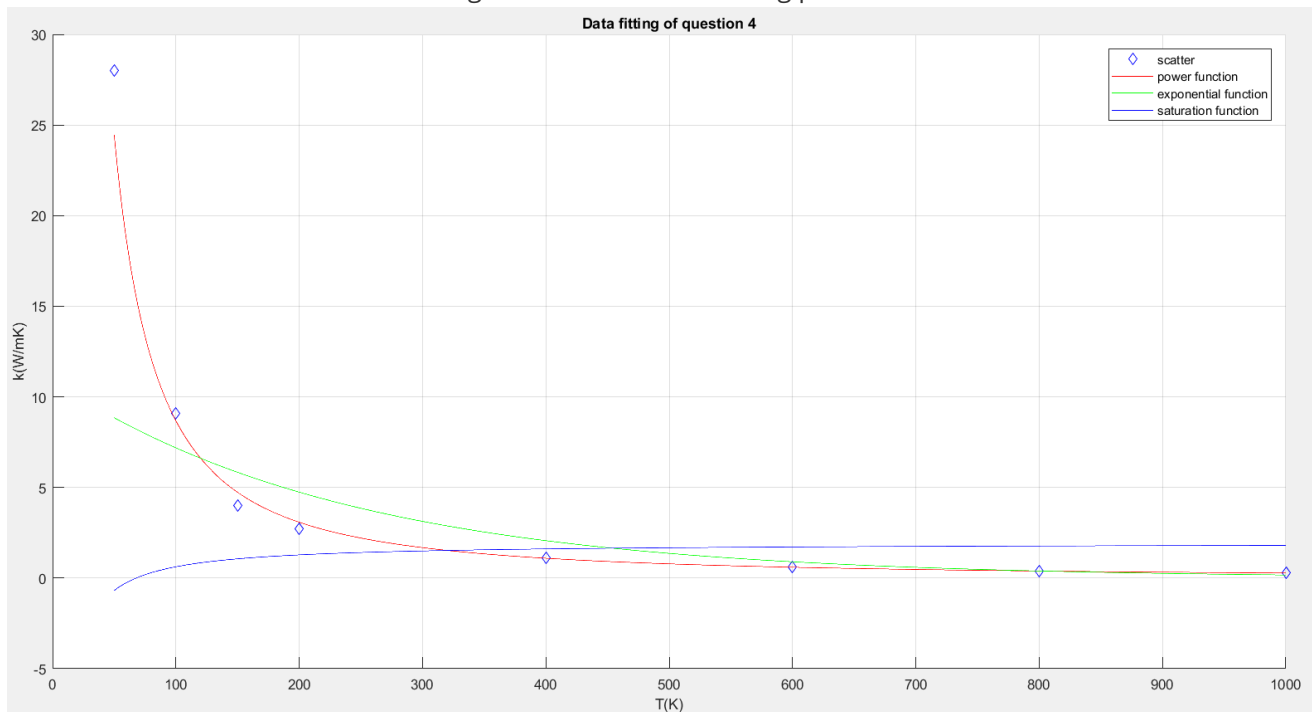
n=8;
T=[50,100,150,200,400,600,800,1000];
k=[28,9.1,4.0,2.7,1.1,0.6,0.4,0.3];
%% Use the power function to fit dataset:
y1=log(k);x1=log(T);Sx1=0;Sy1=0;Sxy1=0;Sxx1=0;
for i=1:8
    Sx1=Sx1+x1(i);Sy1=Sy1+y1(i);
    Sxy1=Sxy1+x1(i)*y1(i);Sxx1=Sxx1+x1(i)*x1(i);
end
% To determine the coefficients:
a1=(n*Sxy1-Sx1*Sy1)/(n*Sxx1-Sx1*Sx1);
a0=(Sxx1*Sy1-Sx1*Sxy1)/(n*Sxx1-Sx1*Sx1);
% Start to plot:
figure(1);
scatter(T,k,'db');hold on;
b=a1;a=exp(a0);
x=linspace(50,1000,100000);y=a.*(x.^b);
plot(x,y,'r-');% the red solidline to represent.
hold on;
%% Use the exponential function to fit dataset:
y2=log(k);x2=T;Sx2=0;Sy2=0;Sxy2=0;Sxx2=0;
for i=1:8
    Sx2=Sx2+x2(i);Sy2=Sy2+y2(i);
    Sxy2=Sxy2+x2(i)*y2(i);Sxx2=Sxx2+x2(i)*x2(i);
end
a1=(n*Sxy2-Sx2*Sy2)/(n*Sxx2-Sx2*Sx2);
a0=(Sxx2*Sy2-Sx2*Sxy2)/(n*Sxx2-Sx2*Sx2);
b=a1;a=exp(a0);
x=linspace(50,1000,100000);y=a.*(exp(b.*x));
plot(x,y,'g-');% the green solidline to represent.
hold on;
%% Use the saturation function to fit dataset:
y3=1./k;x3=1./T;Sx3=0;Sy3=0;Sxy3=0;Sxx3=0;
for i=1:8
    Sx3=Sx3+x3(i);Sy3=Sy3+y3(i);
    Sxy3=Sxy3+x3(i)*y3(i);Sxx3=Sxx3+x3(i)*x3(i);
end
% To determine the coefficients:
a1=(n*Sxy3-Sx3*Sy3)/(n*Sxx3-Sx3*Sx3);
a0=(Sxx3*Sy3-Sx3*Sxy3)/(n*Sxx3-Sx3*Sx3);
b=a1;a=a0;
x=linspace(50,1000,100000);y=b.*(1./x)+a;
plot(x,y,'b-');% the blue solidline to represent.
legend('scatter','power function','exponential function','saturation function');
grid on;
xlabel('T(K)');
ylabel('k(W/mK)');
title('Data fitting of question 4');

```

Result:

Result:

The difference between different fitting is shown as the following picture:



(i) For power function, we get $a_0 = 9.0409$, $a_1 = -1.4940$; We check it by using matlab built-in cftool, and we get the same result. R-square=0.9959; (ii) For exponential function, we get $a_0 = 2.3879$, $a_1 = -0.0042$; We check it by using matlab built-in cftool, and we get the same result. R-square=0.8472; (iii) For exponential function, we get $a_0 = 1.9353$, $a_1 = -131.1678$; We check it by using matlab built-in cftool, and we get the same result. R-square=0.4715; So we can draw a conclusion that power function has a best fit, and the fitting function relation is $k = 8441.6x^{-1.4940}$;

5

Based on our experience, the sales volume y increases with x_1 's increase and decrease with x_2 's increase. We need to find a two-variable function. First, we set up the relation of $y = a_0 + a_1 x_1 + a_2 x_2$.

Now, we use the Least-Square Method to determine parameters a_0, a_1, a_2 ; $r_i = y_i - a_0 - a_1 x_{i1} - a_2 x_{i2}$, $S = \frac{1}{N} \sum_{i=1}^N r_i^2$; Now $S = S(a_0, a_1, a_2)$, the equations need to be solved are: $\frac{\partial S}{\partial a_i} = 0$, where $i=0,1,2$;

Calculate it we get:

$$\begin{cases} S_y - a_0 S - a_1 S_{x_1} - a_2 S_{x_2} = 0 \\ S_{x_1 y} - a_0 S_{x_1} - a_1 S_{x_1 x_1} - a_2 S_{x_1 x_2} = 0 \\ S_{x_2 y} - a_0 S_{x_2} - a_1 S_{x_1 x_2} - a_2 S_{x_2 x_2} = 0 \end{cases}$$

So we define $A = \begin{Bmatrix} S & S_{x_1} & S_{x_2} \\ S_{x_1} & S_{x_1 x_1} & S_{x_1 x_2} \\ S_{x_2} & S_{x_1 x_2} & S_{x_2 x_2} \end{Bmatrix}$ $b = \begin{Bmatrix} S_y \\ S_{x_1 y} \end{Bmatrix}$ then $a = A \backslash b$;

【Code Q5.m】

```
% this script is used to solve question 5.
```

```
clear all;clc;
%% show the dataset and produce the summary of x1,x2,x1*x1,x1*x2,x2*x2,y,x1*y,x2*y;
x1=[110,130,180,125,150,165,120,145,175,155];
x2=[105,125,85,145,200,160,240,260,290,270];
y=[102,99,115,75,55,95,36,78,82,90];
s1=0;s2=0;s11=0;s12=0;s22=0;sy=0;sy1=0;sy2=0;s=10;
for i=1:10
    s1=s1+x1(i);s2=s2+x2(i);
    s11=s11+x1(i)*x1(i);s12=s12+x1(i)*x2(i);s22=s22+x2(i)*x2(i);
    sy=sy+y(i);sy1=sy1+x1(i)*y(i);sy2=sy2+x2(i)*y(i);
end
%% to solve the linear equations:
A=[s s1 s2;s1 s11 s12;s2 s12 s22];
b=[sy;sy1;sy2];
a=A\b;%a=[a0,a1,a2]
%% calculate the model f and compare with y
f=a(1)+a(2).*x1+a(3).*x2;
rr=(f-y)./y;
rr=rr';
```

【Result】

【Result】

Fitting function: $y = 54.4432 + 0.4564x_1 - 0.2029x_2$. Using the matlab built-in function "regress",

```
X=[ones(10,1) x1' x2'];  
regress(y', X);
```

```
>> regress(y', X)
```

ans =

We can get the same result.

Now we calculate the relative error of y_i and f_i .

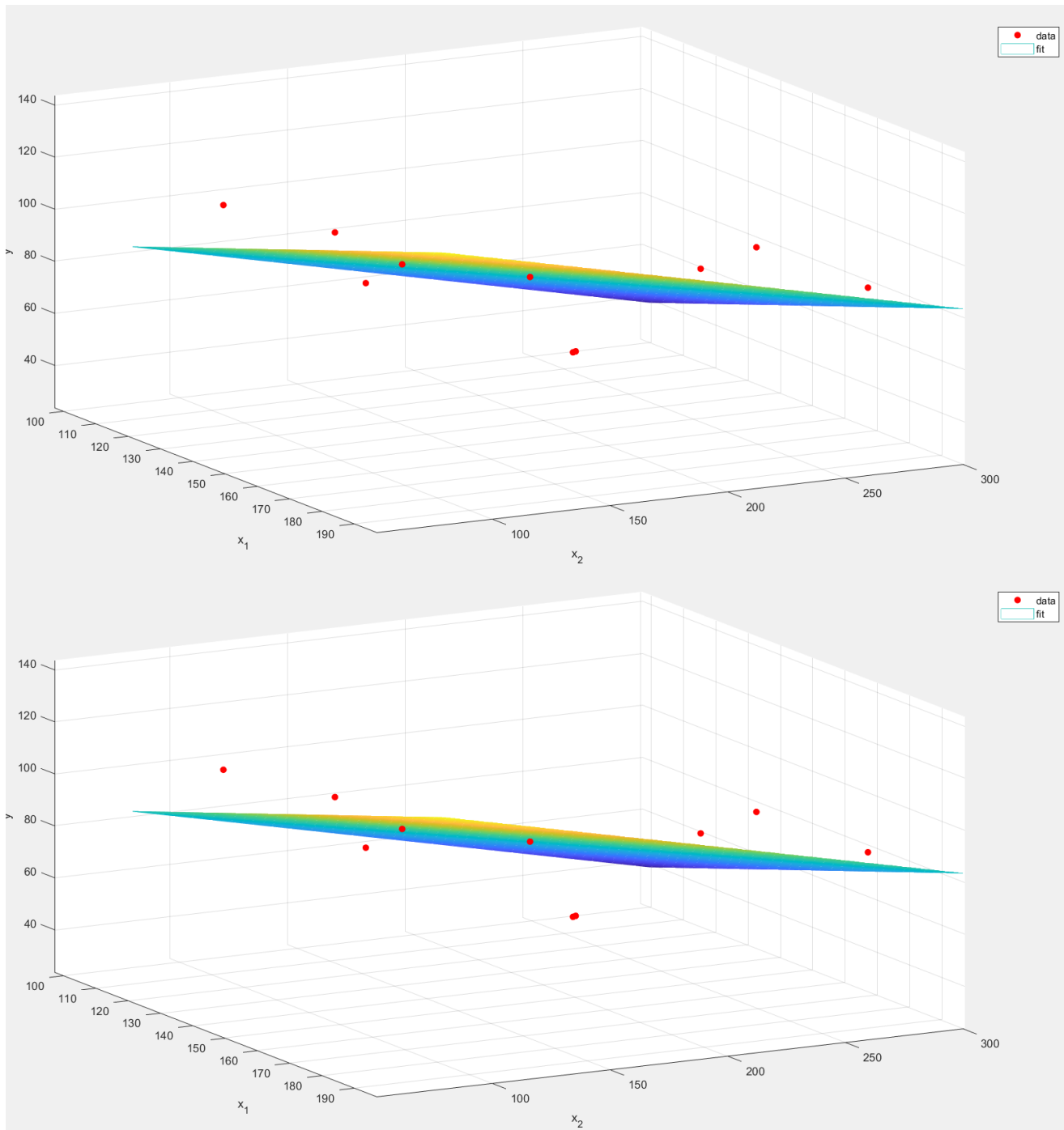
$$\begin{array}{r} 54.4432 \\ 0.4564 \\ -0.2029 \\ \hline \end{array}$$

$$\begin{array}{c|c|c} n & \text{test}\{y_i\} & \text{test}\{f(x_{1i}, x_{2i})\} & \text{relative error} \\ \hline 1 & 102 & 83.3393 & 0.18 \\ 2 & 99 & 88.4087 & 0.11 \\ 3 & 115 & 119.3425 & 0.04 \\ 4 & 75 & 82.0590 & 0.09 \\ 5 & 55 & 82.3189 & 0.50 \\ 6 & 95 & 97.2801 & 0.02 \\ 7 & 36 & 60.5123 & 0.68 \\ 8 & 78 & 67.8635 & -0.13 \\ 9 & 82 & 75.4676 & -0.08 \\ 10 & 90 & 70.3982 & -0.22 \end{array}$$

From the table, we can see that the linear relation $y = 54.4432 + 0.4564x_1 - 0.2029x_2$ fits not so well, because the relative error of $n=5$ and $n=7$ are 0.50 and 0.68, which is so large. But we are pleased to see that other data points fits well, and $a_1 > 0, a_2 < 0$ are consistent with our experience. And the data and fitting plot is shown as the following picture:

code :

```
scatter3(x1,x2,y,'filled','r');  
hold on;  
x1fit =100:2:200;  
x2fit =80:2:300;  
[X1FIT,X2FIT] = meshgrid(x1fit,x2fit);  
YFIT=a(1)+a(2)*X1FIT+a(3)*X2FIT;  
mesh(X1FIT,X2FIT,YFIT)  
view(50,40)  
xlabel('x_1');  
ylabel('x_2');  
zlabel('y');  
legend('data','fit');
```



We can see from the picture that Data is distributed on both sides of the plane. Actually, we can fit it better by adding quadratic polynomials. $y = a_0 + a_1x_1 + a_2x_2 + a_3x_1x_2 + a_4x_1^2 + a_5x_2^2$ Now we just use matlab built-in regress:

```
X = [ones(10,1) x1' x2' (x1.*x2)' (x1.^2)' (x2.^2)'];
[b, bint, r, rint, stats] = regress(y', X);
```

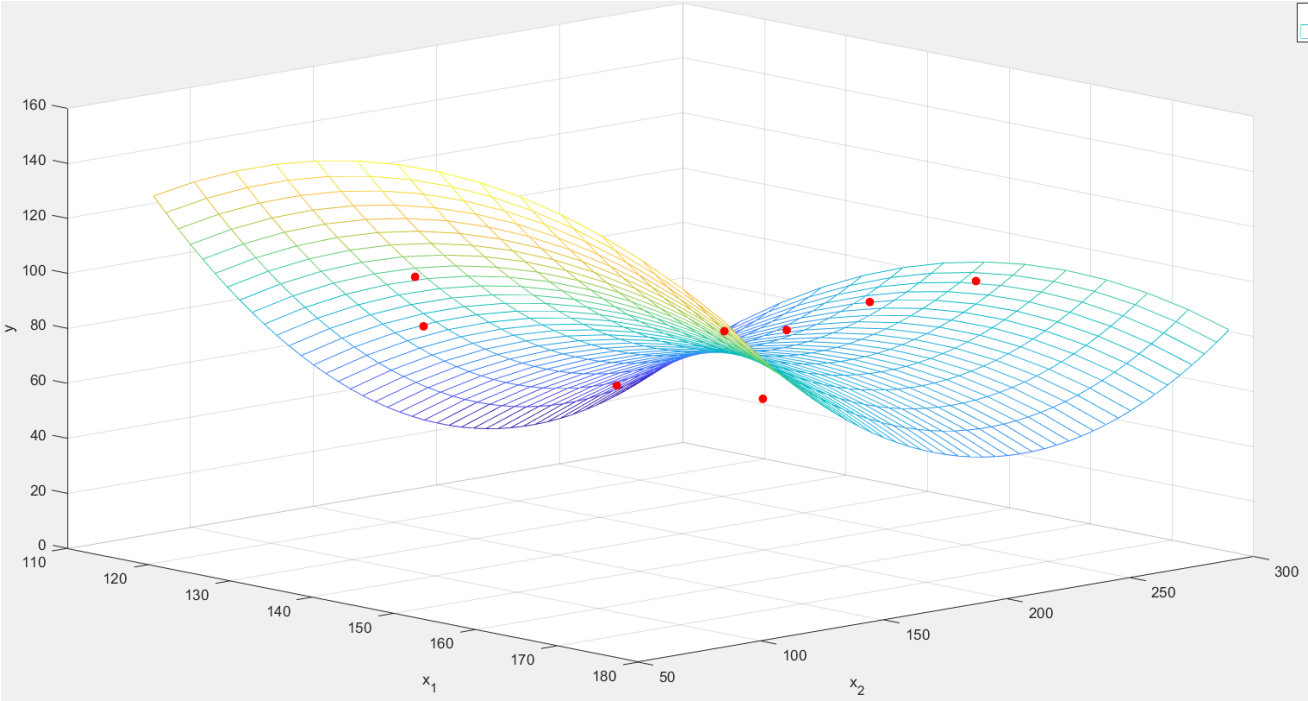
we get the coefficients:

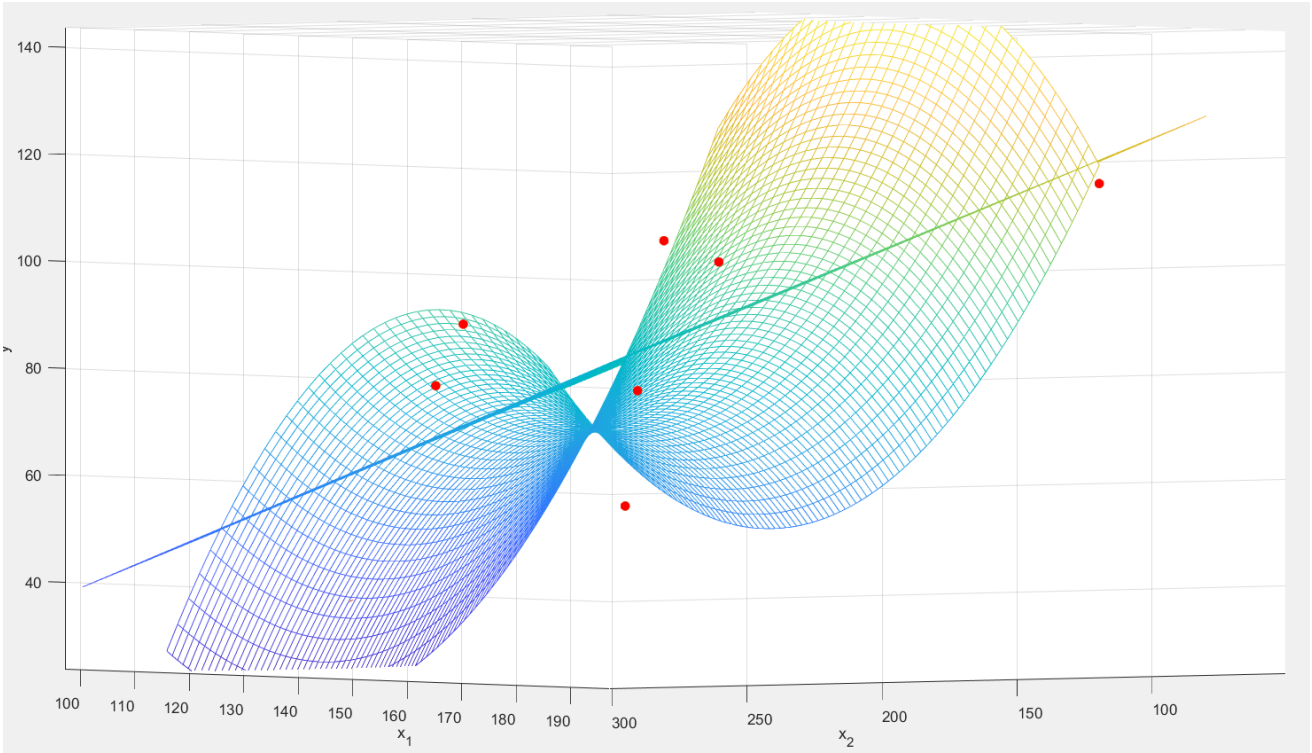
b	
6x1 double	
	1
1	-191.3181
2	7.0712
3	-2.6281
4	0.0042
5	-0.0258
6	0.0045

and its R-square is 0.857822, which is larger than the before

one. the new fitting function:

$y = -191.3181 + 7.0712x_1 - 2.6281x_2 + 0.0042x_1x_2 - 0.0258x_1^2 + 0.0045x_2^2$ the new fitting plot and the Comparison plot are as followings:





we can see that , after we adding quadratic polynomials, we get a better fit, which is good enough.